

Seventh Edition

CHAPTER 6

VECTOR MECHANICS FOR ENGINEERS: STATICS

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Lecture Notes:
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Texas Tech University

تحلیل سازه‌ها
۱- خرپاها

گردآوری و تنظیم: محمدحسین ابوالیشری

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Structural Analysis

Sample Problem 6.2 - Solution

Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.

Given: truss and P

Find: force in each of the members in terms of P

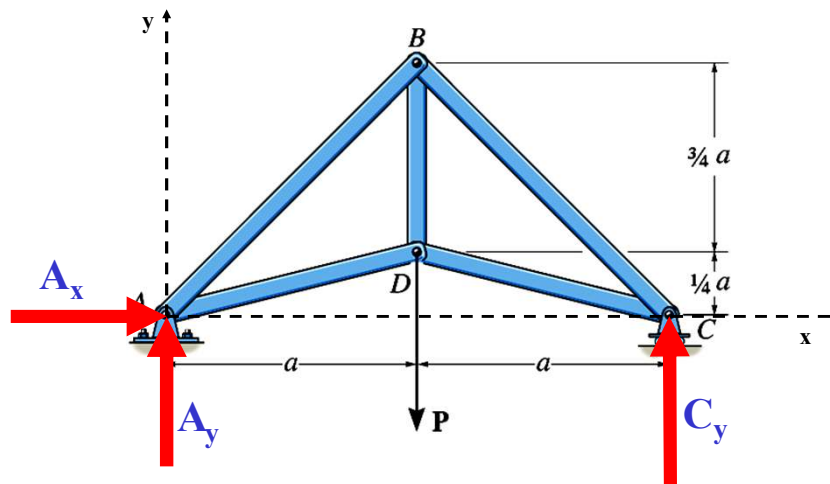
Solution:

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Structural Analysis

Sample Problem 6.2

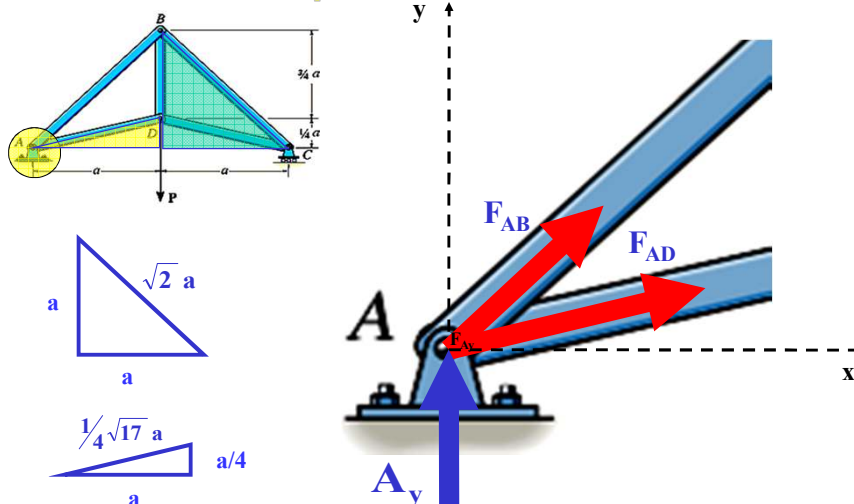


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Structural Analysis

Sample Problem 6.2 - Solution

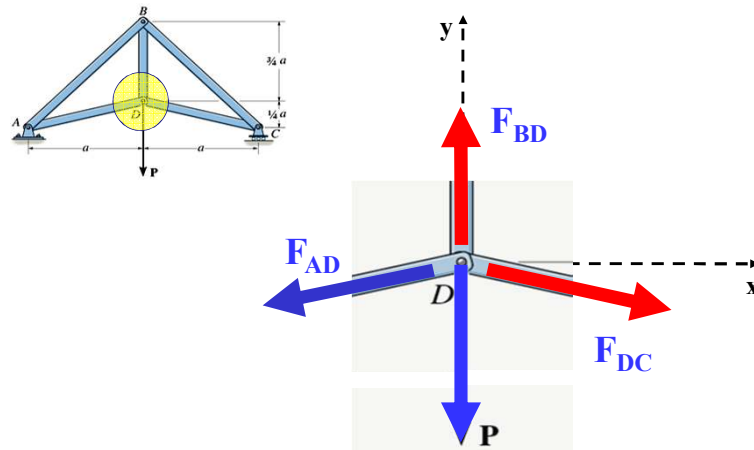


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Structural Analysis

Sample Problem 6.2

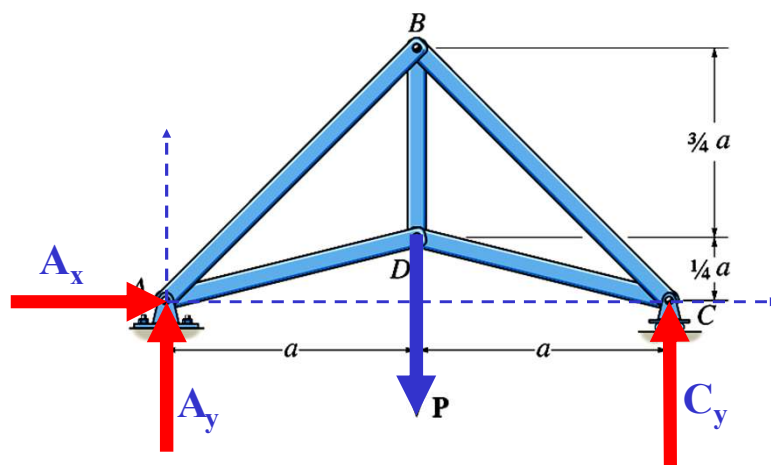


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Sample Problem 6.2



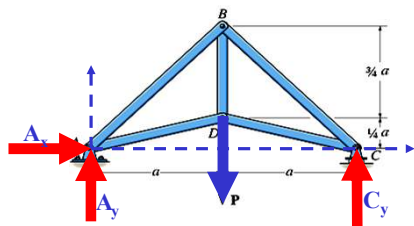
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Structural Analysis

Sample Problem 6.2 - Solution

1. Draw a free body diagram
2. Note that there are three unknown reaction forces F_{BE} , F_{BA} , and F_{DE}
3. There are no zero force members
4. Use that the two force equilibrium equations and the moment equilibrium equation to solve for the three unknown forces



Free Body Diagram

Take moments about joint A to solve for C_y

$$\sum M_A = 0$$

$$C_y(2a) - P(a) = 0$$

$$C_y = \frac{P}{2}$$

Use force equilibrium to solve for the reactions at A

$$\sum F_x = 0 \quad \sum F_y = 0$$

$$A_x = 0 \quad A_y + C_y - P = 0$$

$$A_y = \frac{P}{2}$$

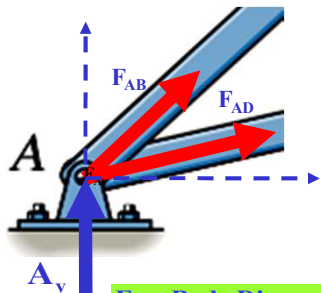
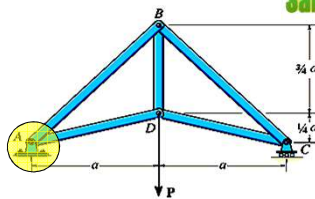
1. Select the joint to begin the calculation of forces in the members
2. Note that since A_x is zero, the truss is symmetric.
3. Therefore, joint A or C is the appropriate point to start the calculations

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Structural Analysis

Sample Problem 6.2 - Solution



Free Body Diagram

Start with joint A

all unknown forces drawn in tension

$$\sum F_x = 0$$

$$F_{AB} \cos 45^\circ + F_{AD} \frac{4}{\sqrt{17}} = 0$$

and

$$\sum F_y = 0$$

$$-\frac{P}{2} + F_{AB} \sin 45^\circ + F_{AD} \frac{1}{\sqrt{17}} = 0$$

Solving

$$F_{AB} = -.943P \quad (c)$$

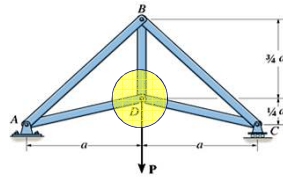
$$F_{AD} = .687P \quad (t)$$

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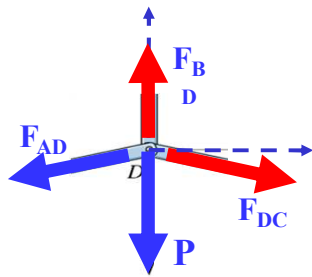
Structural Analysis

Sample Problem 6.2 - Solution



Next move to joint D although B is also appropriate since there are only two unknowns at either joint.

Draw all unknown forces in tension



Free body Diagram

$$\sum F_x = 0$$

$$F_{DC} \frac{4}{\sqrt{17}} - F_{AD} \frac{4}{\sqrt{17}} = 0$$

$$F_{DC} = F_{AD} = .687P \text{ (tension)}$$

and

$$\sum F_y = 0$$

$$F_{BD} - P + 2F_{AD} \frac{1}{\sqrt{17}} = 0$$

$$F_{BD} = 1.33P \text{ (tension)}$$

By symmetry

$$F_{BC} = F_{AB} = .943P \text{ (compression)}$$

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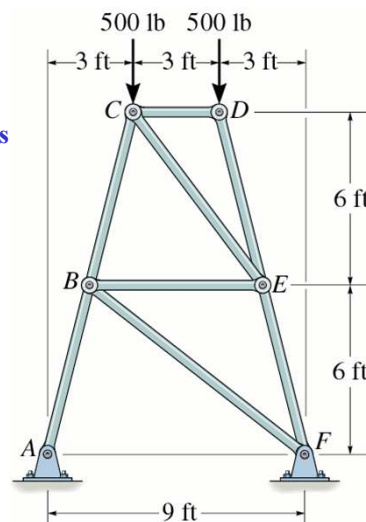
Sample Problem 6.3 - Solution

Determine the force in each member of the truss and state if the members are in tension or compression.

Given: truss and loads

Find: force in each of the members

Solution:

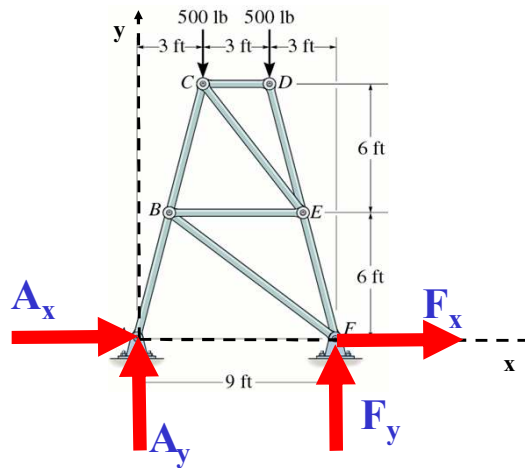


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Sample Problem 6.3

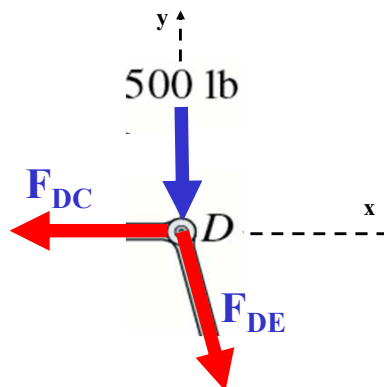
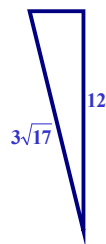
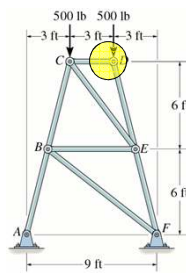


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Sample Problem 6.3

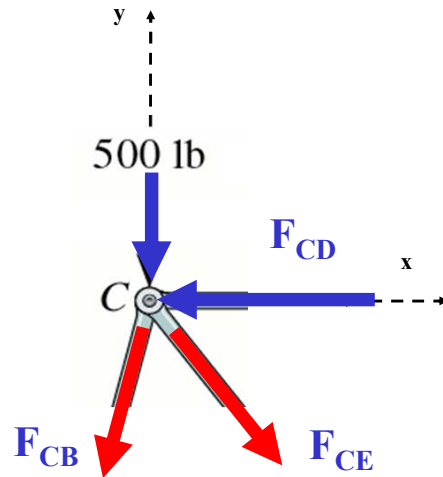
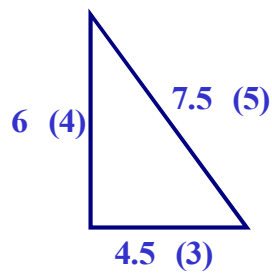
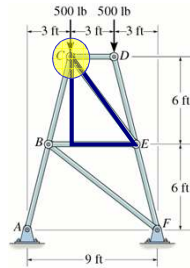


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Sample Problem 6.3

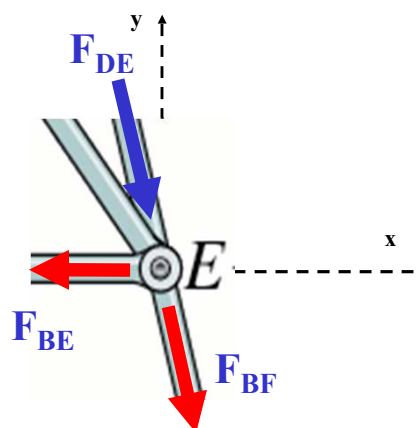
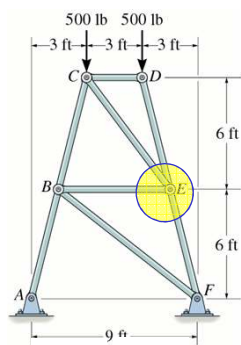


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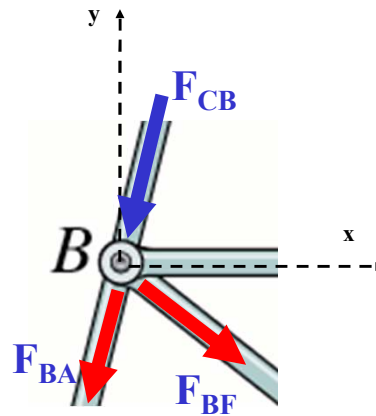
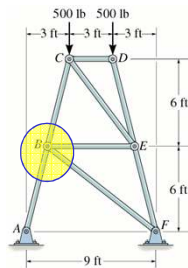


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Structural Analysis

Sample Problem 6.3

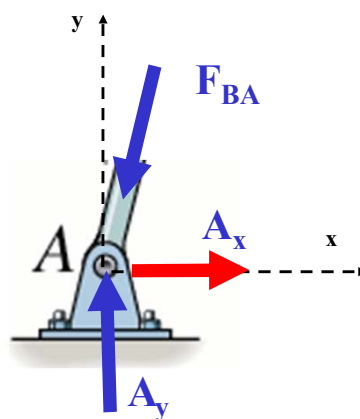
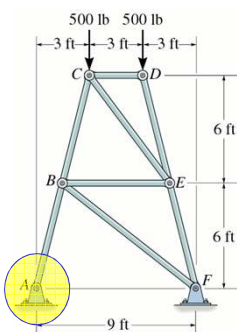


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Structural Analysis

Sample Problem 6.3

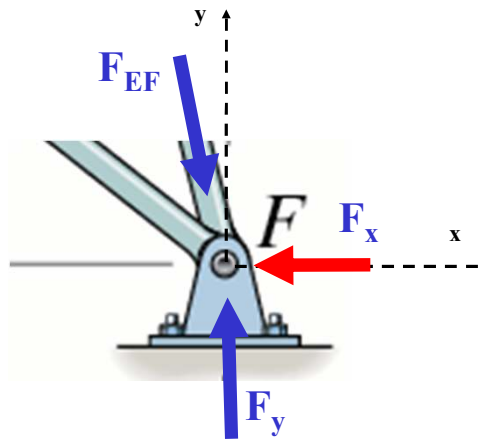
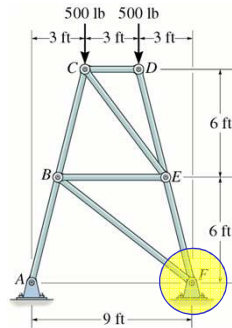


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Structural Analysis

Sample Problem 6.3

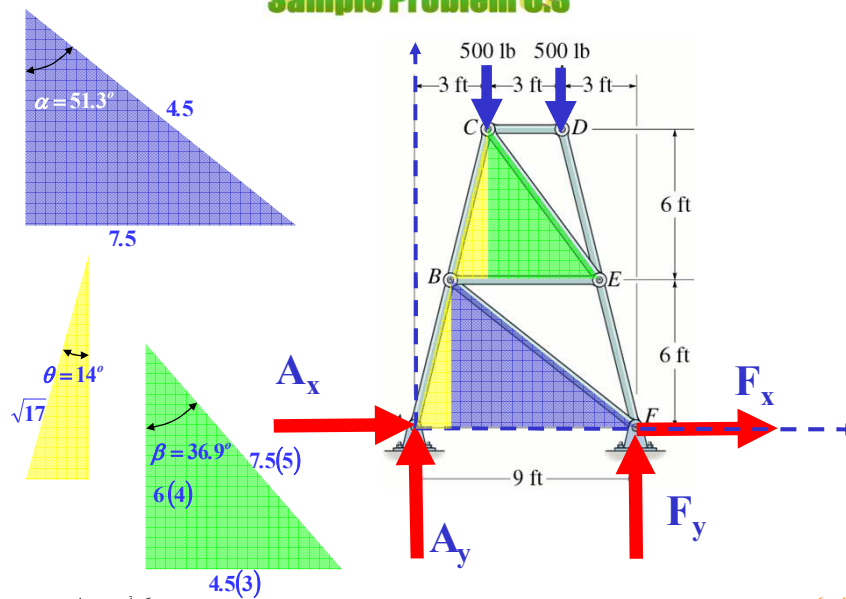


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Structural Analysis

Sample Problem 6.3



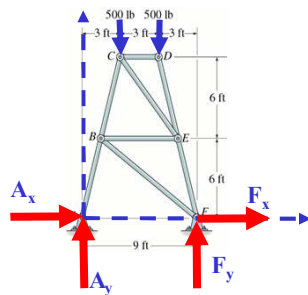
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Structural Analysis

Sample Problem 6.3 - Solution

1. Draw a free body diagram for the structure
2. Note that there are four unknown reaction forces F_x , F_y , A_x and A_y
3. The equilibrium equations cannot resolve the reaction forces for the entire structure.



Free Body Diagram

Use equilibrium equations to solve for three of the unknowns.

$$\sum M_A = 0$$

$$F_y(9) - 500(3) - 500(9) = 0$$

$$F_y = 500 \text{ lb}$$

and

$$\sum F_y = 0$$

$$A_y + F_y - 500 - 500 = 0$$

$$A_y + 500 - 500 - 500 = 0$$

$$A_y = 500 \text{ lb}$$

and

$$\sum F_x = 0$$

$$A_x + F_x = 0$$

$$A_x = -F_x$$

Note there are still two unknowns but the relationship between them is now known.

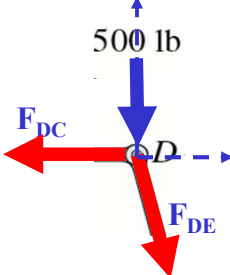
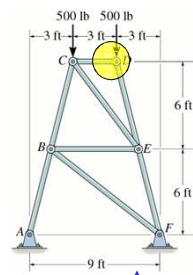
Note: there are only two members connected at joint D. This implies that the forces in all members can be found

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Structural Analysis

Sample Problem 6.3 - Solution



Free Body Diagram

A review of the structure indicates that there are only two unknown forces in the members meeting at joint D. Use equilibrium to solve for the forces in the members.

Draw all unknown forces in tension

$$\sum F_y = 0$$

$$-500 - F_{DE} \frac{4}{\sqrt{17}} = 0$$

$$F_{DE} = -515 \text{ lb (compression)}$$

and

$$\sum F_x = 0$$

$$-F_{DC} + F_{DE} \frac{1}{\sqrt{17}} = 0$$

$$F_{DC} = -125 \text{ lb (compression)}$$

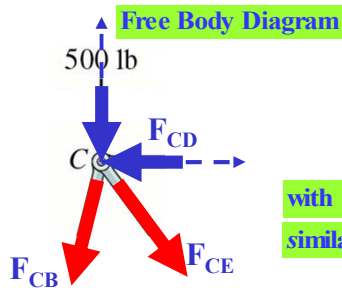
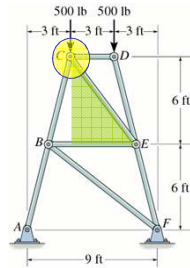
Note: $F_{DC} = -F_{CD}$ and F_{CD} will be in compression for joint C

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Structural Analysis

Sample Problem 6.3 - Solution



Next move to joint C. Now there are only two unknown forces since F_{CD} is known

Draw all unknown forces in tension

$$\sum F_y = 0$$

$$-F_{CB} \left(\frac{4}{\sqrt{17}} \right) - F_{CE} \left(\frac{4}{5} \right) - 500 = 0$$

and

$$\sum F_x = 0$$

$$-F_{CB} \left(\frac{1}{\sqrt{17}} \right) + F_{CE} \left(\frac{3}{5} \right) = 0$$

solving for F_{CB} and F_{CE}

$$F_{CB} = -515 \text{ lb (compression)}$$

$$F_{CE} = 0$$

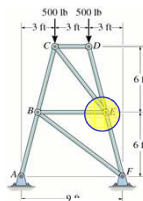
with $F_{CE} = 0$ then F_{EB} becomes a zero force member.

similarly, F_{BF} is also a zero force member.

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Structural Analysis

Sample Problem 6.3 - Solution

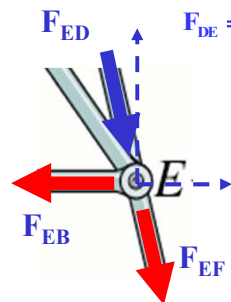


with

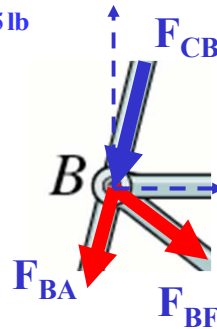
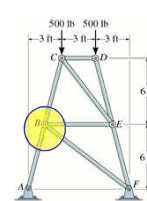
$$F_{CE} = F_{EB} = F_{BF} = 0$$

$$F_{DE} = F_{EF} = F_{CB} = F_{BA} = -515 \text{ lb}$$

Reactions at A and E



Free Body Diagrams

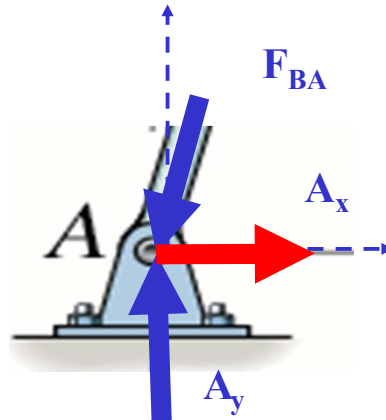
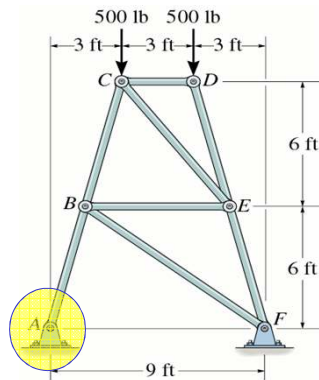


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Structural Analysis

Sample Problem 6.3



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Structural Analysis

Sample Problem 6.3 - Solution

Reactions at A and E are the same by symmetry

with

$$F_{BA} = -515 \text{ lb} \quad A_y = 500 \text{ lb}$$

use force equilibrium in x

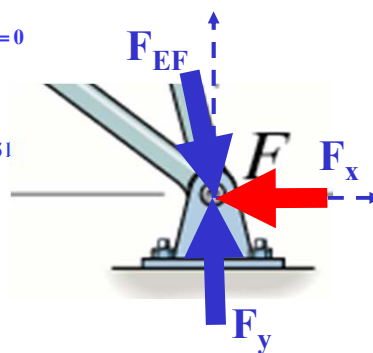
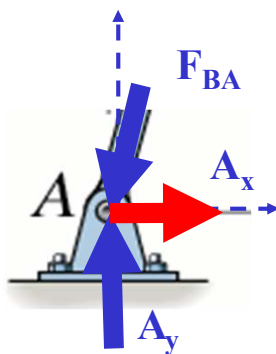
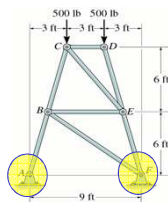
$$\sum F_x = 0$$

$$A_x - F_{AB} \left(\frac{1}{\sqrt{17}} \right) = 0$$

$$A_x = 125 \text{ lb}$$

then

$$A_x = -F_x = -125 \text{ lb}$$



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Structural Analysis

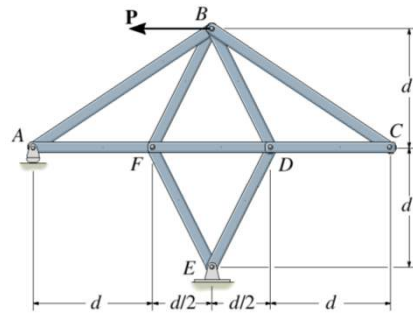
Sample Problem 6.4

If the maximum force that any member can support is 4 kN in compression, determine the maximum force P that can be supported at point B. Take $d=1$ m.

Given: truss and load, $F_{\max}=4$ kN

Find: P_{\max}

Solution:

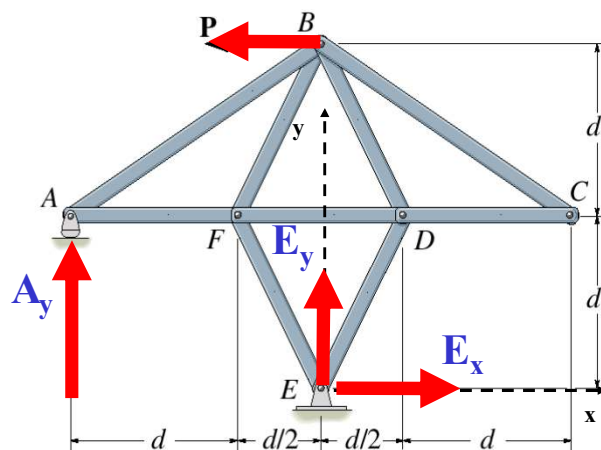


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Structural Analysis

Sample Problem 6.4

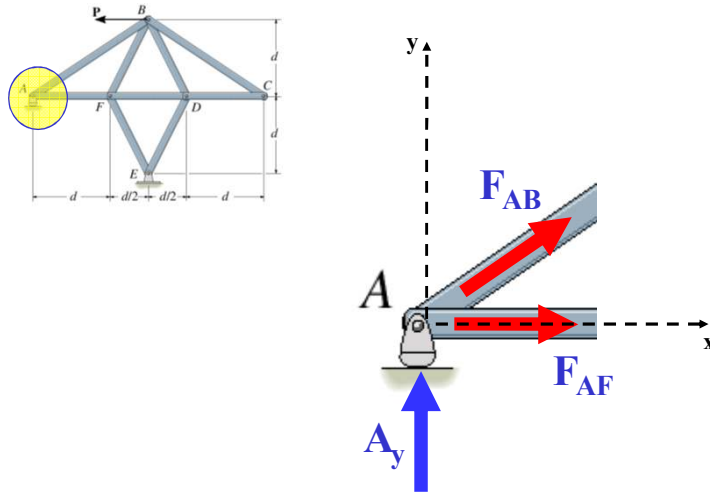


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Sample Problem 6.4

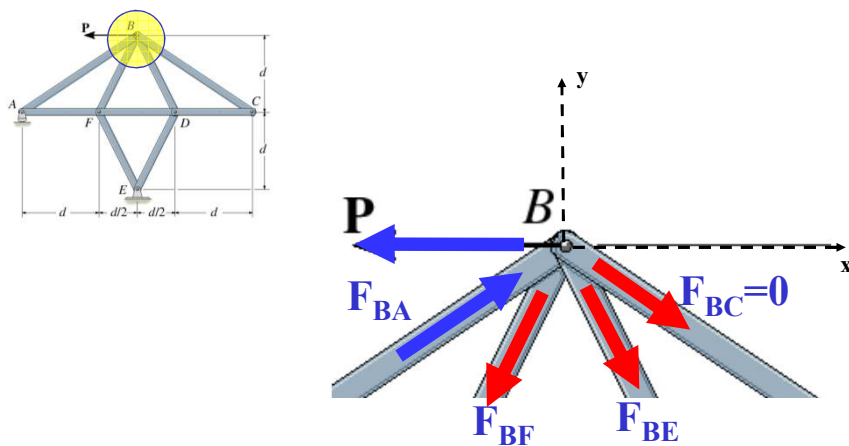


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Structural Analysis

Sample Problem 6.4

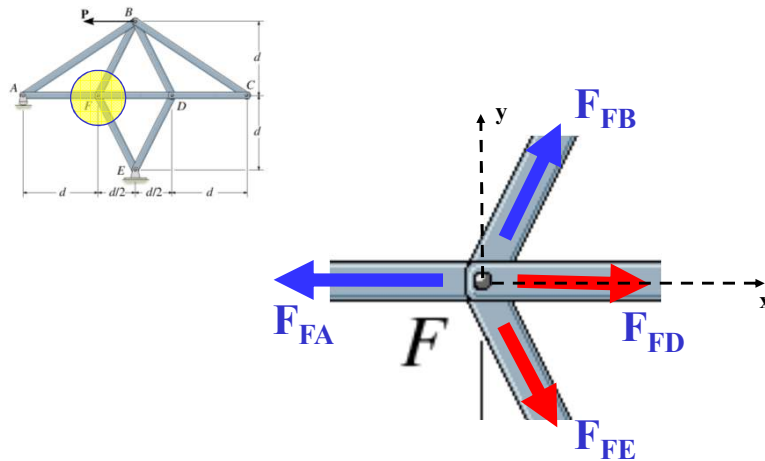


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Structural Analysis

Sample Problem 6.4

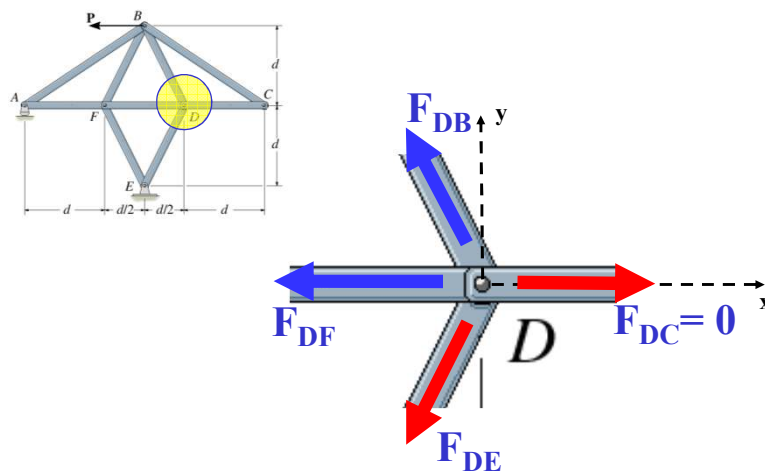


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Structural Analysis

Sample Problem 6.4

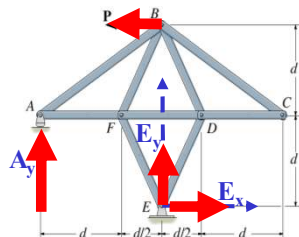


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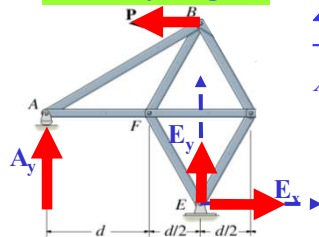
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Structural Analysis

Sample Problem 6.4 - Solution



Free Body Diagram



1. Check for zero force members (CD and CB)
2. Draw a free body diagram for the structure
3. Note that there are three unknown reaction forces
4. The equilibrium equations cannot resolve the reaction forces in terms of force P.

Use equilibrium equations to solve for three of the unknowns.

$$\begin{aligned} \sum M_E = 0 & & \sum F_x = 0 & & \sum F_y = 0 \\ -A_y(1.5)d - P(2)d = 0 & & E_x - P = 0 & & A_y + E_y = 0 \\ A_y = 1.333P & & E_x = P & & E_y = A_y \\ & & & & E_y = -1.333P \end{aligned}$$

Method of Joints

to find forces in the members

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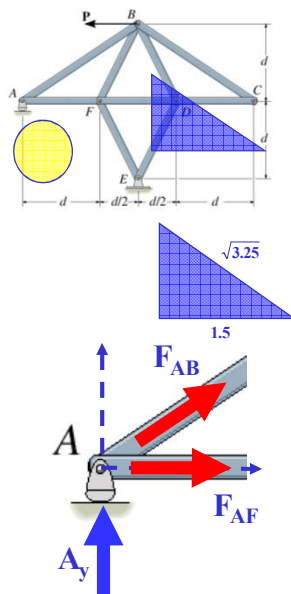
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Structural Analysis

Sample Problem 6.4 - Solution

Method of Joints - Joint A

draw all unknown forces in tension



$$\begin{aligned} \sum F_y = 0 \\ A_y + F_{AB} \frac{1}{\sqrt{3.25}} = 0 \\ F_{AB} = -2.404P \quad (\text{compression}) \end{aligned}$$

$$\begin{aligned} \sum F_x = 0 \\ F_{AF} + F_{AB} \frac{1.5}{\sqrt{3.25}} = 0 \\ F_{AF} = 2P \quad (\text{tension}) \\ E_y = -1.333P \end{aligned}$$

move to joint B

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Structural Analysis

Sample Problem 6.4 - Solution

Method of Joints - Joint B

draw all unknown forces in tension

note $F_{BC} = 0$

$$\sum F_x = 0$$

$$-P + F_{AB} \frac{1.5}{\sqrt{3.25}} - F_{BF} \frac{1}{\sqrt{5}} + F_{BD} \frac{1}{\sqrt{5}} = 0$$

$$\sum F_y = 0$$

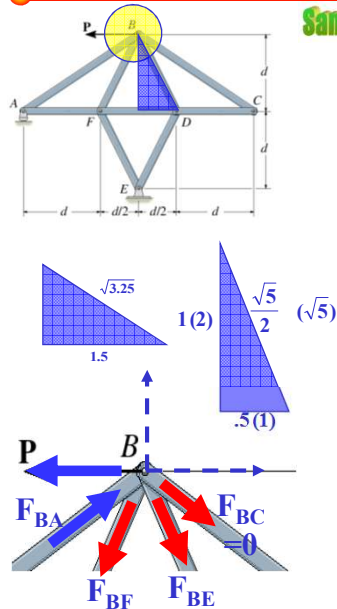
$$F_{BA} \frac{1}{\sqrt{3.25}} - F_{BF} \frac{2}{\sqrt{5}} - F_{BD} \frac{2}{\sqrt{5}} = 0$$

solving

$$F_{BF} = 1.863P \text{ (tension)}$$

$$F_{BD} = -.3727P \text{ (compression)}$$

move to joint F



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6 - ۳۳/84

Structural Analysis

Sample Problem 6.4 - Solution

Method of Joints - Joint F

draw all unknown forces in tension

$$\sum F_y = 0$$

$$F_{FB} \frac{2}{\sqrt{5}} - F_{FE} \frac{2}{\sqrt{5}} = 0$$

$$F_{FE} = 1.863P \text{ (tension)}$$

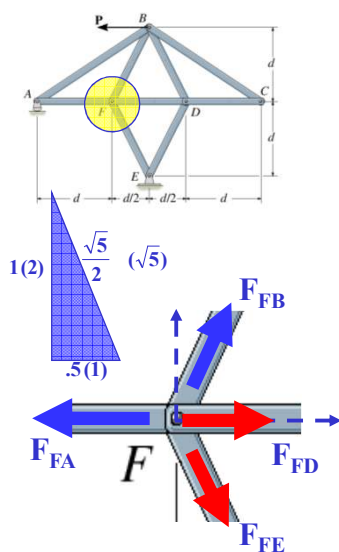
and

$$\sum F_x = 0$$

$$F_{FD} - F_{FA} + F_{FB} \frac{1}{\sqrt{5}} + F_{FE} \frac{1}{\sqrt{5}} = 0$$

$$F_{FD} = .333P \text{ (tension)}$$

move to joint D

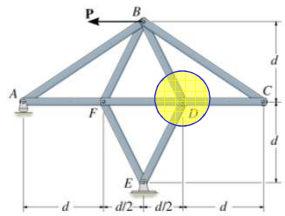


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6 - ۳۴/84

Structural Analysis

Sample Problem 6.4 - Solution



Method of Joints - Joint D

draw all unknown forces in tension

$$\sum F_y = 0$$

$$F_{DB} \frac{2}{\sqrt{5}} - F_{DE} \frac{2}{\sqrt{5}} = 0$$

$$F_{DE} = F_{DB} = .3727P \text{ (compression)}$$

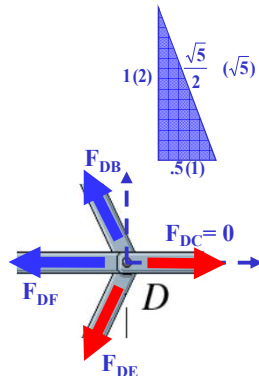
Note $F_{DC} = 0$

maximum compression member is

$$F_{AB} = 2.404P$$

$$P = \frac{F_{AB}}{2.404} = \frac{4}{2.404} = 1.664 \text{ kN}$$

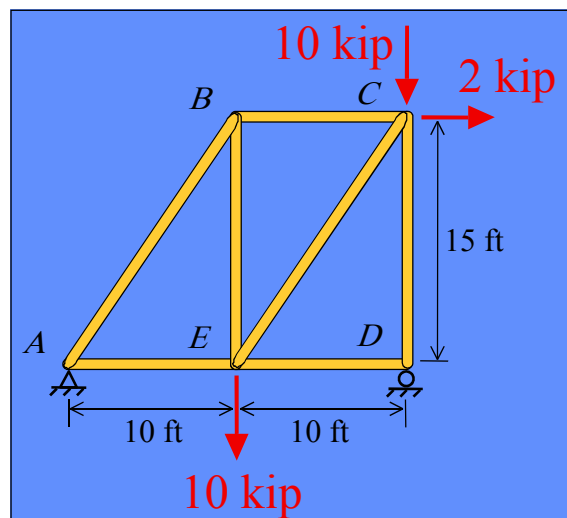
All forces can be solved with this value for P



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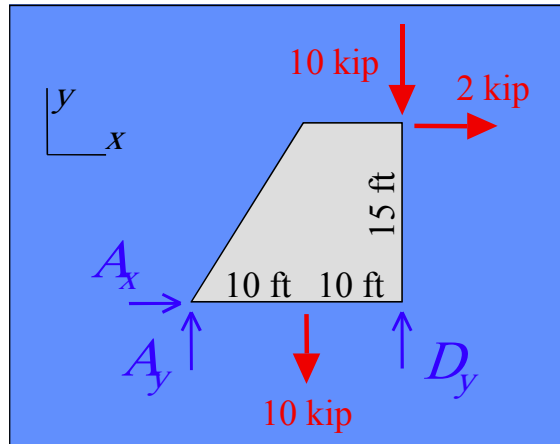
6 - ۳۵/84

Example 1: Determine the force in each member of the truss and indicate whether the members are in tension or compression. Assume that all members are pin-connected.



6 - ۳۶/84

FBD of entire structure to get reactions:



$$\sum F_x = 0 = A_x + 2 \text{ kip}$$

$$A_x = -2 \text{ kip} \quad \Leftarrow$$

$$\sum M_A = 0 = 10 \text{ kip} (10') - D_y (20') + 10 \text{ kip} (20') + 2 \text{ kip} (15')$$

$$D_y = 16.5 \text{ kip} \quad \Leftarrow$$

$$\sum M_D = 0 = -A_y (20') + 10 \text{ kip} (10') - 2 \text{ kip} (15')$$

$$A_y = 3.5 \text{ kip} \quad \Leftarrow$$

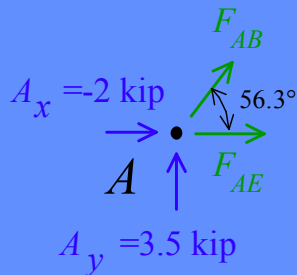
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6 - ۳۷/84

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$ corresponds to tension in member ij .

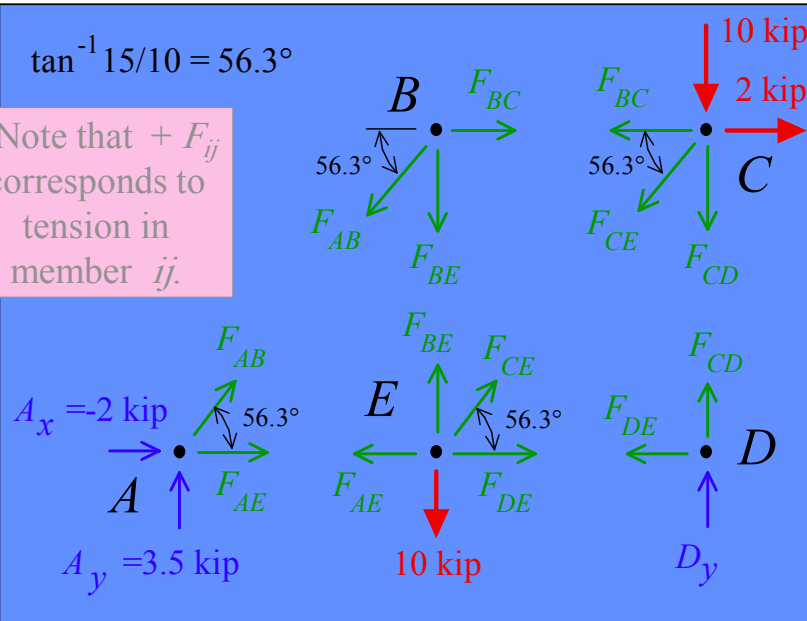


6 - ۳۸/84

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$ corresponds to tension in member ij .



6 - 39/84

@A

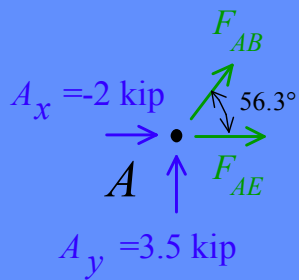
$$\sum F_y = 0 = A_y + F_{AB} \sin 56.3^\circ \quad F_{AB} = -4.21 \text{ kip} \leftarrow$$

$$\sum F_x = 0 = A_x + F_{AB} \cos 56.3^\circ + F_{AE} \quad F_{AE} = 4.33 \text{ kip} \leftarrow$$

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$
corresponds to
tension in
member ij .

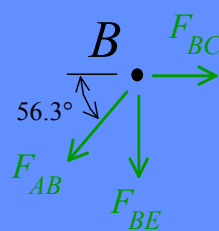
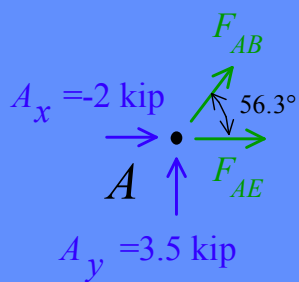


6 - 41/84

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$
corresponds to
tension in
member ij .



6 - 42/84

@A

$$\sum F_y = 0 = A_y + F_{AB} \sin 56.3^\circ \quad F_{AB} = -4.21 \text{ kip} \quad \Leftarrow$$

$$\sum F_x = 0 = A_x + F_{AB} \cos 56.3^\circ + F_{AE} \quad F_{AE} = 4.33 \text{ kip} \quad \Leftarrow$$

@B

$$\sum F_y = 0 = F_{AB} \sin 56.3^\circ + F_{BE} \quad F_{BE} = 3.50 \text{ kip} \quad \Leftarrow$$

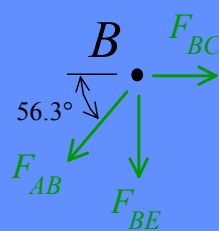
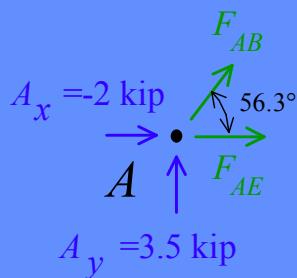
$$\sum F_x = 0 = -F_{AB} \cos 56.3^\circ + F_{BC} \quad F_{BC} = -2.34 \text{ kip} \quad \Leftarrow$$

6 - 43/84

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$
corresponds to
tension in
member ij .

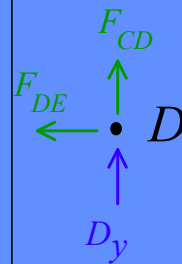
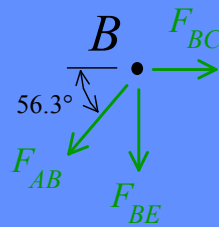
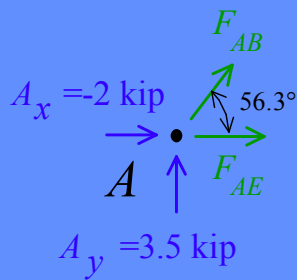


6 - 44/84

FBDs of joints to get member forces (internal forces)

$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$
corresponds to
tension in
member ij .



6 - 49/84

@A

$$\sum F_y = 0 = A_y + F_{AB} \sin 56.3^\circ \quad F_{AB} = -4.21 \text{ kip} \quad \Leftarrow$$

$$\sum F_x = 0 = A_x + F_{AB} \cos 56.3^\circ + F_{AE} \quad F_{AE} = 4.33 \text{ kip} \quad \Leftarrow$$

@B

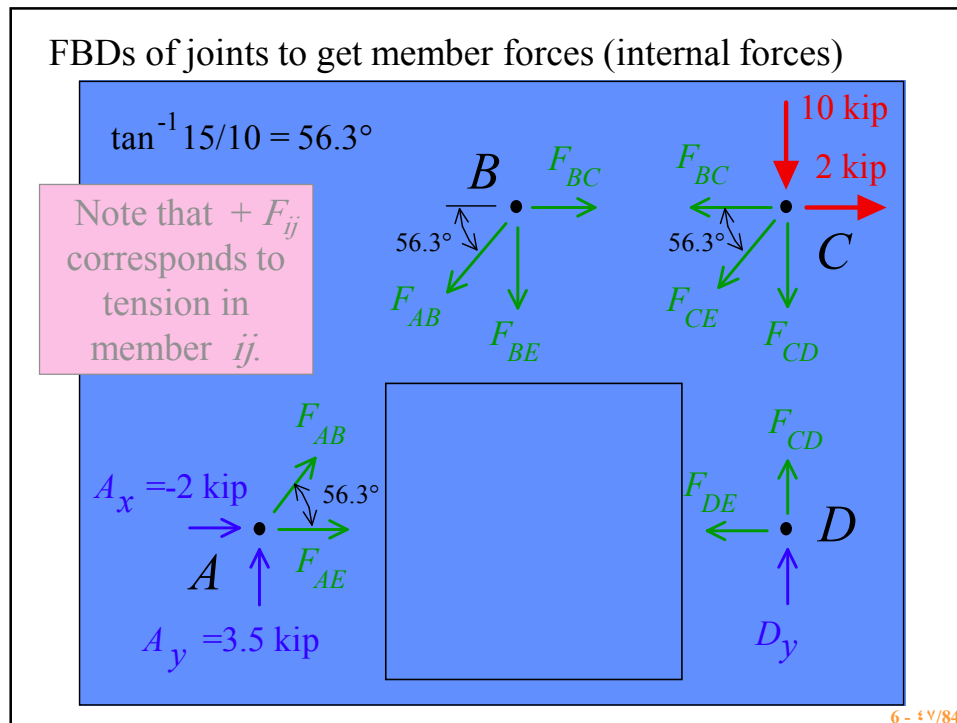
$$\sum F_y = 0 = F_{AB} \sin 56.3^\circ + F_{BE} \quad F_{BE} = 3.50 \text{ kip} \quad \Leftarrow$$

$$\sum F_x = 0 = -F_{AB} \cos 56.3^\circ + F_{BC} \quad F_{BC} = -2.34 \text{ kip} \quad \Leftarrow$$

@D

$$\sum F_x = 0 = F_{DE} \quad F_{DE} = 0 \quad \Leftarrow$$

$$\sum F_y = 0 = D_y + F_{CD} \quad F_{CD} = -16.5 \text{ kip} \quad \Leftarrow$$



@C

$$\sum F_x = 0 = -F_{BC} - F_{CE} \cos 56.3^\circ + 2 \text{ kip} \quad F_{CE} = 7.82 \text{ kip} \quad \Leftarrow$$

$$\sum F_y = 0 = 10 \text{ kip} + F_{CE} \sin 56.3^\circ + F_{CD} \quad F_{CD} = -16.51 \text{ kip} \quad \Leftarrow$$

@E

$$\sum F_x = 0 = \dots$$

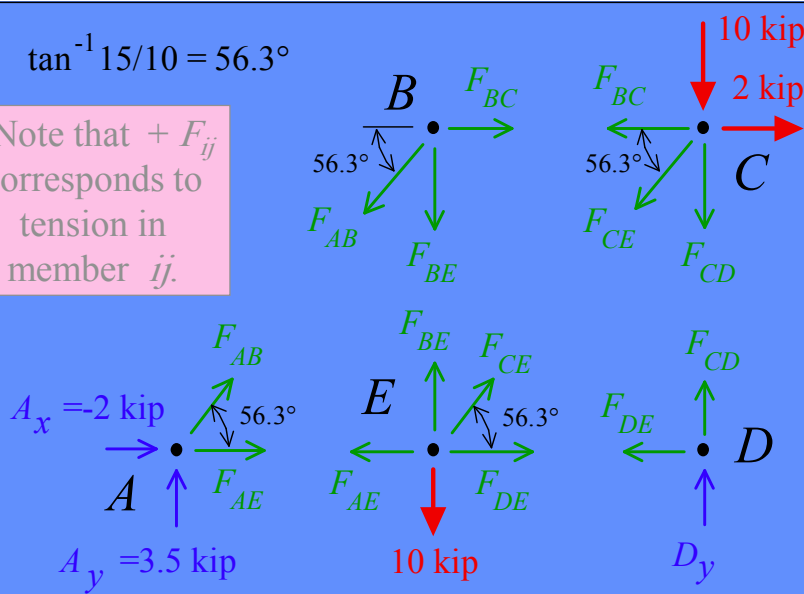
$$\sum F_y = 0 = \dots$$

Could write these equations, but all unknowns have been determined. Good idea to use these equations for a check!

FBDs of joints to get member forces (internal forces)

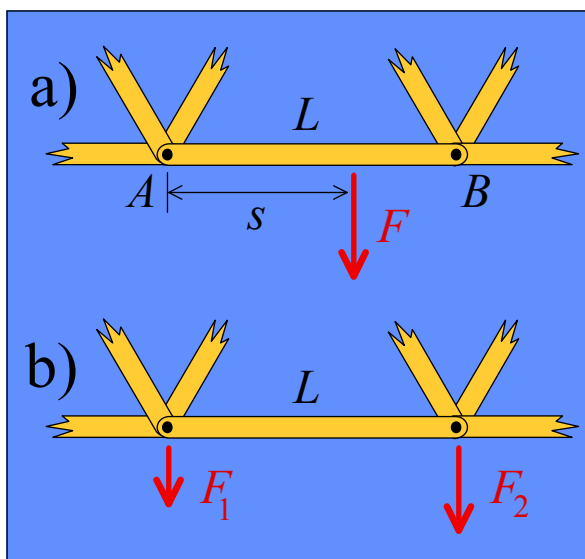
$$\tan^{-1} 15/10 = 56.3^\circ$$

Note that $+F_{ij}$ corresponds to tension in member ij .



6 - 49/84

When a load is not applied at a joint, we usually use an *equivalent load system* ...



6 - 50/84

$$\sum M_B = 0$$

$$\Rightarrow F_1 = \frac{L-s}{L} F$$

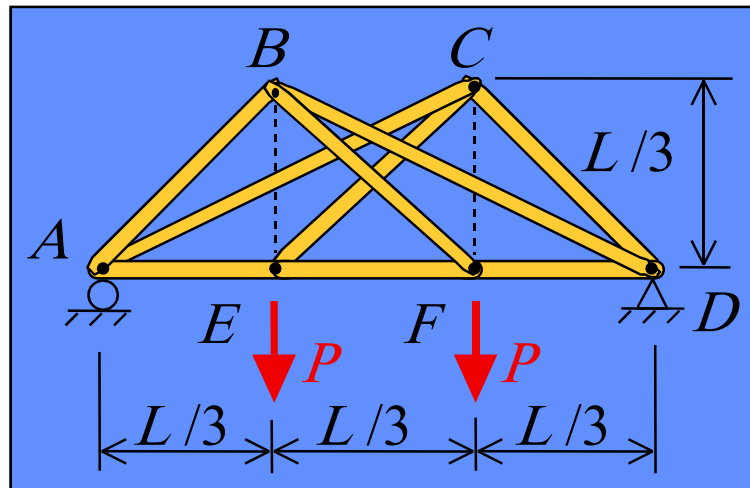
$$\sum M_A = 0$$

$$\Rightarrow F_2 = \frac{s}{L} F$$

The *bending* experienced by member AB is treated separately.

example: Determine the force in each member of the double scissors truss in terms of the load P , and state if the members are in tension or compression.

$$\begin{aligned} F_{BD} &= F_{AC} = -2P/5/3 \\ F_{BF} &= F_{EC} = P/2 \\ F_{AB} &= F_{CD} = -P/2/3 \\ F_{AE} &= F_{FD} = 5P/3 \\ F_{EF} &= 2P/3 \end{aligned}$$



6 - 01/84

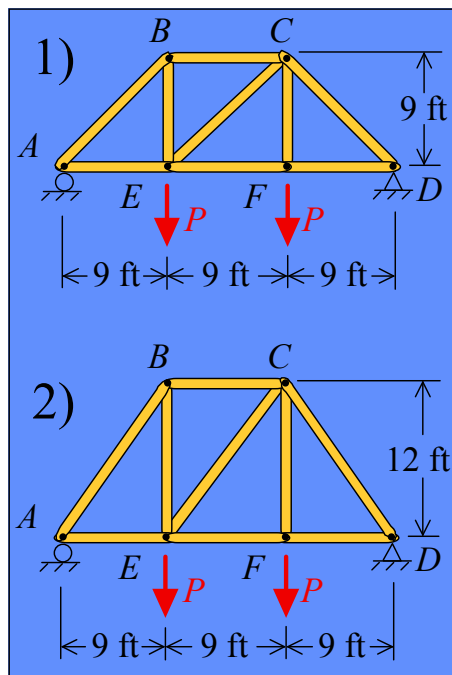
Supplementary HW Problem

(will not be collected)

- 1) Determine all member forces for truss #1.
- 2) Determine all member forces for truss #2.
- 3) Let all members in trusses #1 and #2 be of the same material with the same cross section shape. If buckling is neglected, which of the two trusses do you think will be stronger? If buckling is considered, might your answer change? Explain.

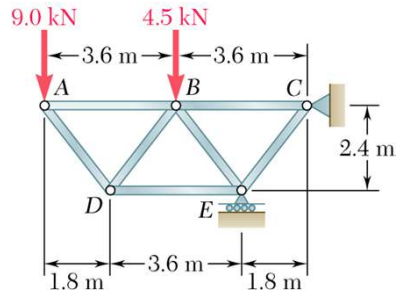
(Buckling will be discussed next lecture!)

6 - 01/84



Vector Mechanics for Engineers: Statics

Sample Problem 6.1



Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C .
- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints D , B , and E from joint equilibrium requirements.
- All member forces and support reactions are known at joint C . However, the joint equilibrium requirements may be applied to check the results.

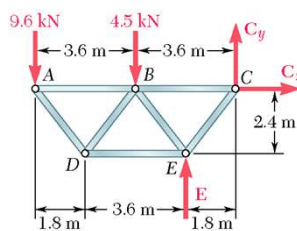
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6 - ۳/134

Vector Mechanics for Engineers: Statics

Sample Problem 6.1



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C .

$$\begin{aligned}\sum M_C = 0 \\ = (9.0 \text{ kN})(7.2 \text{ m}) + (4.5 \text{ kN})(3.6 \text{ m}) - E(1.8 \text{ m})\end{aligned}$$

$$E = 45 \text{ kN} \uparrow$$

$$\sum F_x = 0 = C_x \quad C_x = 0$$

$$\sum F_y = 0 = -90 \text{ kN} - 4.5 \text{ kN} + 45 \text{ kN} + C_y$$

$$C_y = 31.5 \text{ kN} \downarrow$$

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Vector Mechanics for Engineers: Statics

Sample Problem 6.1

- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{9.0 \text{ kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$F_{AB} = 6.75 \text{ kN}$
 $F_{AD} = 11.25 \text{ kN}$
- There are now only two unknown member forces at joint D .

$$F_{DB} = F_{DA}$$

$$F_{DE} = 2\left(\frac{3}{5}\right)F_{DA}$$

$F_{DB} = 11.25 \text{ kN}$
 $F_{DE} = 13.5 \text{ kN}$

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Vector Mechanics for Engineers: Statics

Sample Problem 6.1

- There are now only two unknown member forces at joint B . Assume both are in tension.

$$\sum F_y = 0 = -4.5 - \frac{4}{5}(11.25) - \frac{4}{5}F_{BE}$$

$$F_{BE} = -3750 \text{ lb}$$

$F_{BE} = 16.9 \text{ kN}$

$$\sum F_x = 0 = F_{BC} - 6.75 - \frac{3}{5}(16.9) - \frac{3}{5}(16.9)$$

$$F_{BC} = +23.63 \text{ kN}$$

$F_{BC} = 23.63 \text{ kN}$
- There is one unknown member force at joint E . Assume the member is in tension.

$$\sum F_x = 0 = \frac{3}{5}F_{EC} + 13.5 + \frac{3}{5}(6.9)$$

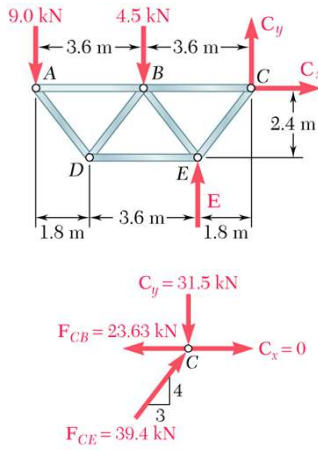
$$F_{EC} = -39.4 \text{ kN}$$

$F_{EC} = 39.4 \text{ kN}$

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Vector Mechanics for Engineers: Statics

Sample Problem 6.1



• All member forces and support reactions are known at joint C . However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -23.63 \text{ kN} + \frac{3}{5}(39.4) = 0 \quad (\text{checks})$$

$$\sum F_y = -31.5 \text{ kN} + \frac{4}{5}(39.4) = 0 \quad (\text{checks})$$

$C_y = 31.5 \text{ kN}$
 $F_{CB} = 23.63 \text{ kN}$
 $C_x = 0$
 $F_{CE} = 39.4 \text{ kN}$

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Structural Analysis

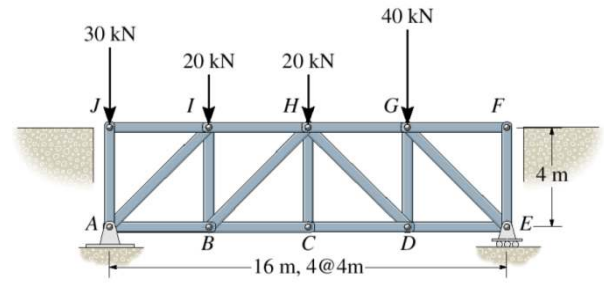
Sample Problem 6.6

The Howe bridge truss is subjected to the loading shown. Determine the force in members HD , CD , and GD , and state if the members are in tension or compression.

Given: truss and loads

Find: force in members HD , CD , GD

Solution:

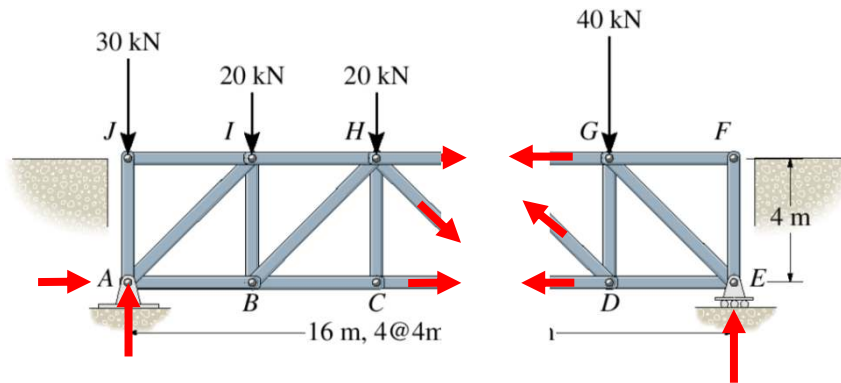


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Structural Analysis

Sample Problem 6.6

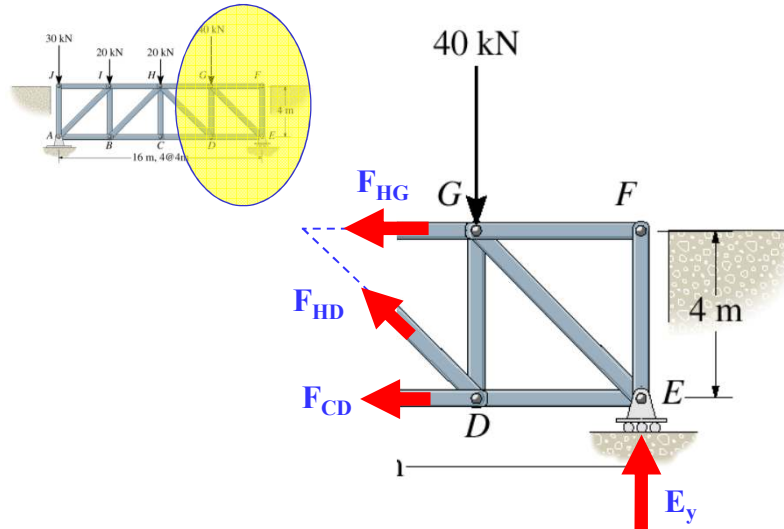


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Structural Analysis

Sample Problem 6.6

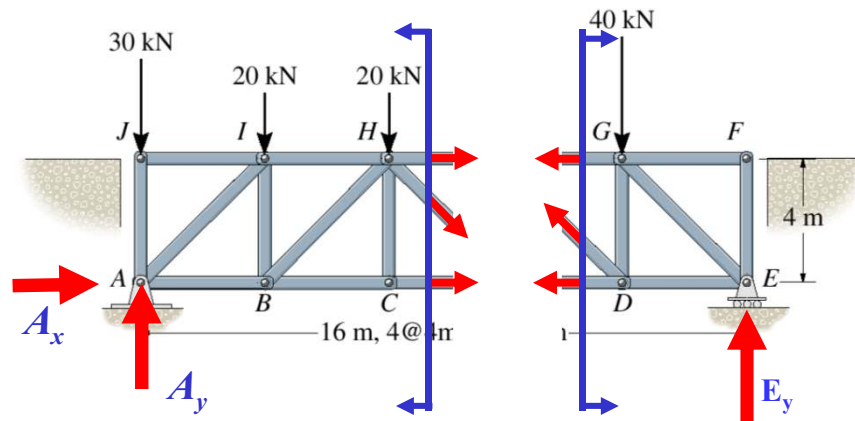


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Structural Analysis

Sample Problem 6.6



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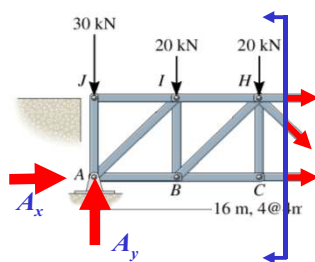
6 - ۶۱/84

Structural Analysis

Sample Problem 6.6 - Solution

1. Draw a line to cut all members of interest CD, HD, HG
2. Note that there are 5 unknowns (1 reaction forces and the forces in 4 members)
3. Use truss equilibrium to solve for E_y and Method of Sections to solve for 3 forces in the members.
4. Use Method of Joints to solve for member GD

Free Body Diagram



Truss Equilibrium

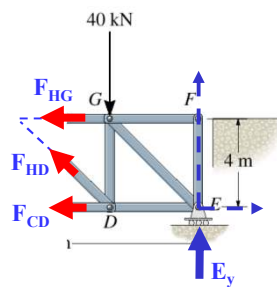
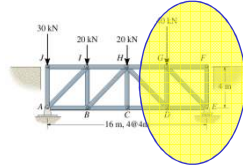
$$\begin{aligned}\sum M_A &= 0 \\ E_y(16) - 20(4) - 40(12) - 20(8) &= 0 \\ E_y &= 45 \text{ kN}\end{aligned}$$

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Structural Analysis

Sample Problem 6.6



Method of Sections

Moment about C

$$\begin{aligned}\sum M_D &= 0 \\ -F_{HG}(4) + E_y(4) &= 0 \\ F_{HG} &= 45 \text{ kN (tension)}\end{aligned}$$

Moment about H

$$\begin{aligned}\sum M_H &= 0 \\ E_y(8) - F_{CD}(4) - 40(4) &= 0 \\ F_{CD} &= 50 \text{ kN (tension)}\end{aligned}$$

Forces in y direction

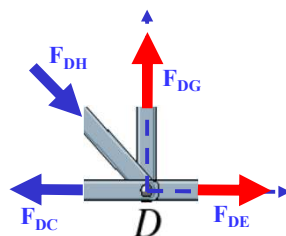
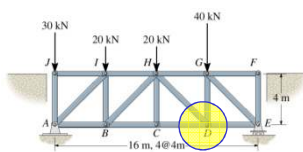
$$\begin{aligned}\sum F_y &= 0 \\ E_y - F_{HD} \cos 45^\circ - 40 &= 0 \\ F_{HD} &= -7.07 \text{ kN (compression)}\end{aligned}$$

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6 - ۶۳/84

Structural Analysis

Sample Problem 6.6



Free Body Diagram

Method of Joints

Joint D

$$\begin{aligned}\sum F_y &= 0 \\ F_{CD} - F_{HD} \cos 45^\circ &= 0 \\ F_{CD} &= 5 \text{ kN (tension)}\end{aligned}$$

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Structural Analysis

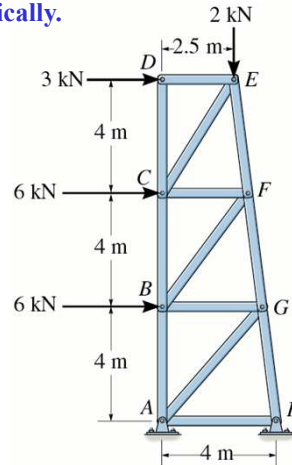
Sample Problem 6.7

The tower truss is subjected to the loads shown. Determine the force in the members BG and CF, and state if the members are in tension or compression. The left side ABCD stands vertically.

Given: truss and loads

Find: BG and CF

Solution:

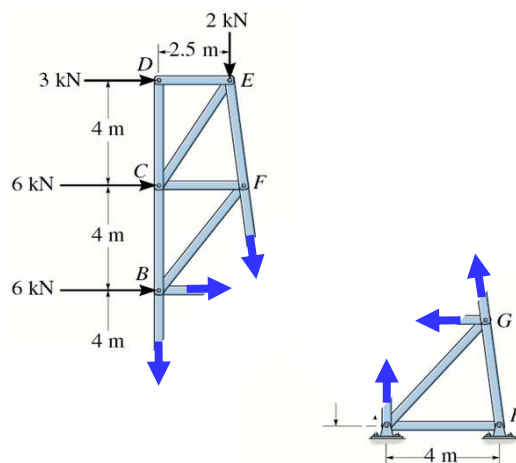


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Structural Analysis

Sample Problem 6.7

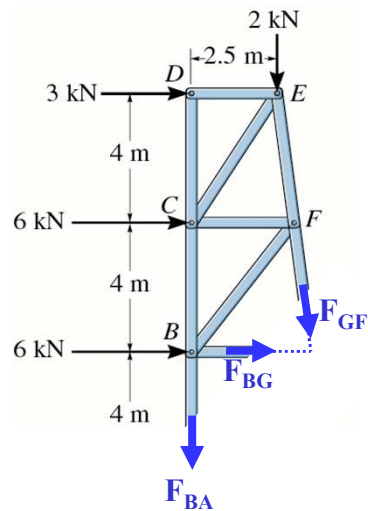


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Structural Analysis

Sample Problem 6.7

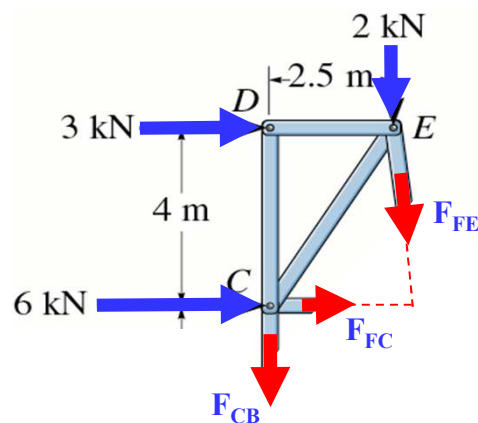
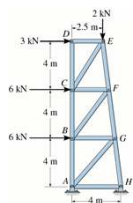


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Structural Analysis

Sample Problem 6.7

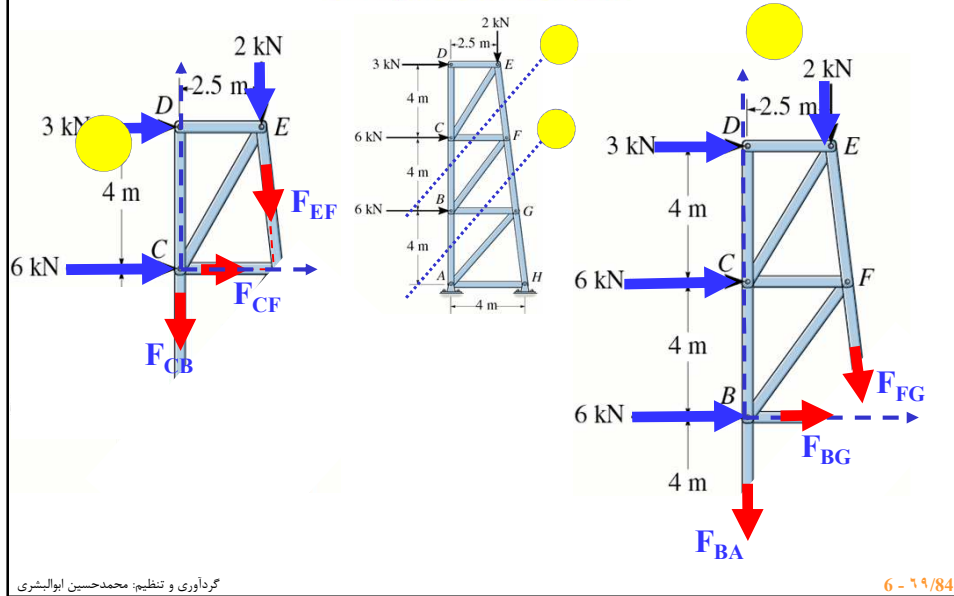


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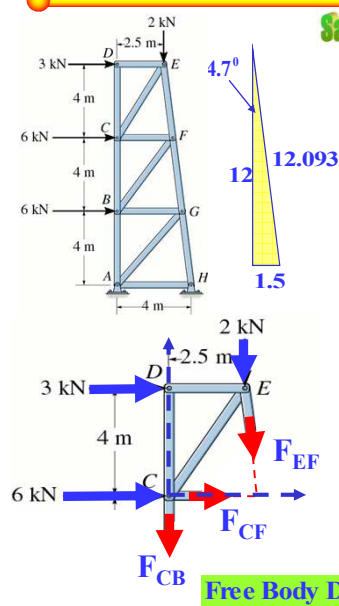
Structural Analysis

Sample Problem 6.7



Structural Analysis

Sample Problem 6.7 - Solution



1. Draw two lines to isolate the members of interest BG, CF
2. Note we only need to solve for two members (not all)
3. Using the method of sections, considering the top of the truss, will result in at most 3 unknowns for each cut.
4. The equilibrium equations for each cut are sufficient to find the desired forces.

$$\sum M_C = 0$$

Cut #1

$$-F_{FE} \left(\frac{12}{12.093} \right) (2.5) - F_{FE} \left(\frac{1.5}{12.093} \right) (4) - (3)(4) - (2)(2.5) = 0$$

$$F_{FE} = -5.71 \text{ kN (compression)}$$

$$\sum F_x = 0$$

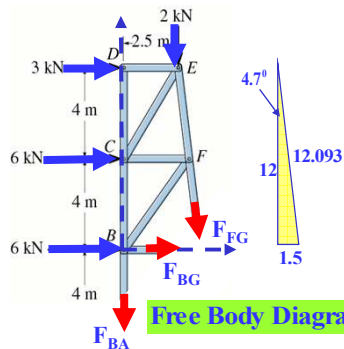
$$F_{FE} \left(\frac{1.5}{12.093} \right) + F_{CF} + 6 + 3 = 0$$

$$F_{CF} = -8.29 \text{ kN (compression)}$$

Structural Analysis

Sample Problem 6.7 - Solution

Cut #2



$$\sum M_B = 0$$

$$-F_{FG} \left(\frac{12}{12.093} \right) \left[2.5 + 4 \left(\frac{1.5}{12.093} \right) \right] - F_{FG} \left(\frac{1.5}{12.093} \right) 4 - 3(8) - 2(2.5) - 6(4) = 0$$

$$F_{FG} = -15.26 \text{ kN (compression)}$$

Free Body Diagram

$$\sum F_x = 0$$

$$F_{FG} \left(\frac{1.5}{12.093} \right) + F_{BG} + 6 + 6 + 3 = 0$$

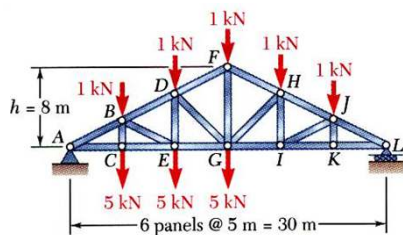
$$F_{BG} = -13.1 \text{ kN (compression)}$$

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6 - ۷۱/84

Vector Mechanics for Engineers: Statics

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L.
- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.
- Apply the conditions for static equilibrium to determine the desired member forces.

Determine the force in members FH, GH, and GI.

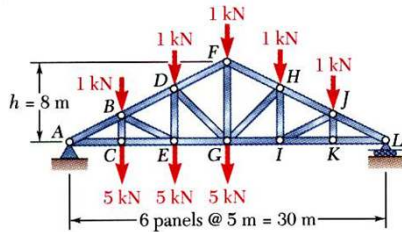
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Vector Mechanics for Engineers: Statics

Sample Problem 6.3



SOLUTION:

- Take the entire truss as a free body. Apply the conditions for static equilibrium to solve for the reactions at A and L .

$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN}) - (20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (25\text{ m})L$$

$$L = 7.5\text{ kN} \uparrow$$

$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

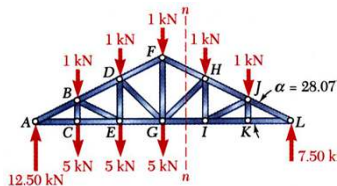
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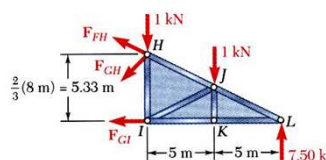
6 - ۷۳/84

Vector Mechanics for Engineers: Statics

Sample Problem 6.3



- Pass a section through members FH , GH , and GI and take the right-hand section as a free body.



- Apply the conditions for static equilibrium to determine the desired member forces.

$$\begin{aligned} \sum M_H &= 0 \\ (7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) &= 0 \\ F_{GI} &= +13.13\text{ kN} \end{aligned}$$

$$F_{GI} = 13.13\text{ kN } T$$

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Vector Mechanics for Engineers: Statics

Sample Problem 6.3

$$\tan \alpha = \frac{FG}{GL} = \frac{8 \text{ m}}{15 \text{ m}} = 0.5333 \quad \alpha = 28.07^\circ$$

$$\sum M_G = 0$$

$$(7.5 \text{ kN})(15 \text{ m}) - (1 \text{ kN})(10 \text{ m}) - (1 \text{ kN})(5 \text{ m}) + (F_{FH} \cos \alpha)(8 \text{ m}) = 0$$

$$F_{FH} = -13.82 \text{ kN}$$

$F_{FH} = 13.82 \text{ kN C}$

$$\tan \beta = \frac{GI}{HI} = \frac{5 \text{ m}}{\frac{2}{3}(8 \text{ m})} = 0.9375 \quad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

$$(1 \text{ kN})(10 \text{ m}) + (1 \text{ kN})(5 \text{ m}) + (F_{GH} \cos \beta)(10 \text{ m}) = 0$$

$$F_{GH} = -1.371 \text{ kN}$$

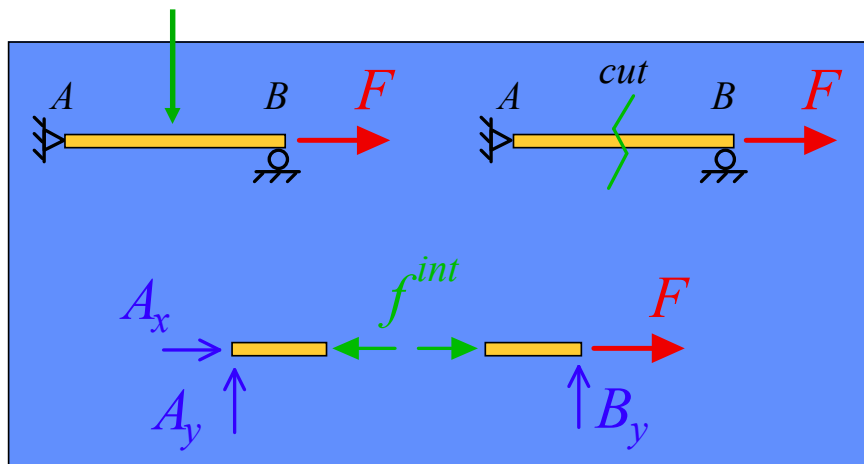
$F_{GH} = 1.371 \text{ kN C}$

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simple example:

*Say we want the
internal forces here*

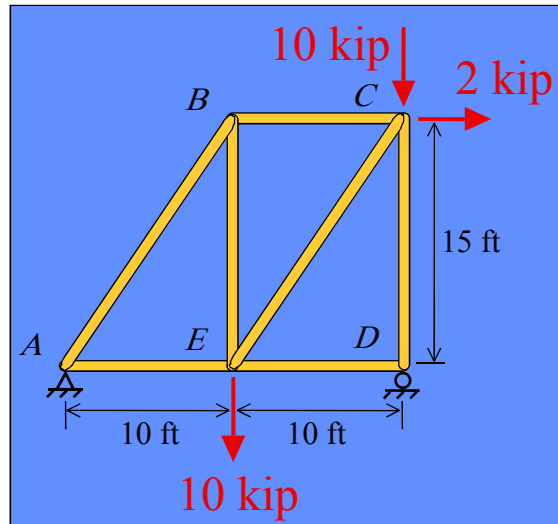


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Procedure for truss analysis:

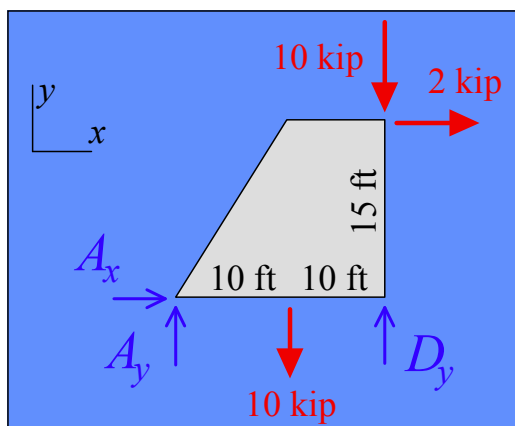
Example 1: Find the internal force in member BE .



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1) Find the support reactions.

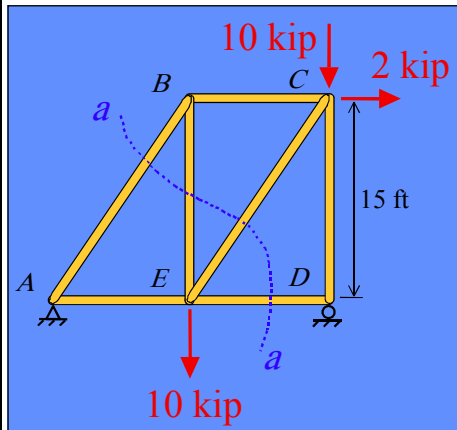


$$\left. \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_A = 0 \end{array} \right\} \rightarrow \begin{array}{l} A_x \\ A_y \\ D_y \end{array}$$

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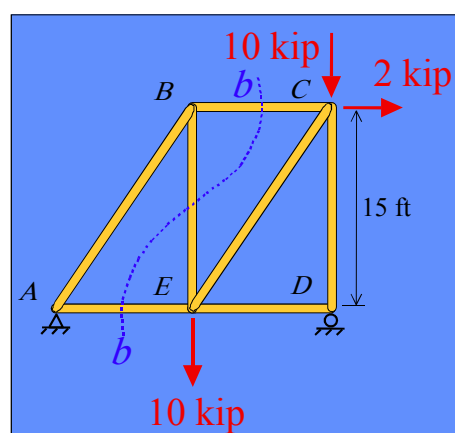
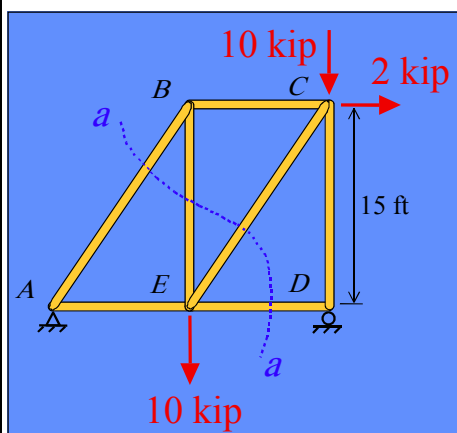
2) Make a cut that passes through the member(s) of interest. On FBD, introduce the unknown internal forces.



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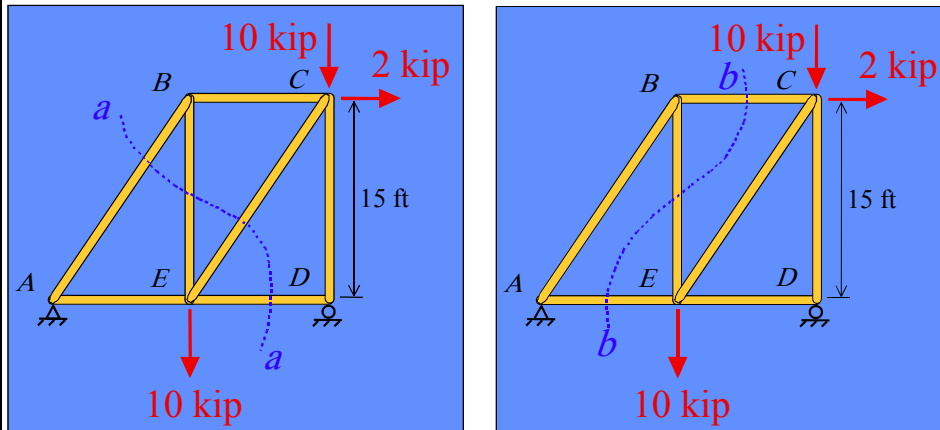
2) Make a cut that passes through the member(s) of interest. On FBD, introduce the unknown internal forces.



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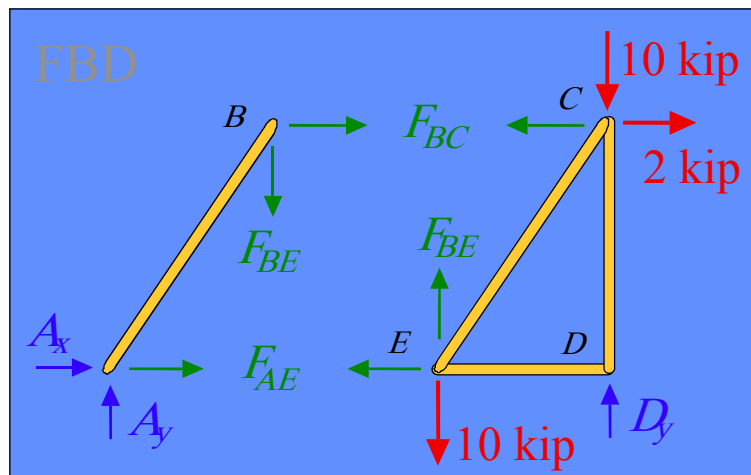
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2) Make a cut that passes through the member(s) of interest. On FBD, introduce the unknown internal forces.



Cut *aa* introduces four unknowns whereas cut *bb* introduces three. Since we have only three equilibrium equations, cut *bb* is the better choice.

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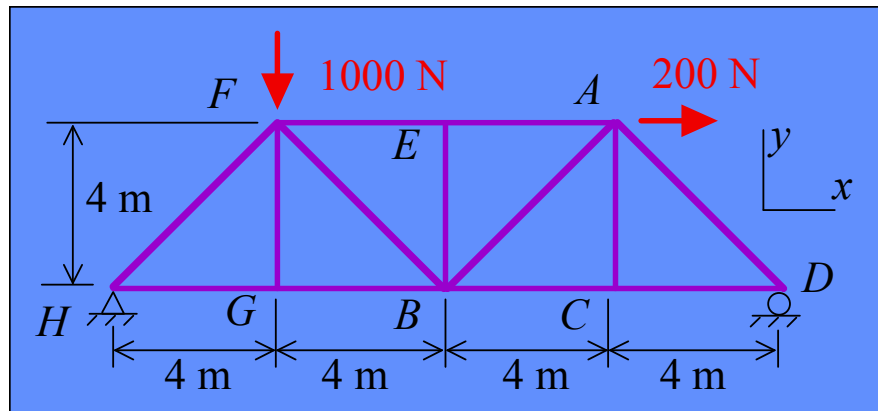


Using either
FBD, solve
equilibrium
equations

$$\left. \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_p &= 0 \end{aligned} \right\} \rightarrow \begin{aligned} F_{AE} \\ F_{BE} \\ F_{BC} \end{aligned}$$

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example: Using the method of sections, determine the internal force in member AB .



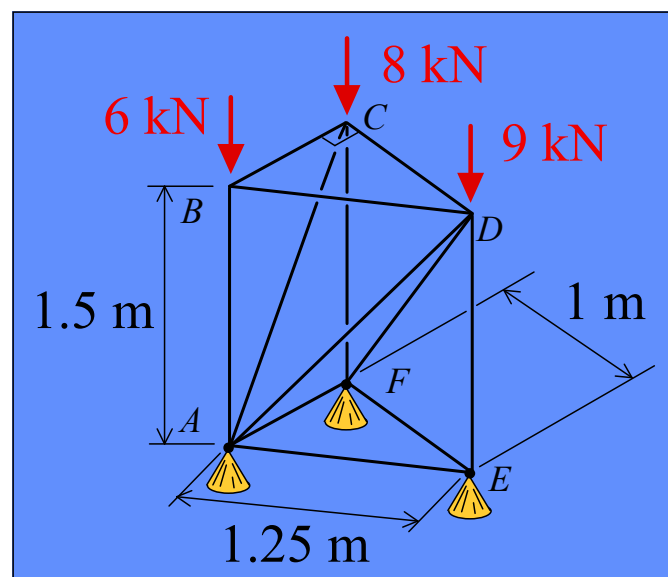
$$T_{AB} = 424 \text{ N (tension)}$$

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MF4

example: Determine the force in each member of the truss.



$$m=12, n=6$$

$$m+6=3n$$

But first, is the truss S.D., S.I., or a M?

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Slide 84

MF4

S.D. Statically Determined
S.I. Statically Indetermined
M. Mechanism

My Friend... :)

