Multivariable Control Systems

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References

References

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- Multivariable Feedback Control, S. Skogestad, I. Postlethwaite, Wiley,2005.

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Introduction

Topics to be covered include:

- Introduction
- Interaction
- Stability
- Analysis and design in multivariable systems
- Some examples of multivariable systems

Introduction

What is multivariable control?

MIMO systems are considered as multivariable systems.

A General multivariable plant

$$Y(s) = G(s)U(s)$$

$$Y(s) = \begin{bmatrix} y_1(s) \\ y_2(s) \\ \vdots \\ y_l(s) \end{bmatrix}$$

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1m}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2m}(s) \\ \vdots & \vdots & \vdots & \vdots \\ g_{l1}(s) & g_{l2}(s) & \cdots & g_{lm}(s) \end{bmatrix}$$

$$U(s) = \begin{bmatrix} u_1(s) \\ u_2(s) \\ \vdots \\ u_m(s) \end{bmatrix}$$

Why they are different from SISO?

Since of interaction and design procedure.

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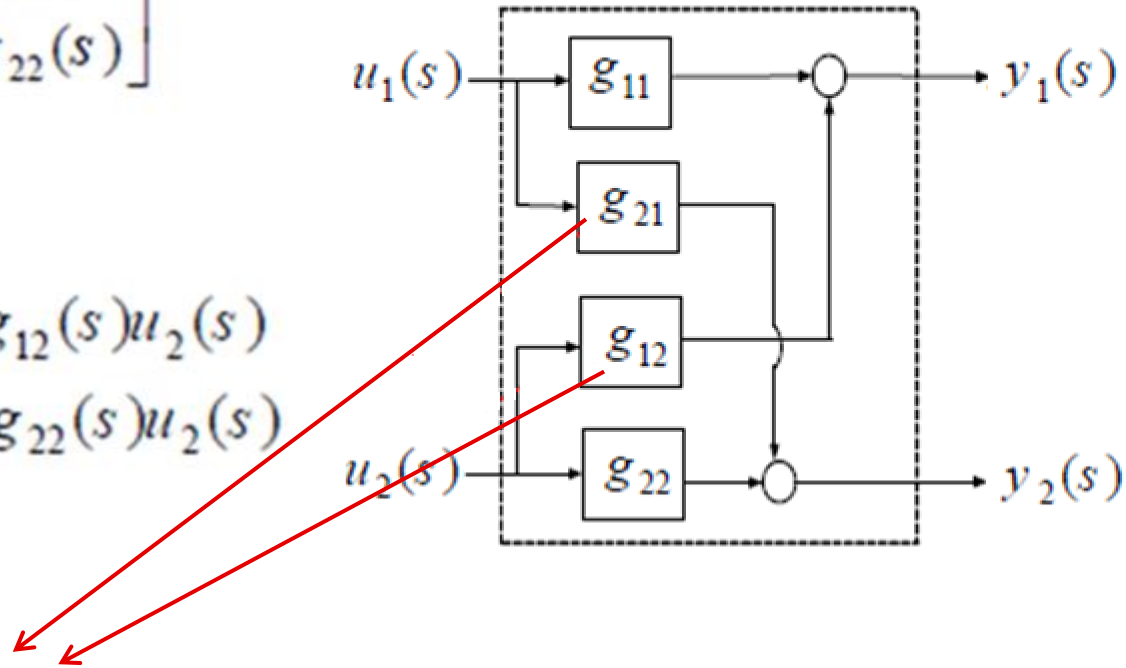
Interaction

$$G(s)=\begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

$$\Downarrow$$

$$y_1(s)=g_{11}(s)u_1(s)+g_{12}(s)u_2(s)$$
$$y_2(s)=g_{21}(s)u_1(s)+g_{22}(s)u_2(s)$$

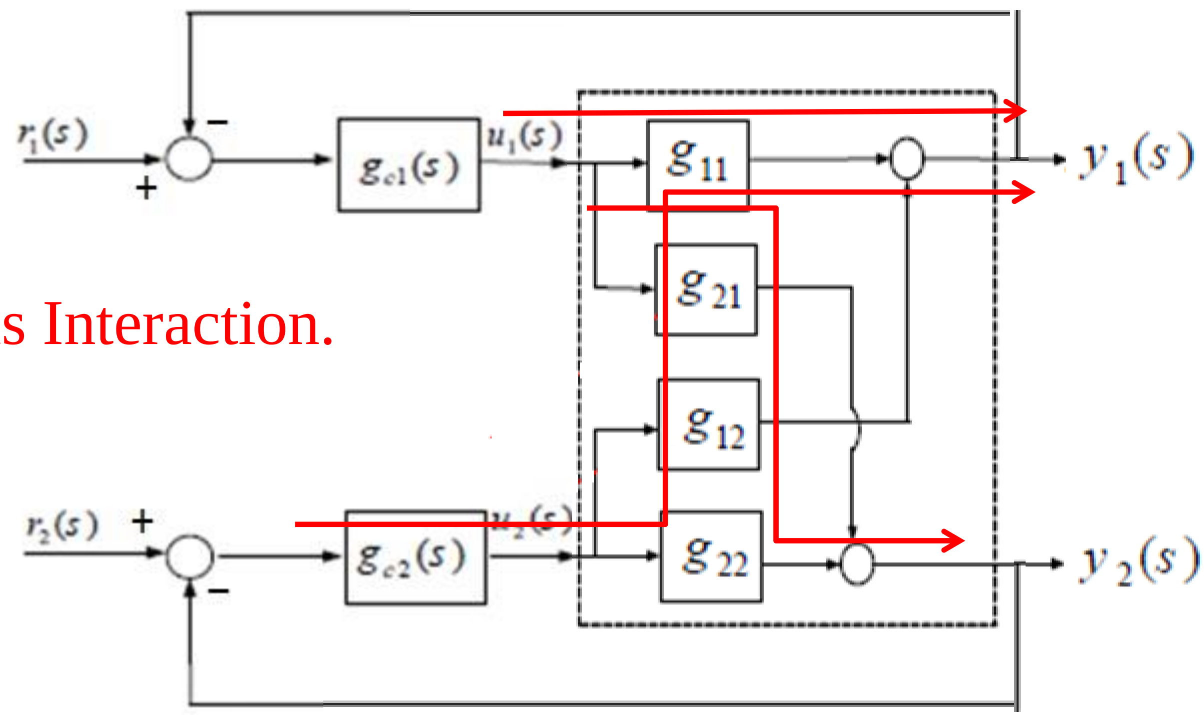


Interaction

Interaction

Interaction on a system

Loop 1



This is Interaction.

Loop 2

Direct effect: The first controller will get the first output ….

Indirect effect: The first controller will disturb the second output and ….

Interaction

One way or single direction interaction (Neutralization process)

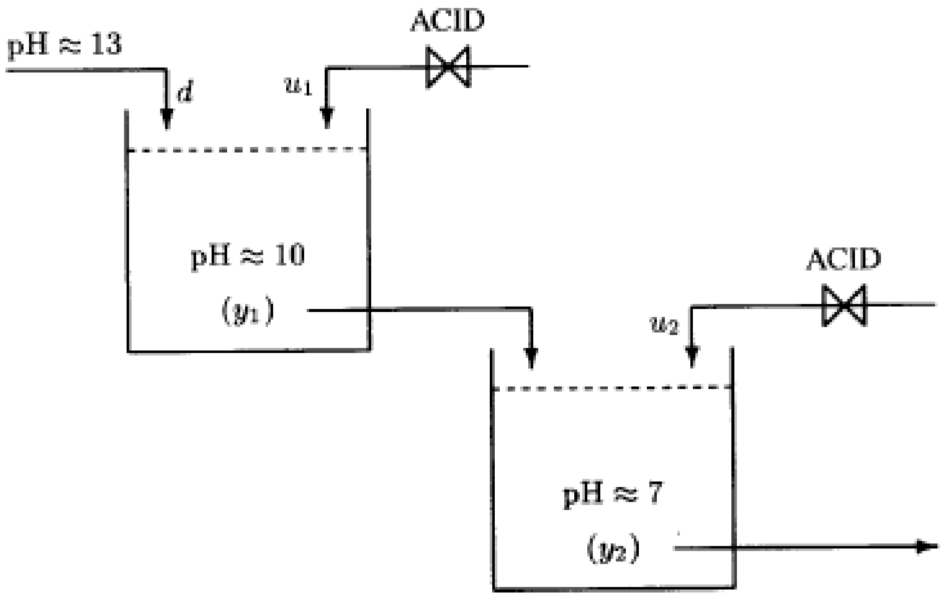
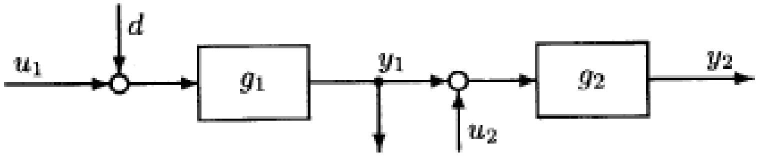


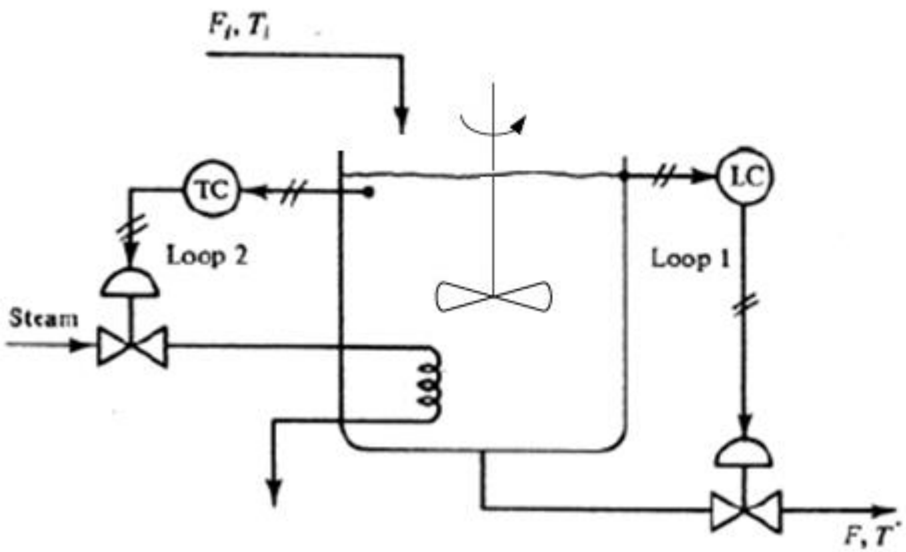
Figure B.1: Neutralization process



$$G(s) = \begin{bmatrix} g_1(s) & 0 \\ g_2(s)g_1(s) & g_2(s) \end{bmatrix}$$

Interaction

One way or single direction interaction (Stirred tank heater)



y_1 is the level of tank.

y_2 is the temperature of tank.

$$G(s) = \begin{bmatrix} g_{11}(s) & 0 \\ g_{21}(s) & g_{22}(s) \end{bmatrix}$$

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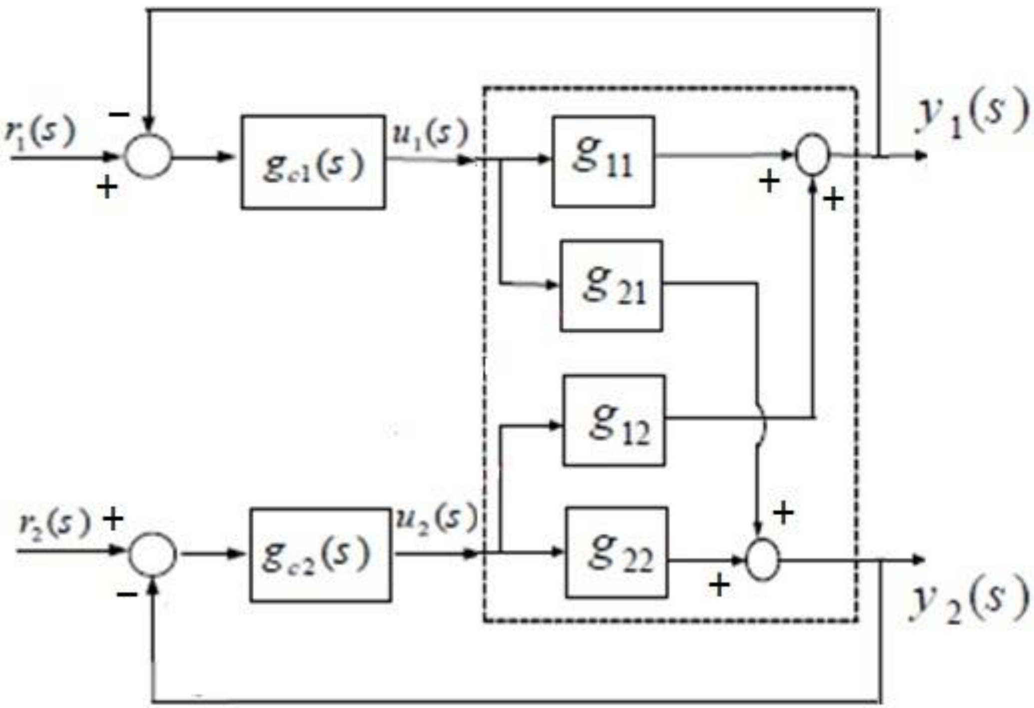
Stability



$$\Delta(s) = \\ = 1 - \left(-g_{11}(s)g_{c1}(s) - g_{22}(s)g_{c2}(s) + g_{12}(s)g_{21}(s)g_{c1}(s)g_{c2}(s)\right) + \left(g_{11}(s)g_{c1}(s)g_{22}(s)g_{c2}(s)\right)$$

$$t_{12}(s) = \frac{g_{12}(s)g_{c2}(s)}{\Delta(s)}$$

$$t_{21}(s) = \frac{g_{21}(s)g_{c1}(s)}{\Delta(s)}$$

$$t_{11}(s) = \frac{g_{11}(s)g_{c1}(s) + g_{c1}(s)g_{c2}(s)\left(g_{11}(s)g_{22}(s) - g_{12}(s)g_{21}(s)\right)}{\Delta(s)}$$

$$t_{22}(s) = \frac{g_{22}(s)g_{c2}(s) + g_{c1}(s)g_{c2}(s)\left(g_{11}(s)g_{22}(s) - g_{12}(s)g_{21}(s)\right)}{\Delta(s)}$$

For stability analysis (BIBO) in the case of no interaction (g12(s)=g21(s)=0) check:

$$1 + g_{11}(s)g_{c1}(s) = 0 \quad \text{and} \quad 1 + g_{22}(s)g_{c2}(s) = 0$$

For stability analysis (BIBO) with interaction check:

$$\Delta(s) = 0$$

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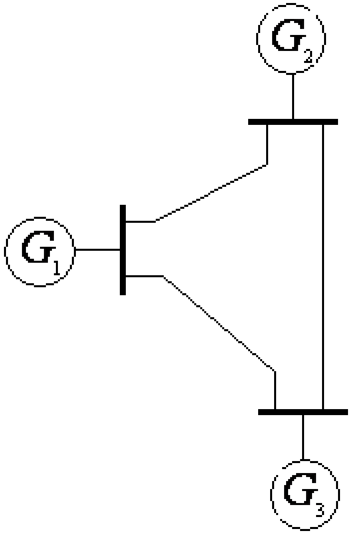
Analysis and design in multivariable systems

Analysis of Multivariable Systems

- Multivariable system representation
- Multivariable poles and zeros
- Controllability and observability
- State space realization
- Multivariable system stability
- Multivariable system robustness analysis
- Control structure design
- Control system design strategy
  - Diagonal or decentralized
  - Block diagonal
  - Fully centralized

Control system design strategy

A multi machine power system



Diagonal or decentralized

$$\begin{bmatrix} Ex_1 \\ Gov_1 \\ Ex_2 \\ Gov_2 \\ Ex_3 \\ Gov_3 \end{bmatrix} = \begin{bmatrix} * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \begin{bmatrix} V_{t1} \\ f_1 \\ V_{t2} \\ f_2 \\ V_{t3} \\ f_3 \end{bmatrix}$$

Block diagonal

$$\begin{bmatrix} Ex_1 \\ Gov_1 \\ Ex_2 \\ Gov_2 \\ Ex_3 \\ Gov_3 \end{bmatrix} = \begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \begin{bmatrix} V_{t1} \\ f_1 \\ V_{t2} \\ f_2 \\ V_{t3} \\ f_3 \end{bmatrix}$$

Fully centralized • • • • • •

Analysis and design in multivariable systems

Multivariable Design Methodologies

- State space methods
- Multivariable root loci approach
- Rosenbrock frequency response approach or Robust approaches
- Pole placement methods
- Eigenstructure assignment
- Multivariable PID controllers
- The classical robust control methods
- QFT
- Soft computing approach

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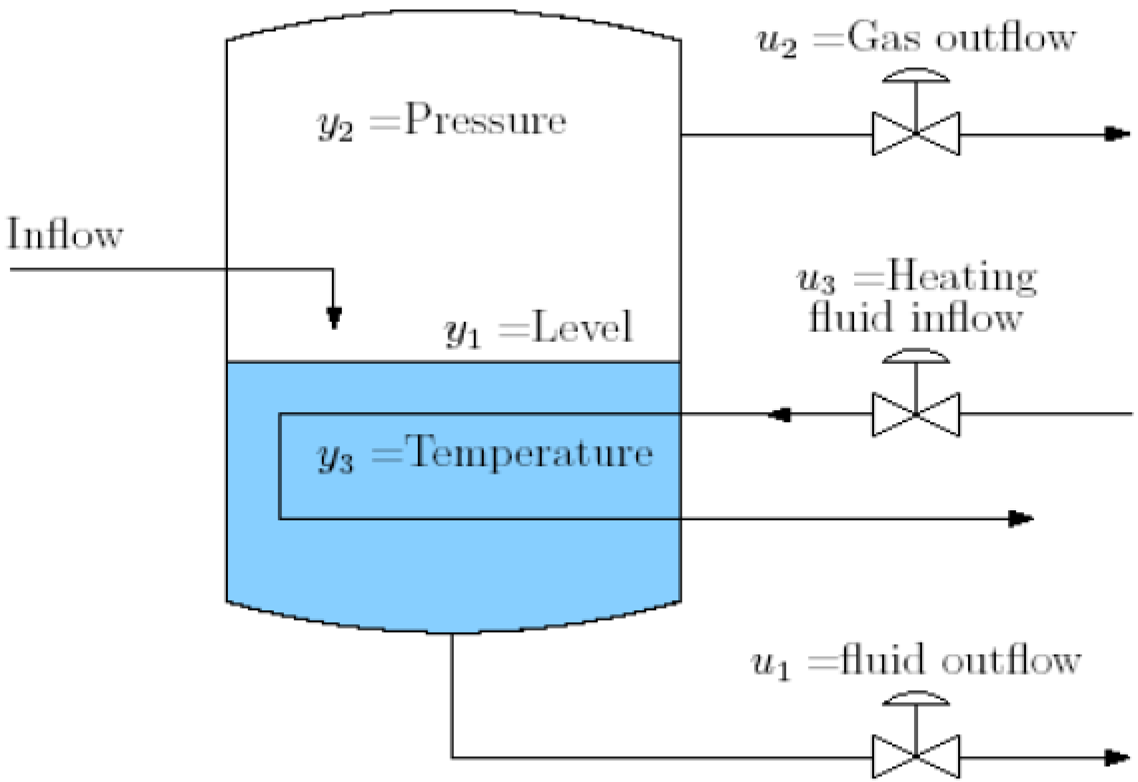
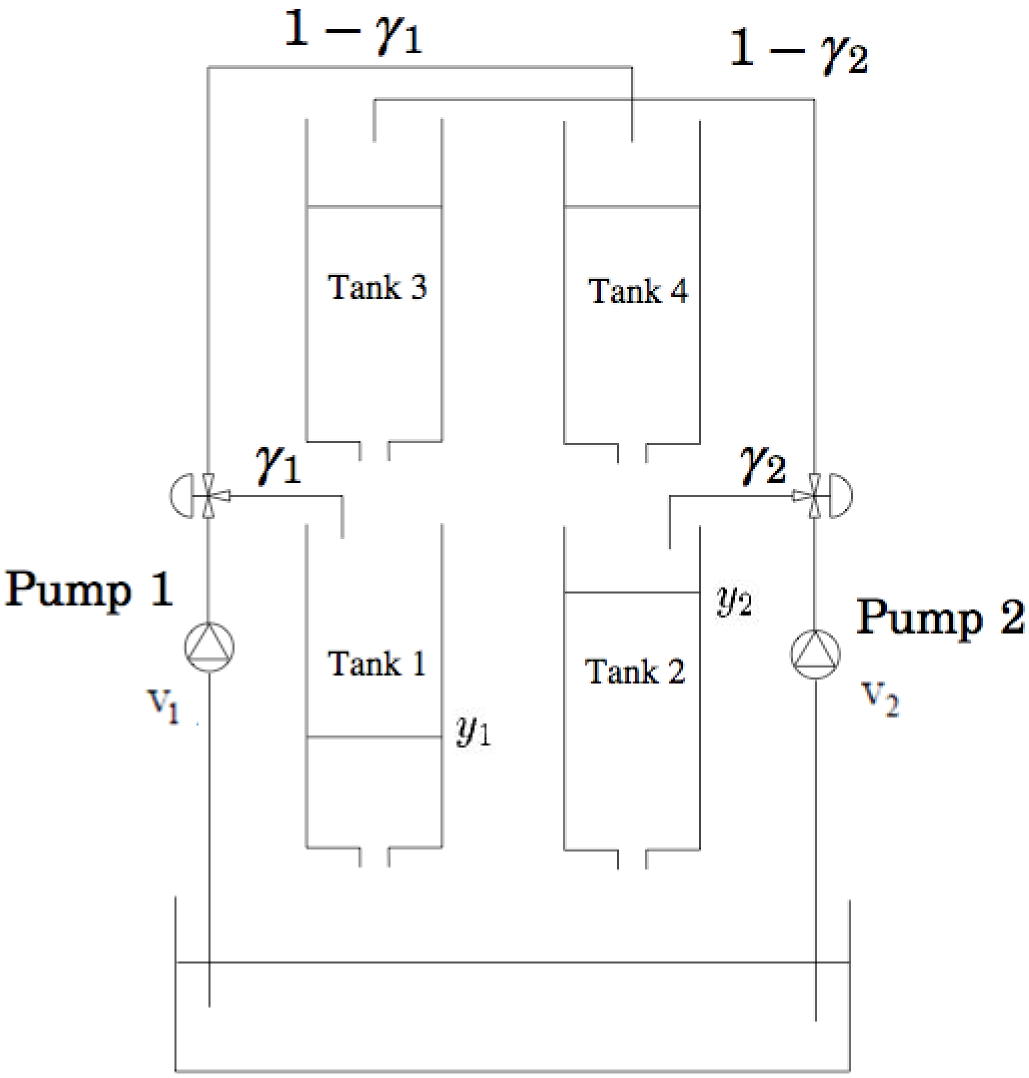
Some examples of multivariable systems

A 3 inputs / 3 outputs chemical process



Some examples of multivariable systems



A_i -Cross sectional area of the tank (m^2)
a_i -Cross sectional area of the outlet hole (m^2)
h_i- Water level (m)
k_i -valve coefficient

$$\frac{dh_1}{dt}=\frac{-a_1}{A_1}\sqrt{2gh_1}+\frac{a_3}{A_1}\sqrt{2gh_3}+\frac{\gamma_1 k_1}{A_1}v_1$$

$$\frac{dh_2}{dt}=\frac{-a_2}{A_2}\sqrt{2gh_2}+\frac{a_4}{A_2}\sqrt{2gh_4}+\frac{\gamma_2 k_2}{A_2}v_2$$

$$\frac{dh_3}{dt}=\frac{-a_3}{A_3}\sqrt{2gh_3}+\frac{(1-\gamma_2)k_2}{A_3}v_2$$

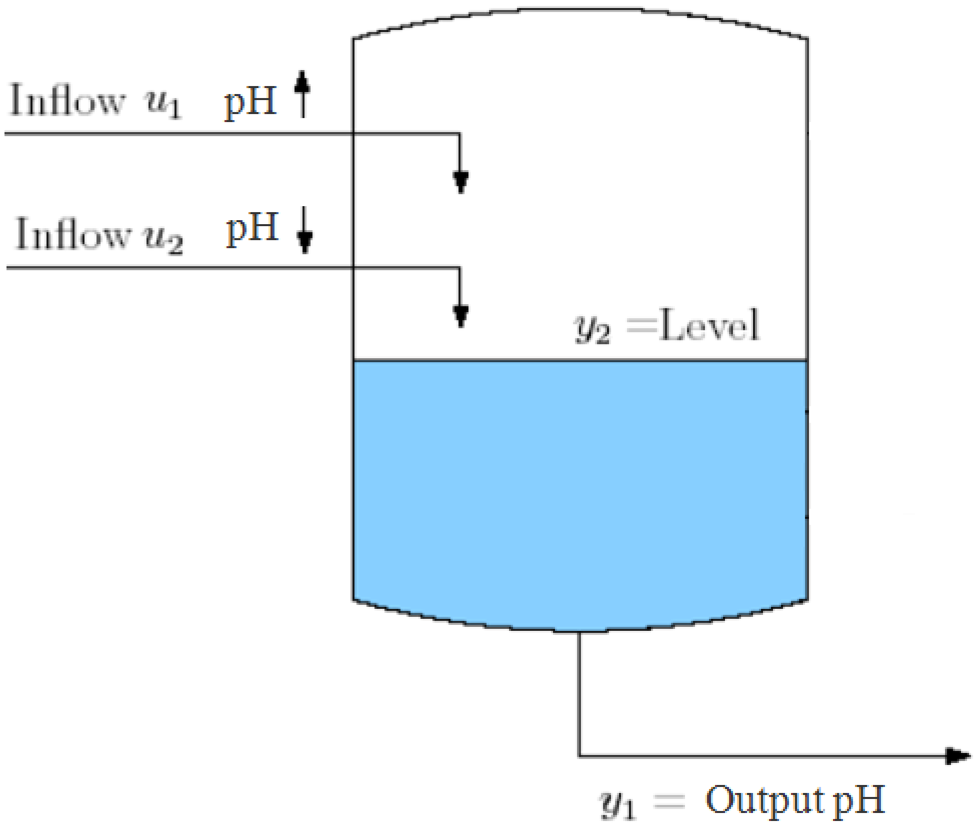
$$\frac{dh_4}{dt}=\frac{-a_4}{A_4}\sqrt{2gh_4}+\frac{(1-\gamma_1)k_1}{A_4}v_1$$

$\gamma_1=\gamma_2=1$ No interaction.

$\gamma_1=\gamma_2=0$ No interaction!?

Some examples of multivariable systems

Example 1-1: Consider following two input two output system.



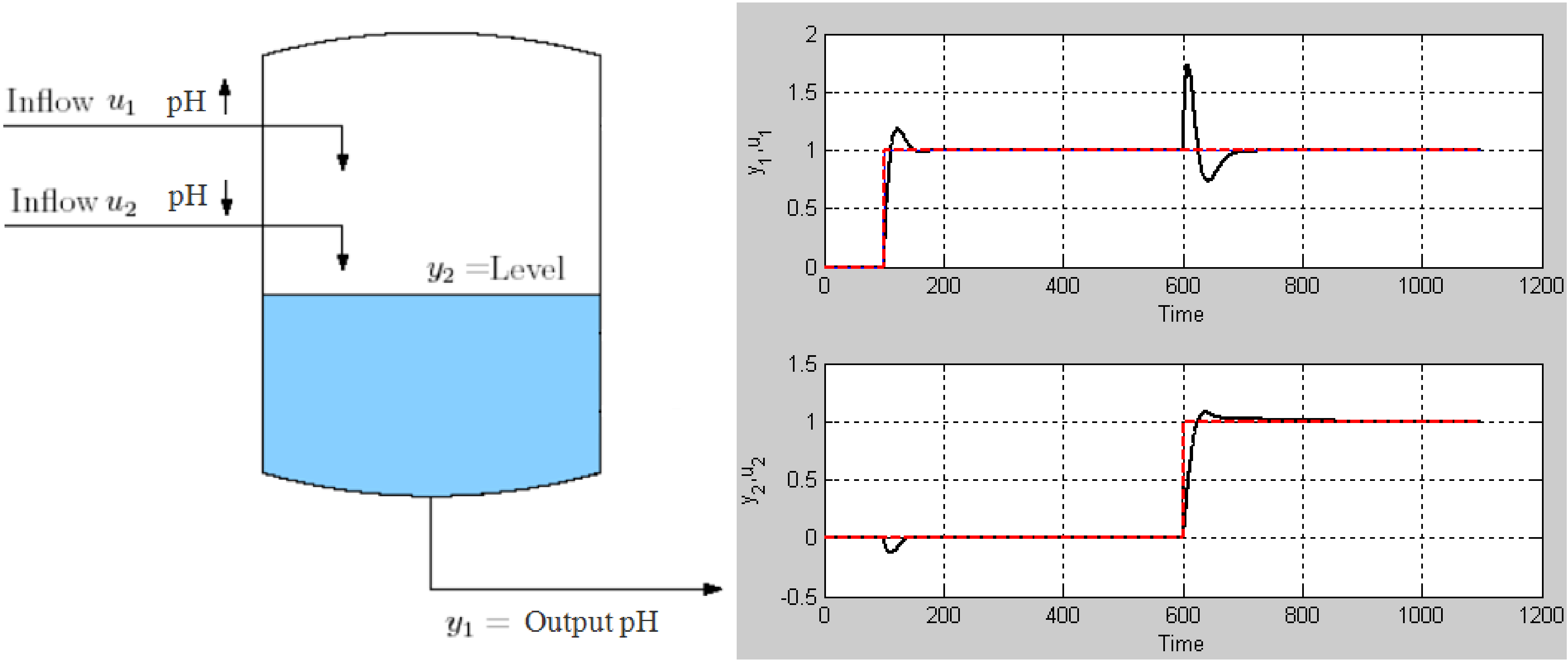
$$G(s)=\begin{bmatrix} \frac{-2.17}{85s+1} & \frac{2.13}{85s+1} \\ \frac{1}{175s+1} & \frac{1}{175s+1} \end{bmatrix}$$

Try a PI controller for g_{11}

Try a PI controller for g_{22}

$$G_c(s)=\begin{bmatrix} \frac{-5s-0.2}{s} & 0 \\ 0 & \frac{8s+0.1}{s} \end{bmatrix}$$

Some examples of multivariable systems



Exercise 1-1: Derive Matlab code for above figure(m-file).

Exercise 1-2: Derive an MIMO example by yourself and explain it and do the same procedure as Example 1-1. (m-file is necessary).

Exercises

1-1 Mentioned in the lecture.

1-2 Mentioned in the lecture.

1-3 Derive a linear state space model for quadruple tank process. Let $A_1=A_2=A_3=A_4=1$ and $h_1=0.5$, $h_2=0.6$, $h_3=h_4=0.4$

1-4 Derive transfer function model for quadruple tank process around the linearized model.