Multivariable Control Systems

Ali Karimpour
Associate Professor
Ferdowsi University of Mashhad

Lecture 8

References are appeared in the last slide.

Multivariable Control System Design

Topics to be covered include:

- **Control structure design**
- **Sequential loop closing**

The process of control system design

1- Study the plant under control and obtain control objectives.

2- Model the system and simplify it.

3- Analyse the model and derive its properties.

4- Select of controlled output to achieve the specific objectives.

5- Select of manipulation and measurements.

6- Select the control configuration

7- Select the controller type.

(Steps 4–7: **Control structure design**)

8- Design performance specification.

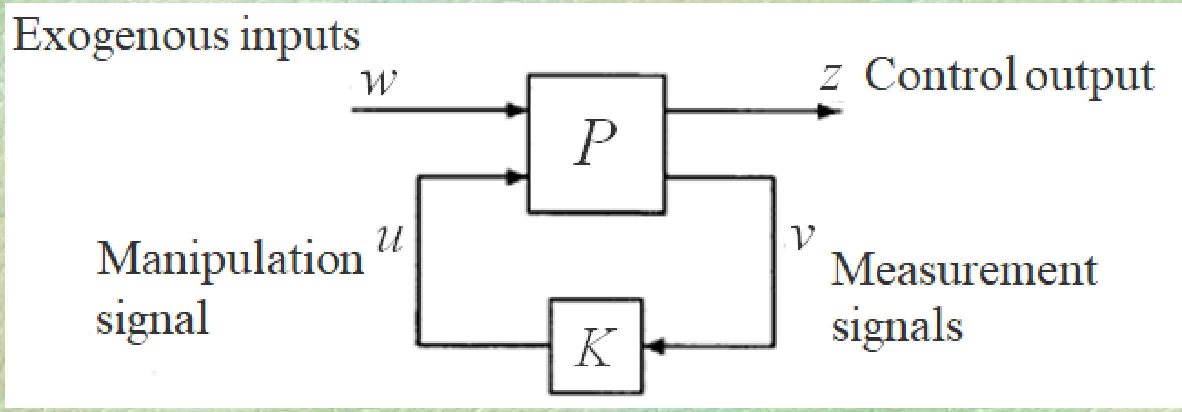
9- Design a controller.

10- Analyse the controller.

11- Simulate the resulting system either on a computer or a pilot plant.

Selection of control output (step 4)

A controlled output is an output variable (usually measured) with an associated control objective (usually a reference value).



In many cases, it is clear from physical understanding of the process for example, heating a furnace.

As another example that it is not clear, consider backing a cake.

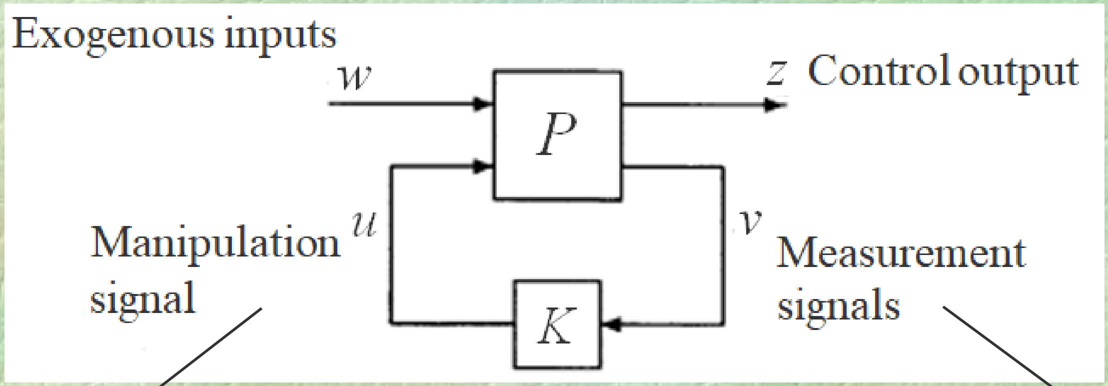
Selection of manipulation and measurements(Step 5)

In some cases there are a large number of candidate measurements and/or manipulations. The need for control has three origin

1- To stabilize an unstable plant,

2- To reject disturbance,

3- To track reference changes,



Large effect on the controlled outputs, and should be close (in terms of dynamic response) to the output and measurements

Strong relationship with the control output, or which may quickly detect a major disturbance

Selection of manipulation and measurements(Step 5)

For more general analysis let:

$$y_{all} = G_{all} u_{all} + G_{dall} d$$

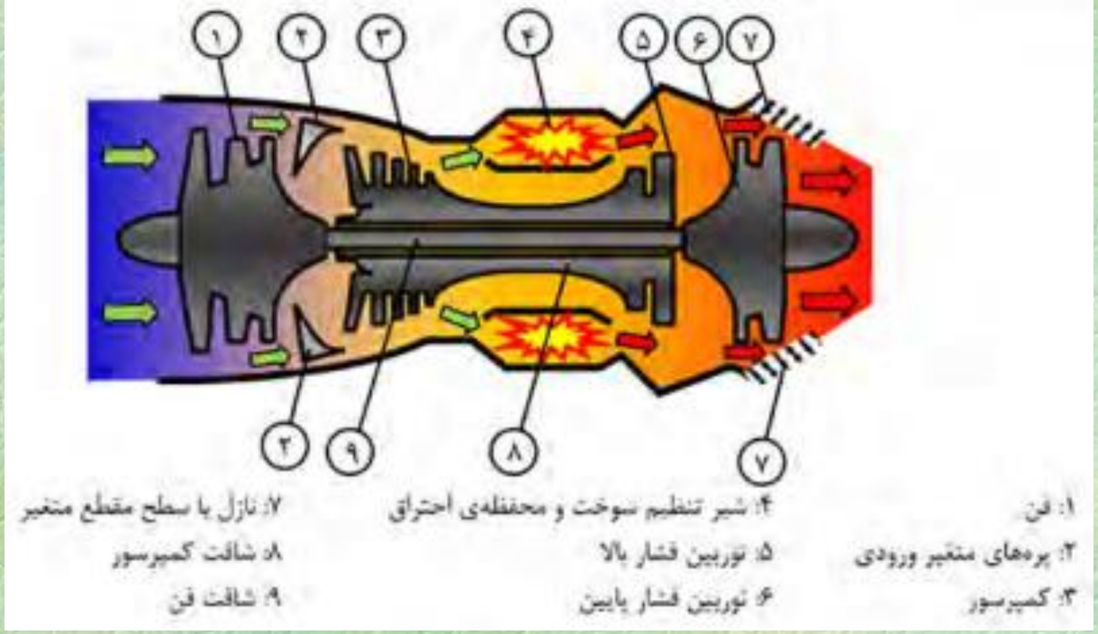
y_{all} is all candidate output and, u_{all} is all candidate input.

Selection of IO set

- Attention to RHP zeros.
- High input-output controllability.
  - Maximize $\underline{\sigma}(G(jw))$
  - Minimize RGA number $RGA_{number} = \|\Lambda - I\|_{sum}$
- High Hankel singular values.

Selection of manipulation and measurements(Step 5)

Example 1: A jet turbine



1- Selection of control output

2- Selection of candidate input (manipulation)

3- Selection of candidate output (measurements)

Trust,

Surge, and

Compressor speed

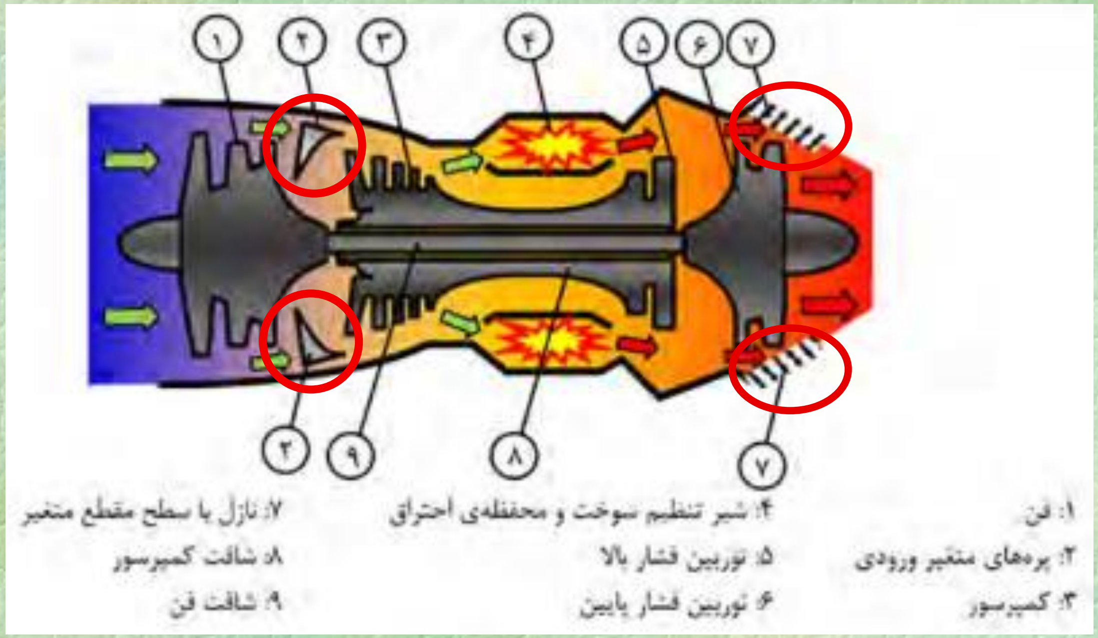
Selection of manipulation and measurements(Step 5)

Example 1 : A jet turbine

1- Selection of control output

2- Selection of candidate input (manipulation)

3- Selection of candidate output (measurements)



I-Fuel flow valve

$$u_{all} = [WFE \quad AJ \quad IGV]^T$$

II- Nozzle with variable area

III- Inlet guide vanes

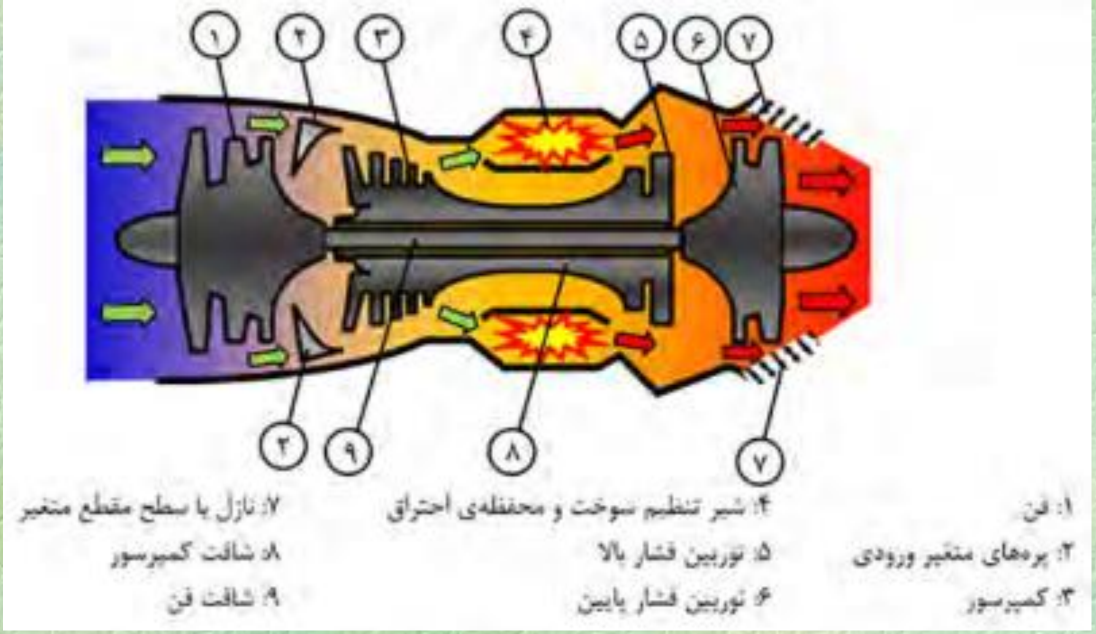
Selection of manipulation and measurements(Step 5)

Example 1 : A jet turbine

1- Selection of control output

2- Selection of candidate input (manipulation)

3- Selection of candidate output (measurements)



$$y_{all} = [\underbrace{NL \quad OPR1 \quad OPR2}_{\text{Trust}} \quad \underbrace{LPPR \quad LPEMN}_{\text{Surge}} \quad \underbrace{NH}_{\text{Compressor speed}}]^T$$

NL = compressor spool speed

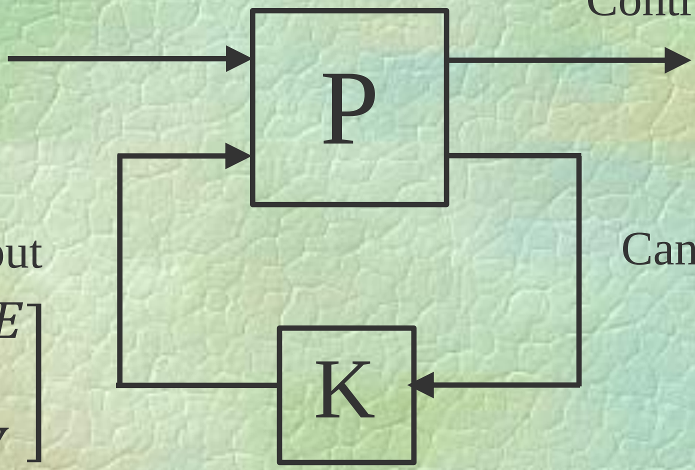
$OPR1$ = the ratio of HP compressor's outlet pressure to engine

$OPR2$ = engine overall pressure ratio

$LPPR$ = LP compressor's pressure ratio

$LPEMN$ = LP compressor's exit Mach number

Selection of manipulation and measurements(Step 5)



Control output $z = \begin{bmatrix} Trust \\ Surge \\ Compressor\ speed \end{bmatrix}$

Candidate input

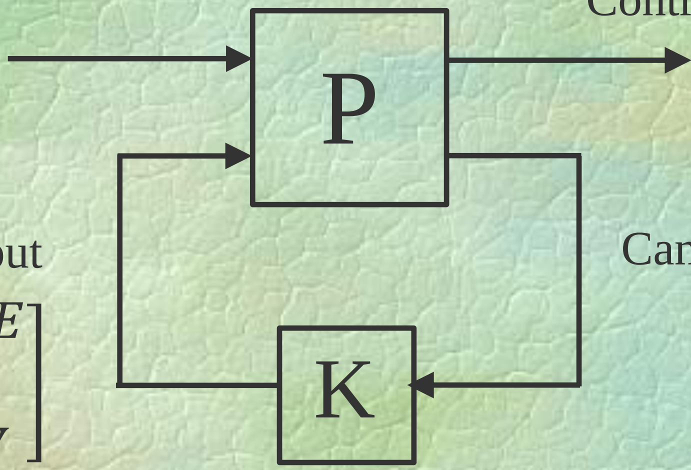
$$u_{all} = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}$$

Candidate output

$$y_{all} = \begin{bmatrix} NL \\ OPR1 \\ OPR2 \\ LPPR \\ LPEMN \\ NH \end{bmatrix}$$

Candidate input selection $u_{all} = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}$

Selection of manipulation and measurements(Step 5)



Control output $z = \begin{bmatrix} Trust \\ Surge \\ Compressor\ speed \end{bmatrix}$

Candidate input

$$u_{all} = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}$$

Candidate output

$$y_{all} = \begin{bmatrix} NL \\ OPR1 \\ OPR2 \\ LPPR \\ LPEMN \\ NH \end{bmatrix}$$

Candidate output selection

$$y_1 = \begin{bmatrix} NL \\ LPPR \\ NH \end{bmatrix} \quad y_2 = \begin{bmatrix} OPR1 \\ LPPR \\ NH \end{bmatrix} \quad y_3 = \begin{bmatrix} OPR2 \\ LPPR \\ NH \end{bmatrix}$$

$$y_4 = \begin{bmatrix} NL \\ LPEMN \\ NH \end{bmatrix} \quad y_5 = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix} \quad y_6 = \begin{bmatrix} OPR2 \\ LPEMN \\ NH \end{bmatrix}$$

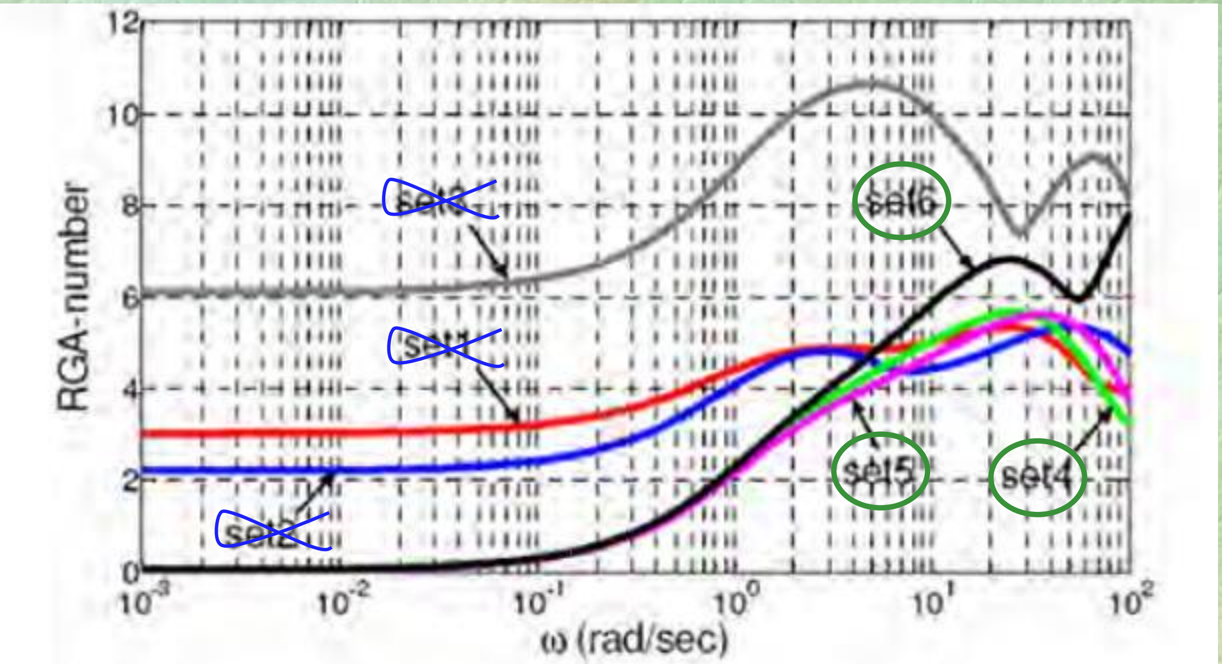
Selection of manipulation and measurements(Step 5)

Set no.	Candidate controlled output	RHP zeros <100 rad/sec	$\underline{\sigma}(G(0))$
1	NL, LPPR, NH	none	0.060
2	OPR1, LPPR, NH	none	0.049
3	OPR2, LPPR, NH	30.9	0.056
4	NL, LPEMN, NH	none	0.366
5	OPR1, LPEMN, NH	none	0.409
6	OPR2, LPEMN, NH	27.7	0.392

Selection of IO set

- Attention to RHP zeros.
- High input-output controllability.
  - Maximize $\underline{\sigma}(G(jw))$
  - Minimize RGA number
- High Hankel singular values.

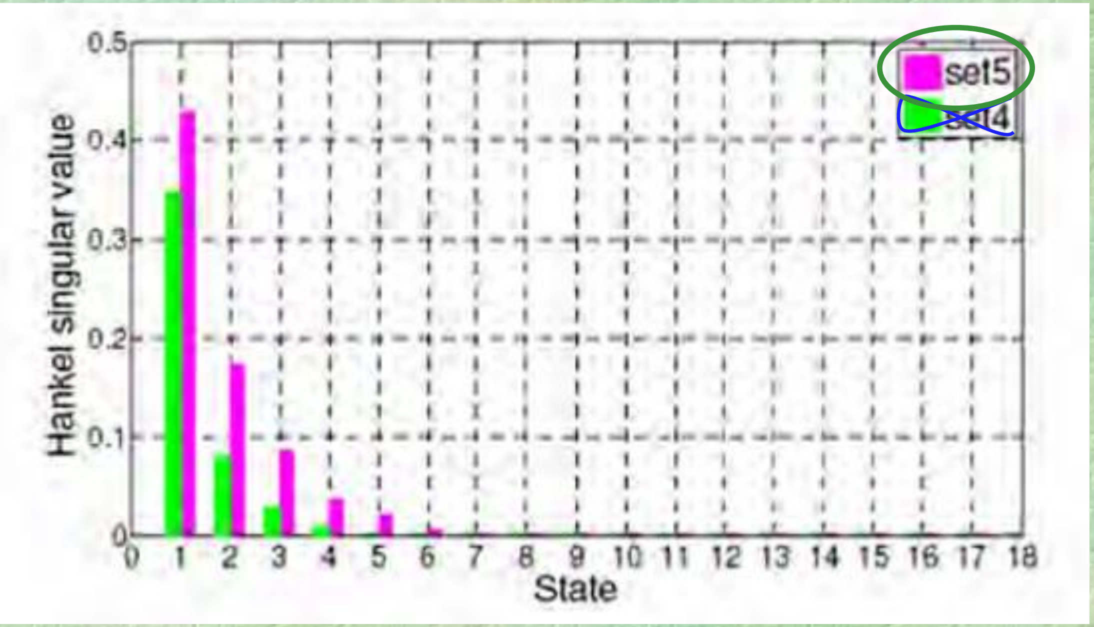
Selection of manipulation and measurements(Step 5)



Selection of IO set

- Attention to RHP zeros.
- High input-output controllability.
  - Maximize $\underline{\sigma}(G(jw))$
  - Minimize RGA number
- High Hankel singular values.

Selection of manipulation and measurements(Step 5)

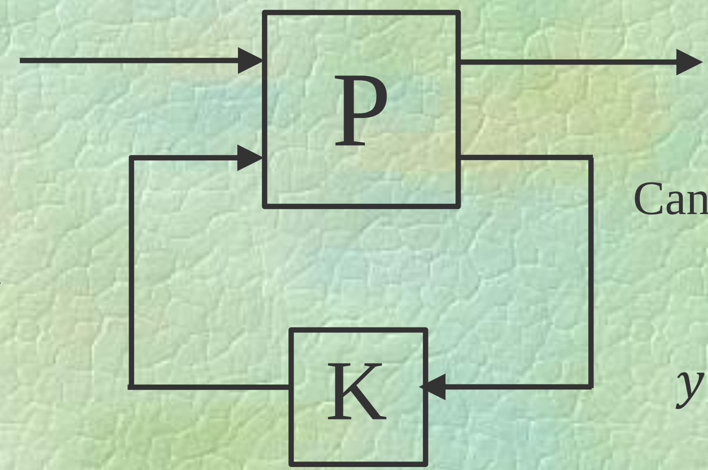


$$y_5 = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix}$$

Selection of IO set:

- Attention to RHP zeros.
- High input-output controllability.
  - Maximize $\underline{\sigma}(G(jw))$
  - Minimize RGA number
- High Hankel singular values.

Select the control configuration (Step 6)



Candidate input

$$u = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}$$

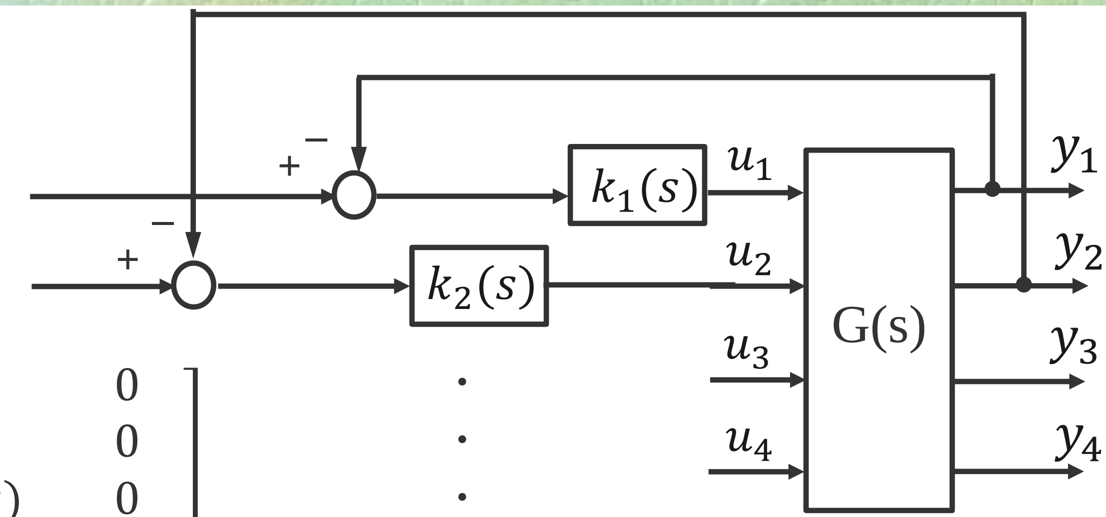
Candidate output

$$y = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix}$$

Structure of K

- Centralized control. $K = \begin{bmatrix} .. & .. & .. \\ .. & .. & .. \\ .. & .. & .. \end{bmatrix}$ ✗
- Decentralized control. $K = \begin{bmatrix} .. & 0 & 0 \\ 0 & .. & 0 \\ 0 & 0 & .. \end{bmatrix}$ ✓

Select the control configuration (Step 6)



$$K(s) = \begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & k_3(s) & 0 \\ 0 & 0 & 0 & k_4(s) \end{bmatrix}$$

The design of decentralized control systems involves some steps:

1- The choice of pairings (control configuration selection)

loop-assignment problem or input-output pairing

(RGA and NI index)

2- How many control loops is necessary?

Select the control configuration (Step 6)

Definition of RGA (Relative Gain Array)

Physical Meaning of RGA: Let

$$y_1 = g_{11}u_1 + g_{12}u_2$$
$$y_2 = g_{21}u_1 + g_{22}u_2$$

g_{ij} relation between y_i and u_j if other inputs $= 0$ or $\left(\frac{\partial y_i}{\partial u_j}\right)_{u_k=0,k\neq j}$

h_{ij} relation between y_i and u_j if other outputs $= 0$ or $\left(\frac{\partial y_i}{\partial u_j}\right)_{y_k=0,k\neq i}$

$$y_1 = g_{11}u_1 + g_{12}u_2$$
$$0 = g_{21}u_1 + g_{22}u_2$$

$$u_2 = -\frac{g_{21}}{g_{22}}u_1$$

$$y_1 = \left(g_{11} + g_{12}\left(-\frac{g_{21}}{g_{22}}\right)\right)u_1 = h_{11}u_1$$

Relative gain? $\lambda_{ij} = g_{ij} / h_{ij}$

$$\Lambda(G) = RGA(G) = G \times G^{-T}$$

Select the control configuration (Step 6)

Let

$$y_1 = g_{11}u_1 + g_{12}u_2$$
$$y_2 = g_{21}u_1 + g_{22}u_2$$

$$\Lambda(G) = G \times G^{-T} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}$$

λ=1 → Open loop and closed loop gains are the same, so interactions has no effect.

λ=0 → g_{11}=0 so u_1 has no effect on y_1.

0<λ → Closing second loop, no sign change the gain between y_1 and u_1.

λ<0 → Closing second loop leads to changing the sign of the gain between y_1 and u_1.(Very Bad)

Select the control configuration (Step 6)

RGA property:

1- It is independent of input and output scaling.

2- Its rows and columns sum to 1.

3- The RGA is identity matrix if G is upper or lower triangular.

4- Plant with large RGA elements are ill conditioned.

5- Suppose G(s) has no zeros or poles at s=0. If $\lambda_{ij}(\infty)$ and $\lambda(0)$ exist and have different signs then one of the following must be true.

* G(s) has an RHP zeros. * $G^{ij}(s)$ has an RHP zeros.

* $g_{ij}(s)$ has an RHP zeros.

6- If $g_{ij} \rightarrow g_{ij}(1-1/\lambda_{ij})$ then the perturbed system is singular.

7- Changing two columns/rows of G leads to same changes to its RGA

Select the control configuration (Step 6)

Definition of Niederlinski index.

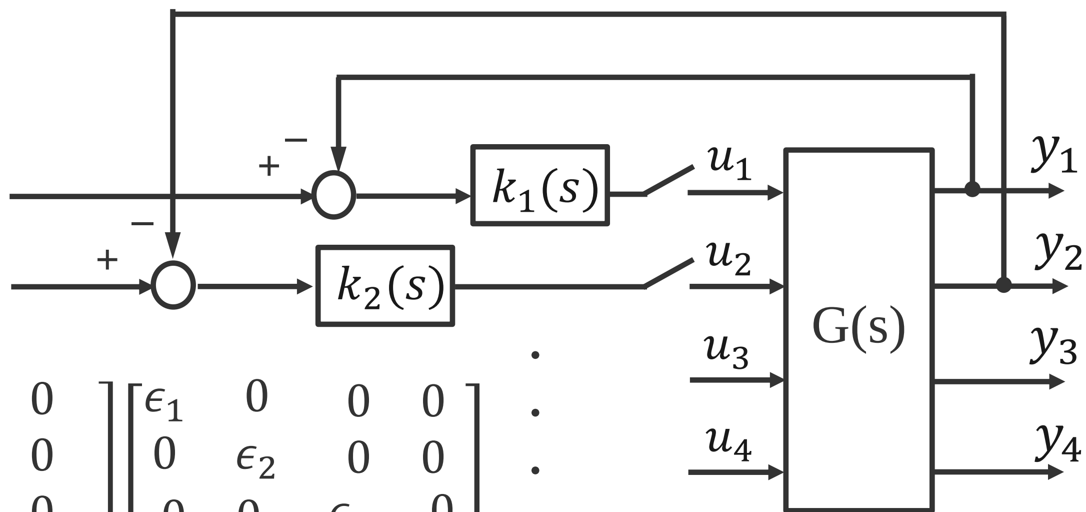
$$NI\big(G(0)\big) = \frac{\det(G(0))}{\prod_i g_{ii}}$$

Select the control configuration (Step 6)

1- The choice of pairings (control configuration selection) loop-assignment problem or input-output pairing

a) Integrity

A decentralized control system has integrity if the closed loop system should remain stable as subsystem controllers are brought in and out of service or when input saturates



$$K(s) = \begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & k_3(s) & 0 \\ 0 & 0 & 0 & k_4(s) \end{bmatrix} \begin{bmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & 0 \\ 0 & 0 & \epsilon_3 & 0 \\ 0 & 0 & 0 & \epsilon_4 \end{bmatrix}$$

$\epsilon_1 \in \{0,1\}$

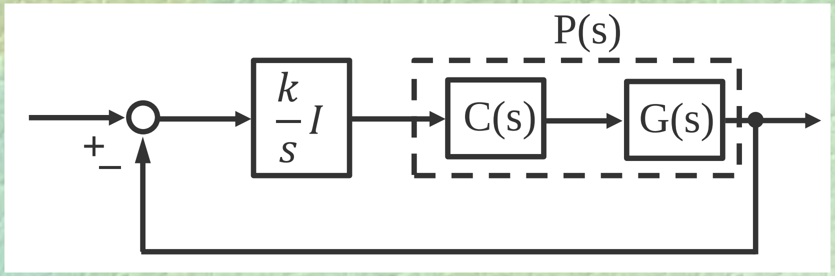
A necessary condition for integrity is that the Niederlinski index of G(0) and the Niederlinski indices of all the principal submatrices $G^{ii}(0)$ of G(0) are positive.

Select the control configuration (Step 6)

1- The choice of pairings (control configuration selection)

loop-assignment problem or input-output pairing

b) Integral stabilizable



System P(s)=G(s)C(s) is integral stabilizable if there exist a k>0 such that closed loop system is stable and its steady-state error to all constant inputs are zero.

Theorem: The necessary condition that a proper real rational stable matrix P(s) is integral stabilizable is: det[P(0)]>0

Exercise: For both following system det[P(0)]=3, show that just one of them is integral stabilizable.

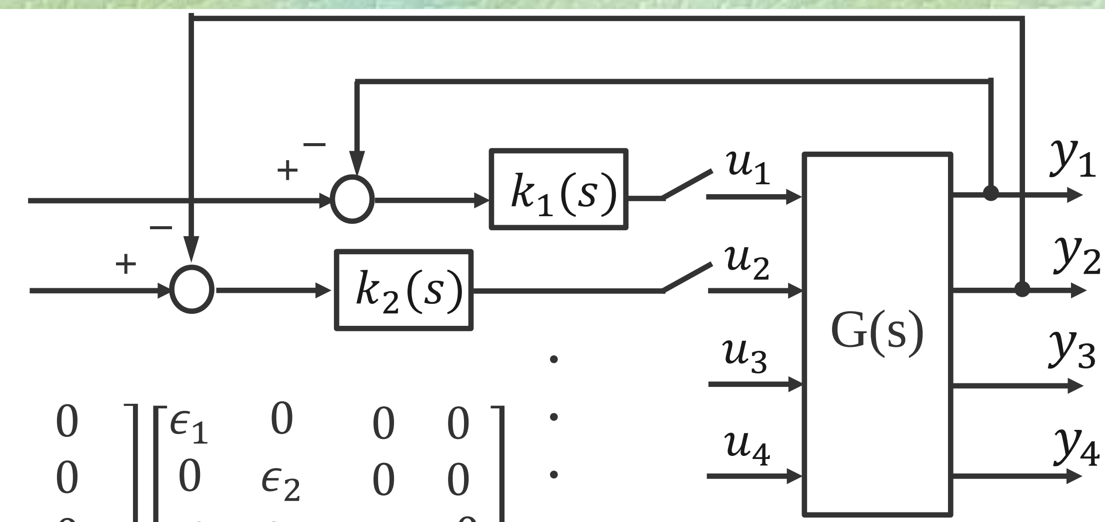
$$a)\ P(s) = \frac{1}{s+1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad b)\ P(s) = \frac{1}{s+1}\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

Select the control configuration (Step 6)

1- The choice of pairings (control configuration selection)
loop-assignment problem or input-output pairing

c) Decentralized integral controllability(DIC)

The plant G(s) (corresponding to a given pairing with the paired elements along its diagonal) is DIC if there exist a stabilizing controller with integral action in each loop such that each individual loop may be detuned by a factor $\epsilon_1 \in [0,1]$ without introducing instability.



$$K(s) = \begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & k_3(s) & 0 \\ 0 & 0 & 0 & k_4(s) \end{bmatrix} \begin{bmatrix} \epsilon_1 & 0 & 0 & 0 \\ 0 & \epsilon_2 & 0 & 0 \\ 0 & 0 & \epsilon_3 & 0 \\ 0 & 0 & 0 & \epsilon_4 \end{bmatrix}$$

$\epsilon_1 \in \{0,1\}$ *integrity*

$0 \leq \epsilon_1 \leq 1$ *DIC*

Select the control configuration (Step 6)

1- The choice of pairings (control configuration selection)

loop-assignment problem or input-output pairing

c) Decentralized integral controllability(DIC)

The plant $G(s)$ (corresponding to a given pairing with the paired elements along its diagonal) is DIC if there exist a stabilizing controller with integral action in each loop such that each individual loop may be detuned by a factor $\epsilon_1 \in [0,1]$ without introducing instability.

Theorem: Steady-state RGA and DIC

Consider a stable square plant G and a diagonal controller K with integral action in all elements, and assume that the loop transfer function GK is strictly proper. If a pairings of outputs and manipulated inputs corresponds to a negative steady-state relative gain, then the closed-loop system has at least one of the following properties.

a) The overall closed-loop system is unstable.
b) The loop with the negative relative gain is unstable by itself.
c) The closed-loop system is unstable if the loop with the negative relative gain is open.

Select the control configuration (Step 6)

1- The choice of pairings (control configuration selection) loop-assignment problem or input-output pairing

Summary of pairing rules

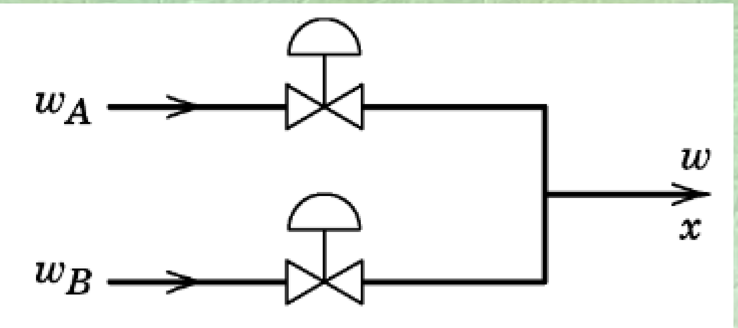
Pairing rule 1: Prefer pairing such that the rearranged system, with the selected pairings along the diagonal, has an RGA matrix close to identity at frequency around the closed-loop bandwidth.

Pairing rule 2: For a stable plant avoid pairings that correspond to negative steady-state RGA elements, $\lambda_{ij} < 0$.

Pairing rule 3: Prefer pairing *ij* where g_{ij} puts minimal restrictions on the achievable bandwidth. Specifically, the effective delay θ_{ij} in $g_{ij}(s)$ should be small.

Select the control configuration (Step 6)

Example 2: Select suitable pairing for the following blending system. (ω is output flow and x is the composition and defined as percent of of ω_A to total flow)



Solution:

$$w = w_A + w_B$$

$$x = \frac{w_A}{w_A + w_B}$$

$$\begin{bmatrix} w \\ x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{1-x_0}{w_0} & \frac{-x_0}{w_0} \end{bmatrix} \begin{bmatrix} w_A \\ w_B \end{bmatrix}$$

RGA of the system is $\Lambda = \begin{bmatrix} x_0 & 1-x_0 \\ 1-x_0 & x_0 \end{bmatrix}$

If $x_0=0.1$ $\Lambda = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}$ $w \leftrightarrow w_B \;\&\; x \leftrightarrow w_A$

If $x_0=0.9$ $\Lambda = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$ $w \leftrightarrow w_A \;\&\; x \leftrightarrow w_B$

Select the control configuration (Step 6)

Example 3: Select suitable pairing for the following plant

$$G(0)=\begin{bmatrix}10.2 & 5.6 & 1.4\\ 15.5 & -8.4 & -0.7\\ 18.1 & 0.4 & 1.8\end{bmatrix}$$

Solution: RGA of the system is

$$\Lambda(0)=\begin{bmatrix}0.96 & \underline{1.45} & -1.41\\ \underline{0.94} & -0.37 & 0.43\\ -0.9 & -0.07 & \underline{1.98}\end{bmatrix}$$

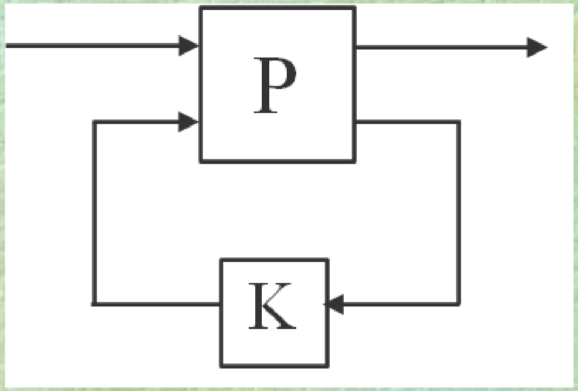
For a 3×3 plant there are 6 alternative pairings, but from the steady-state RGA we see that there is only one positive element in column 2 and only one positive element in row 3 and therefore there is only one possible pairing with all RGA-elements positive ($\{1\leftrightarrow 2,\ 2\leftrightarrow 1,\ 3\leftrightarrow 3\}$).

Select the control configuration (Step 6)

Example 1(Continue) : A jet turbine

Candidate input

$$u = \begin{bmatrix} WFE \\ AJ \\ IGV \end{bmatrix}$$



Candidate output

$$y = \begin{bmatrix} OPR1 \\ LPEMN \\ NH \end{bmatrix}$$

$$y = \begin{bmatrix} 1.076 & -0.027 & 0.004 \\ -0.064 & -0.412 & 0 \\ 1.474 & -0.093 & 0.983 \end{bmatrix} u$$

$$RGA(G(0)) = \begin{bmatrix} 1.002 & 0.004 & -0.006 \\ 0.004 & 0.996 & 0 \\ -0.006 & 0 & 1.006 \end{bmatrix} u$$

So, {WFE, OPR1}, {AJ, LPEMN}, and {IGV, NH} is a suitable pairing.

Select the control configuration (Step 6)

2- How many control loops is necessary?

Combination of SVD, C.N. and RGA can help in this matter.

Select the control configuration (Step 6)

Example 4 Determine the preferred multiloop control strategy for a process with the following steady-state gain matrix, which has been scaled by dividing the process variables by their maximum values.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.48 & 0.9 & -0.006 \\ 0.52 & 0.95 & 0.008 \\ 0.90 & -0.95 & 0.020 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = G(0) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Solution:

$$\Lambda = \begin{bmatrix} 0.71 & -0.16 & 0.45 \\ -0.36 & 0.79 & 0.56 \\ 0.65 & 0.37 & -0.01 \end{bmatrix}$$

$$y_1 \leftrightarrow u_1 \ \ \& \ \ y_2 \leftrightarrow u_3 \ \ \& \ \ y_3 \leftrightarrow u_2$$

$$y_1 \leftrightarrow u_3 \ \ \& \ \ y_2 \leftrightarrow u_2 \ \ \& \ \ y_3 \leftrightarrow u_1$$

$$G(0) = \begin{bmatrix} -0.57 & 0.38 & -0.73 \\ -0.60 & 0.41 & 0.68 \\ 0.56 & 0.83 & -0.01 \end{bmatrix} \begin{bmatrix} 1.62 & 0 & 0 \\ 0 & 1.14 & 0 \\ 0 & 0 & 0.01 \end{bmatrix} \begin{bmatrix} -0.05 & 0.99 & -0.02 \\ -0.99 & -0.05 & 0.01 \\ 0.01 & 0.02 & 0.99 \end{bmatrix}$$

$$C.N. = \frac{\bar{\sigma}}{\underline{\sigma}} = 162$$

Select the control configuration (Step 6)

$y_1 \leftrightarrow u_1$ & $y_2 \leftrightarrow u_3$ & $y_3 \leftrightarrow u_2$ $\qquad$ $y_1 \leftrightarrow u_3$ & $y_2 \leftrightarrow u_2$ & $y_3 \leftrightarrow u_1$

$$G(0)=\begin{bmatrix}-0.57 & 0.38 & -0.73\\ -0.60 & 0.41 & 0.68\\ 0.56 & 0.83 & -0.01\end{bmatrix}\begin{bmatrix}1.62 & 0 & 0\\ 0 & 1.14 & 0\\ 0 & 0 & 0.01\end{bmatrix}\begin{bmatrix}-0.05 & 0.99 & -0.02\\ -0.99 & -0.05 & 0.01\\ 0.01 & 0.02 & 0.99\end{bmatrix} \qquad C.N.=\frac{\bar{\sigma}}{\underline{\sigma}}=162$$

Determine three control loop is not suitable so:

$$y_1, y_2 \quad and \begin{cases} u_1,u_2 & C.N.=184 & \lambda_{11}=-38\\ u_1,u_3 & C.N.=72 & \lambda_{11}=0.55\\ u_2,u_3 & C.N.=133 & \lambda_{11}=0.55\end{cases}$$

$$y_1, y_3 \quad and \begin{cases} u_1,u_2 & C.N.=1.51 & \lambda_{11}=0.36\\ u_1,u_3 & C.N.=69 & \lambda_{11}=0.64\\ u_2,u_3 & C.N.=139 & \lambda_{11}=1.46\end{cases}$$

$y_1 \leftrightarrow u_2$ & $y_3 \leftrightarrow u_1$

$$y_2, y_3 \quad and \begin{cases} u_1,u_2 & C.N.=1.45 & \lambda_{11}=0.37\\ u_1,u_3 & C.N.=338 & \lambda_{11}=3.25\\ u_2,u_3 & C.N.=68 & \lambda_{11}=0.71\end{cases}$$

$y_2 \leftrightarrow u_2$ & $y_3 \leftrightarrow u_1$

Multivariable Control System Design

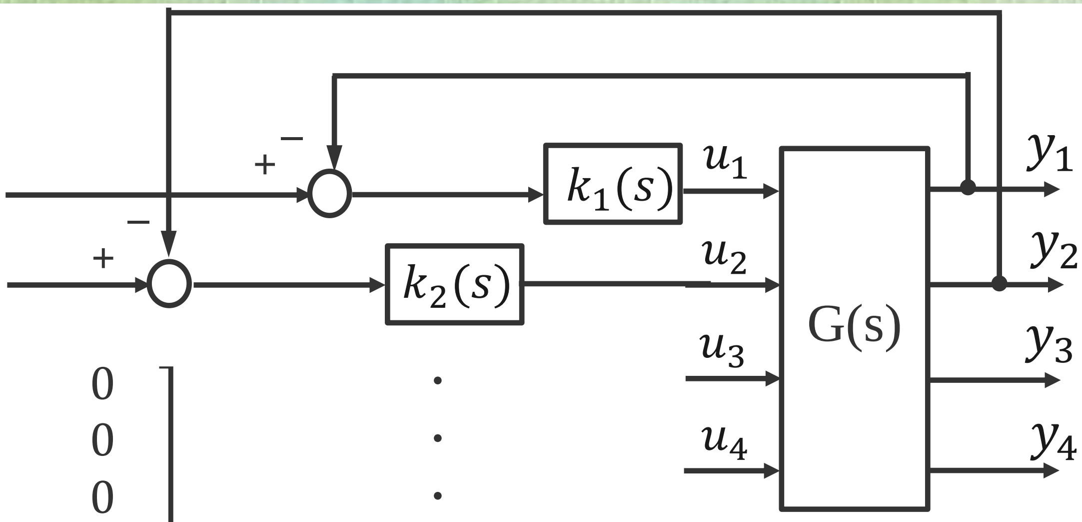
Topics to be covered include:

- **Control structure design**
- **Sequential loop closing**

Sequential loop closing

The simplest approach to multivariable design is to ignore its multivariable nature.

- Choose suitable pairing.
- A SISO controller is designed for one pair of input and output variables.
- When this design has been successfully completed another SISO controller is designed for a second pair of variables and so on.
- In each part we consider the previous loop in system and we check the previous one.



$$K(s)=\begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & k_3(s) & 0 \\ 0 & 0 & 0 & k_4(s) \end{bmatrix}$$

Sequential loop closing

Example 5: Consider following system and derive a sequential loop closing control.

$$G(s) = \frac{1}{s+1}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$$

Solution: First we must choose suitable pairing.

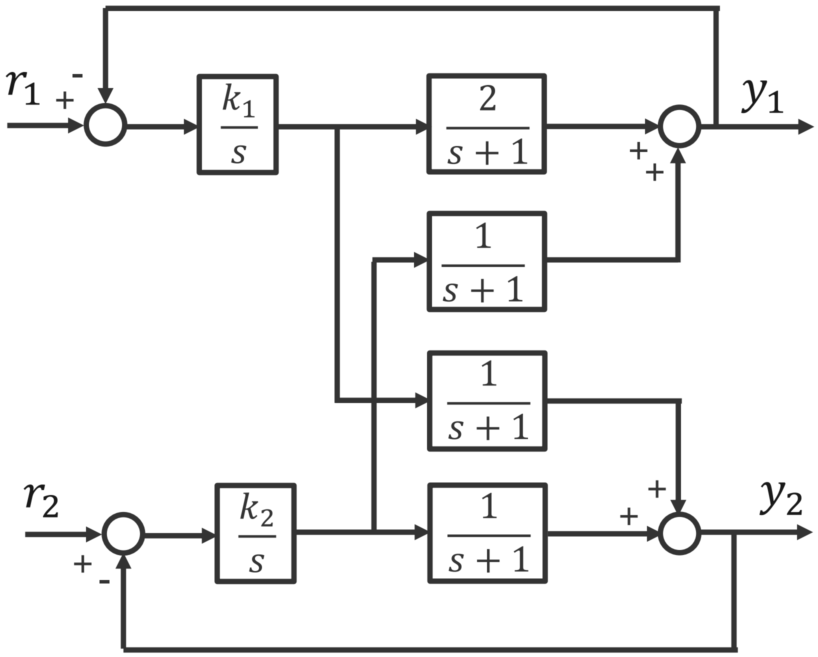
$$RGA(G(0)) = \begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

So, we must rearrange system as $u_1 \leftrightarrow y_2$ and $u_2 \leftrightarrow y_1$

$$G^{new}(s) = \frac{1}{s+1}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Sequential loop closing

$$G^{new}(s) = \frac{1}{s+1}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$



So, we try the first loop with an integrator

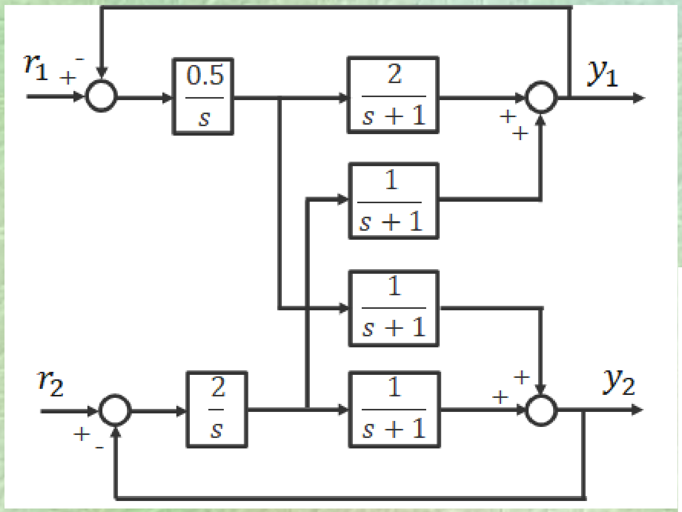
$$\frac{y_1}{r_1} = \frac{2k_1}{s^2 + s + 2k_1} \qquad k_1 = 0.5$$

Now, we try the second loop with first loop in system:

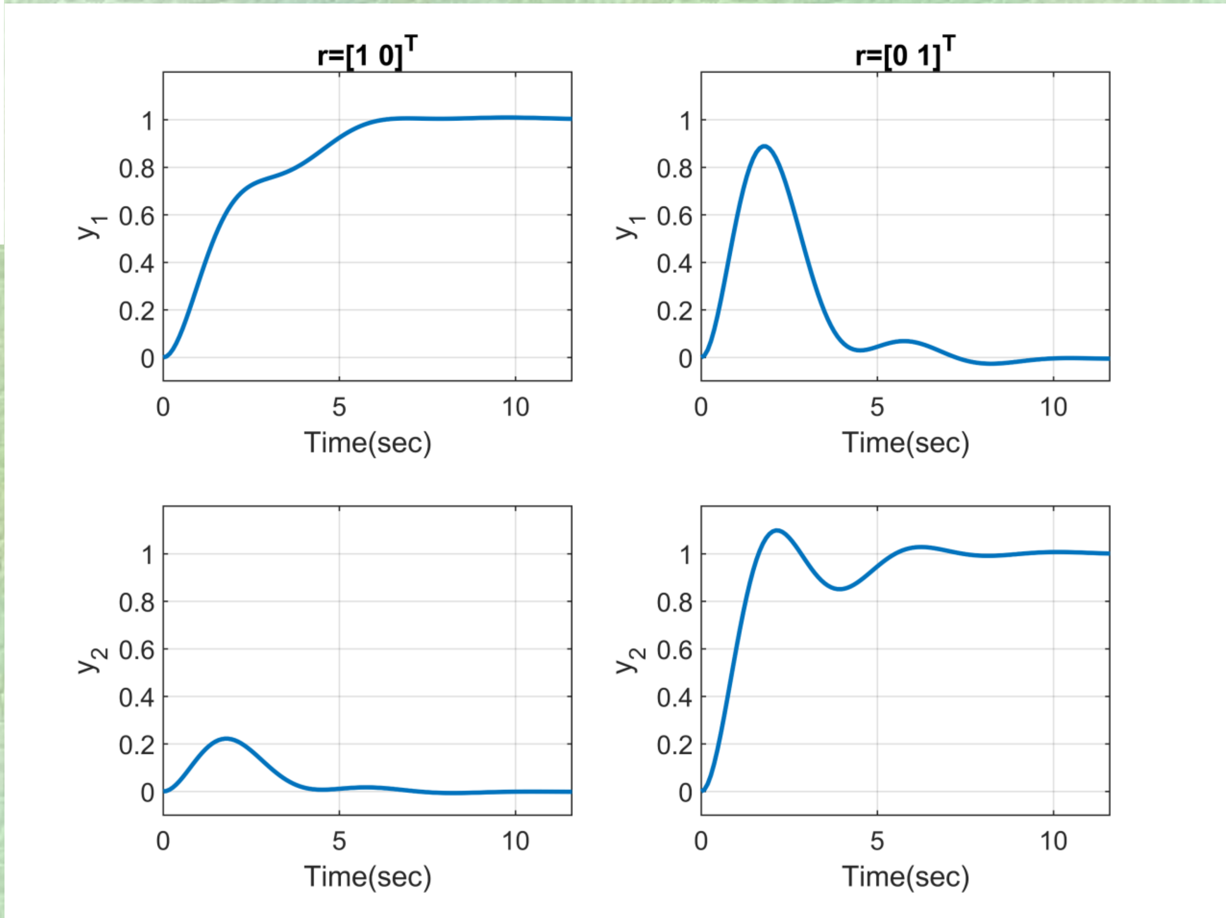
$$\frac{y_2}{r_2} = \frac{k_2(s^2 + s + 1)}{s^4 + 2s^3 + (2 + k_2)s^2 + (1 + k_2)s + 0.5k_2}$$

$$k_2 = 2$$

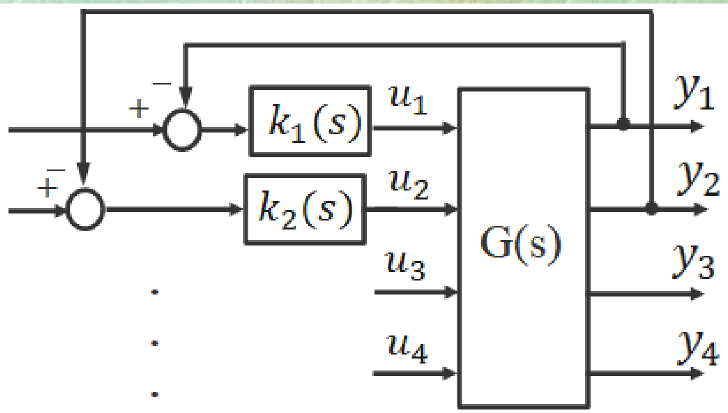
Sequential loop closing



See reference no. 4 for $k_1 = k_2 = 1$



Sequential loop closing



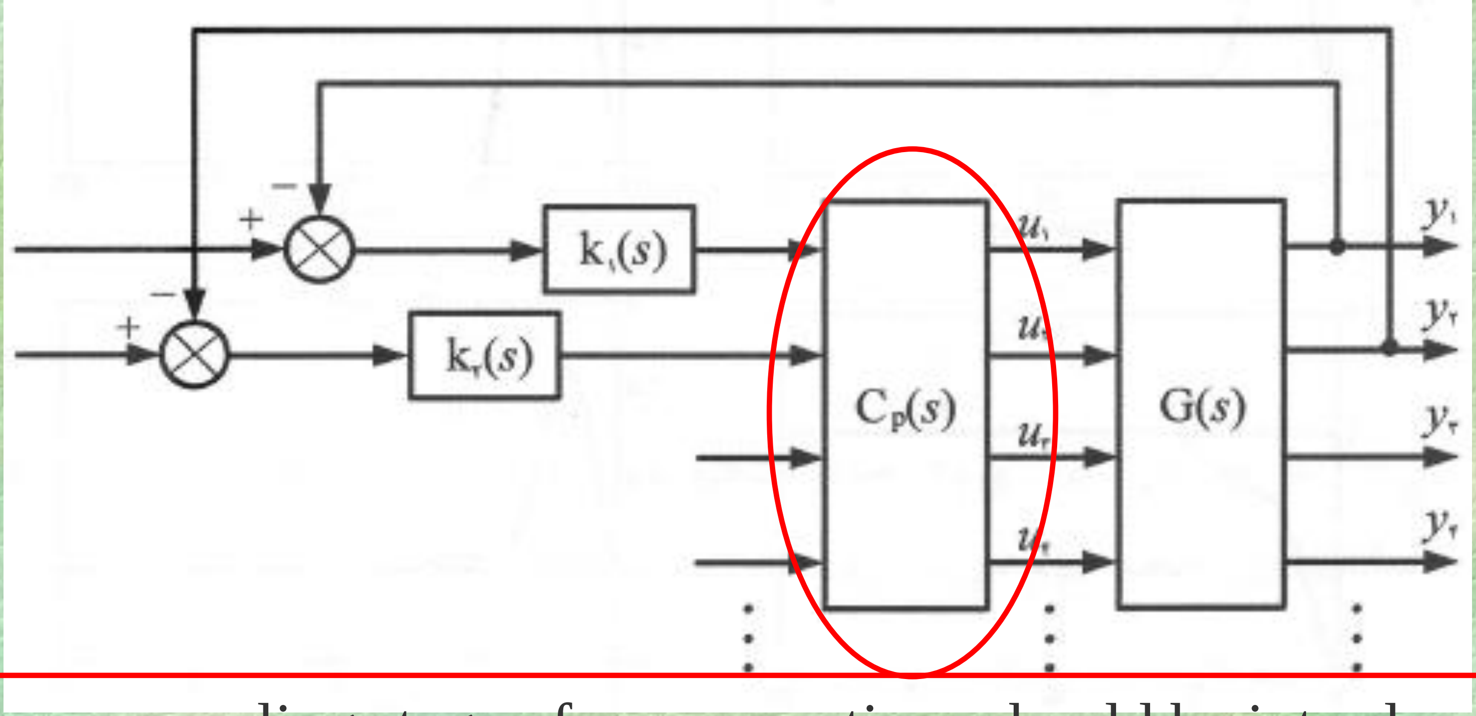
$$K(s) = \begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & k_3(s) & 0 \\ 0 & 0 & 0 & k_4(s) \end{bmatrix}$$

Sequential loop closing specification

- Pairing is an important issues.
- It is working well for low interaction systems.
- It is a decentralized controller.
- Each time one loop is designed.
- It is working well for loop with different bandwidths, start with fast loops.
- Not suitable for control difficulty issue.
  - RHP element zero in minimum phase systems.
  - High interaction system.

Sequential loop closing

A more sophisticated version of sequential loop closing is called **sequential return-difference** method



A cross-coupling stage of compensations should be introduced

This stage should consist of either a constant-gain matrix

or a sequence of elementary operations. $C_p(s) = \frac{1}{d(s)} U(s)$

Sequential loop closing

Example 6:

$$G(s) = \frac{1}{(s+1)^2}\begin{bmatrix} 1-s & \frac{1}{3}-s \\ 2-s & 1-s \end{bmatrix}$$

If we try to design a SISO controller for either the first or second loop here, we have difficulties, if the required bandwidth is close to unity, or greater, because the transfer function 'seen' for the design, namely the (1,1) or (2,2) element of $G(s)$, has a zero at 1.

However, $G(s)$ itself has a transmission zero at - 1 only, so there should be no inherent difficulty of this kind.

If we choose

$$C_p = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

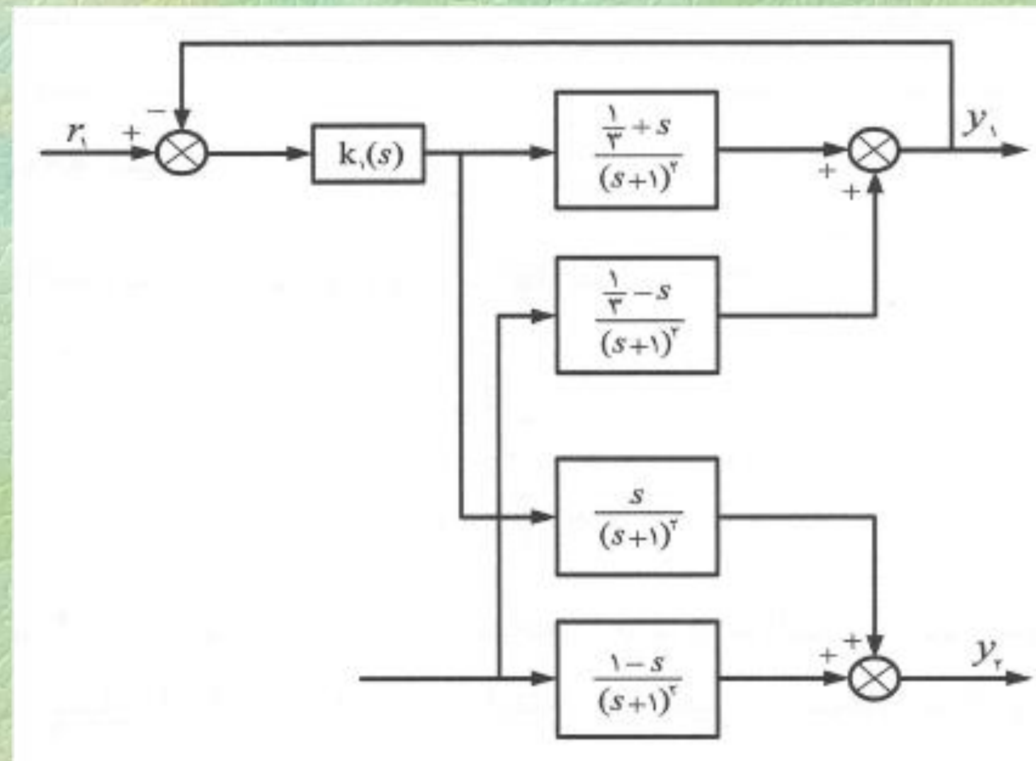
$$Q(s) = G(s)C_p = \frac{1}{(s+1)^2}\begin{bmatrix} \frac{1}{3}+s & \frac{1}{3}-s \\ s & 1-s \end{bmatrix}$$

We see that no right half-plane zero 'appears' when a SISO compensator is being designed for the first loop.

Sequential loop closing

Example 6:

$$Q(s)=G(s)K_a=\frac{1}{(s+1)^2}\begin{bmatrix}\frac{1}{3}+s & \frac{1}{3}-s\\ s & 1-s\end{bmatrix}$$



Once the first loop has been closed by

$$k_1(s)$$

The transfer function 'seen' in the second loop is

$$q_{22}^{1}(s)=\frac{1-s}{(1+s)^2}+\frac{s}{(s+1)^2}h(s)\frac{\frac{1}{3}-s}{(s+1)^2}$$

where $$h(s)=\frac{-k_1(s)}{1+\left[\left(\frac{1}{3}+s\right)/(s+1)^2\right]k_1(s)}$$

Sequential loop closing

Example 6:

$$q_{22}^{1}(s)=\frac{1-s}{(1+s)^2}+\frac{s}{(s+1)^2}h(s)\frac{\frac{1}{3}-s}{(s+1)^2}$$

where $$h(s)=\frac{-k_1(s)}{1+\left[\left(\frac{1}{3}+s\right)/(s+1)^2\right]k_1(s)}$$

Now, if we assume high gain in the first loop

$$|k_1(s)|>>\left|\frac{(s+1)^2}{\frac{1}{3}+s}\right| \quad \Rightarrow \quad h(s)\approx-\frac{(s+1)^2}{\frac{1}{3}+s}$$

and hence

$$q_{22}^{1}(s)\approx\frac{1}{(s+1)(3s+1)}$$

so that no right half-plane zero is seen when the compensator for the second loop is designed.

Sequential loop closing

- The main idea that of using a first stage of compensation to make subsequent loop compensation easier

- The main weakness of the method is that little help is available for choosing that first stage of compensation.

The available analysis relies on the assumption that there are high gains in the loops which have already been closed, and such an assumption can rarely be justified, except at low frequencies.

⇓

One rather special case, in which the assumption of high gains is justified, arises when different bandwidth is required for each loop, and all the bandwidths are well separated from each other.

Sequential loop closing

Mayne (1979) suggests that, if the plant has a state-space realization (*A*, *B*, *C*), the product *CB* being non-singular (and the matrix *D* being zero), then the first stage of compensation can be chosen to be

$$C_p = (CB)^{-1}$$

Since $G(s) \rightarrow \dfrac{CB}{s} \quad as \quad |s| \rightarrow \infty$ so $G(s)C_p \rightarrow \dfrac{I}{s} \quad as \quad |s| \rightarrow \infty$

so that each loop looks like a first-order SISO system at high frequencies.

However, an alternative choice, such as

$$C_p \approx G^{-1}(j\omega_b) \qquad \text{or} \qquad C_p \approx j\omega_b G^{-1}(j\omega_b)$$

DNA is another approach

References

References

- Multivariable Feedback Design, J M Maciejowski, Wesley,1989.
- Multivariable Feedback Control, S.Skogestad, I. Postlethwaite, Wiley,2005.
- Control Configuration Selection in Multivariable Plants, A. Khaki-Sedigh, B. Moaveni, Springer Verlag, 2009.
- تحلیل و طراحی سیستم های چند متغیره، دکتر علی خاکی صدیق

Web References

- http://www.um.ac.ir/~karimpor
- http://saba.kntu.ac.ir/eecd/khakisedigh/Courses/mv/