

STATICS

Chapter 5 Equilibrium of a Rigid Body

Sample Problems

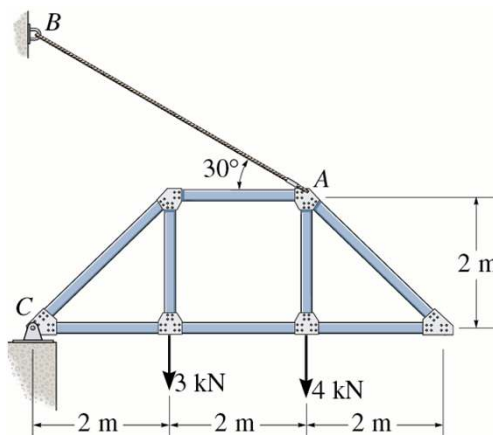
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Equilibrium of a Rigid Body

Sample Problem 5.1

A trussed bridge is pinned at C and supported by the cable AB. If it is loaded as shown, find the reaction forces at the pin and the tension in the cable AB.

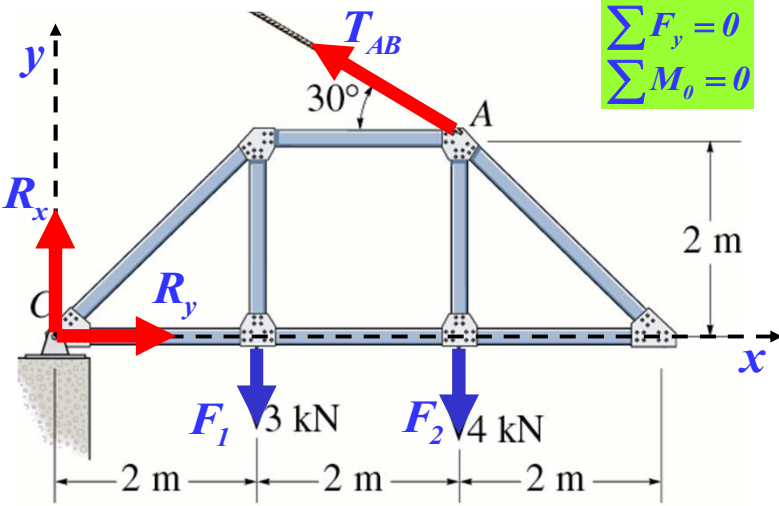
Solution:



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Equilibrium of a Rigid Body

Sample Problem 5.1



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_o = 0$$

2 m

2 m 2 m 2 m

F_1 3 kN F_2 4 kN

C A

R_x R_y

T_{AB} 30°

x y

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Equilibrium of a Rigid Body

Sample Problem 5.1 - Solution

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_o = 0$$

Free Body Diagram

Apply Equations of Equilibrium

To find T_{AB} use moment Equation

$$\sum M_C = 0$$

$$T_{AB} \cos 30^\circ (2) + T_{AB} \sin 30^\circ (4) - 3(2) - 4(4) = 0$$

$$T_{AB} = 5.89 \text{ kN}$$

To find R_x and R_y use force equations

$$\sum F_x = 0$$

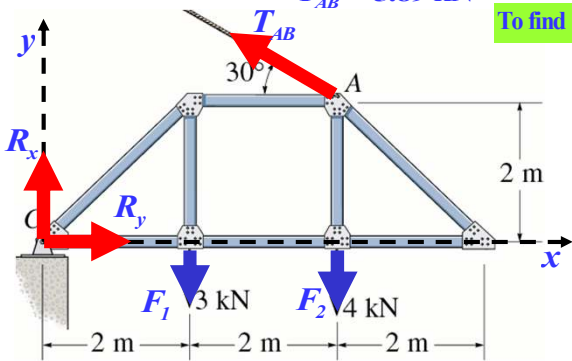
$$R_x - T_{AB} \cos 30^\circ = 0$$

$$R_x = 5.11 \text{ kN}$$

$$\sum F_y = 0$$

$$R_y + T_{AB} \sin 30^\circ - 3 - 4 = 0$$

$$R_y = 4.05 \text{ kN}$$



2 m

2 m 2 m 2 m

F_1 3 kN F_2 4 kN

C A

R_x R_y

T_{AB} 30°

x y

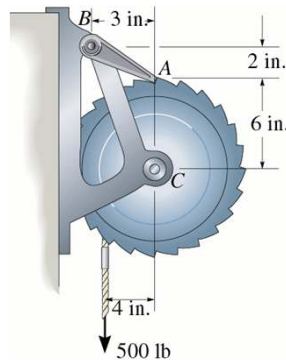
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Equilibrium of a Rigid Body

Sample Problem 5.2

The winch consists of the drum radius 4 in., which is pin connected at its center C. At its outer rim is a ratchet gear having a mean radius of 6 in. The pawl AB serves as a two-force member and holds the drum from rotating. If the suspended load is 500 lb, determine the horizontal and vertical components of the reaction at the pin C.

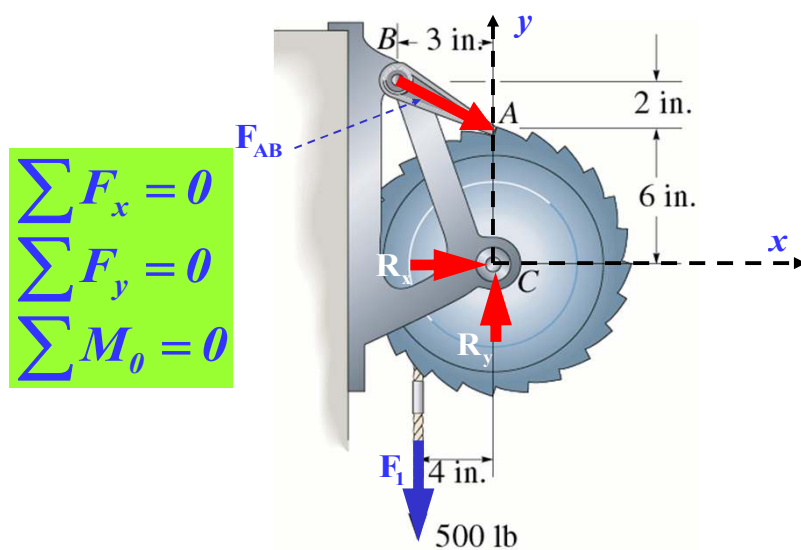
Solution:



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Equilibrium of a Rigid Body

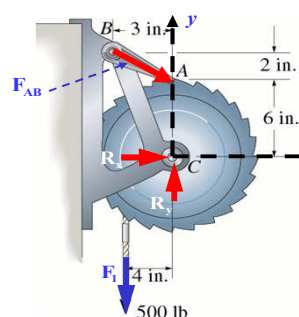
Sample Problem 5.2



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Equilibrium of a Rigid Body

Sample Problem 5.2 - Solution



Free Body Diagram

Apply Equations of Equilibrium

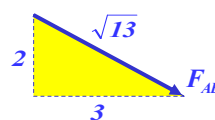
To find F_{AB} use moment equation about C

$$\sum M_C = 0$$

$$F_{AB} \left(\frac{3}{\sqrt{13}} \right) (2) - F_l (4) = 0$$

$$F_{AB} = \frac{(500)(4)\sqrt{13}}{(3)(6)}$$

$$F_{AB} = 401 \text{ lb}$$



To find R_x and R_y use force equations

$$\sum F_x = 0$$

$$-R_x + F_{AB} \left(\frac{3}{\sqrt{13}} \right) = 0$$

$$R_x = 333 \text{ lb}$$

$$\sum F_y = 0$$

$$-F_l + R_y - F_{AB} \left(\frac{2}{\sqrt{13}} \right) = 0$$

$$R_y = F_{AB} \left(\frac{2}{\sqrt{13}} \right) + F_l$$

$$R_y = 722 \text{ lb}$$

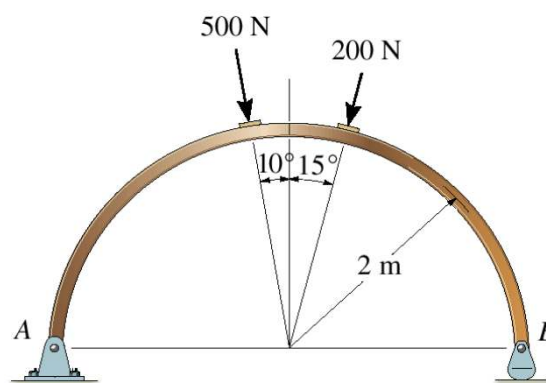
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Equilibrium of a Rigid Body

Sample Problem 5.3

Determine the horizontal and vertical components of force at the pin A and the reaction at the rocker B of the curved beam.

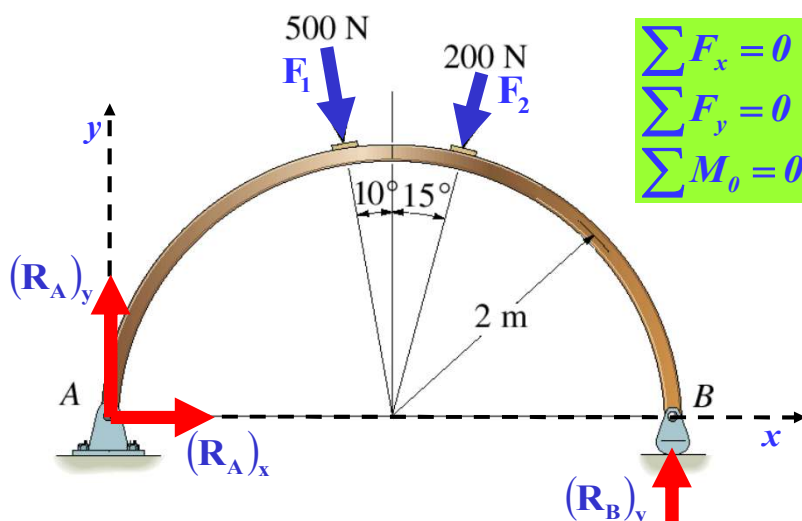
Solution:



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Equilibrium of a Rigid Body

Sample Problem 5.3



$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_o &= 0\end{aligned}$$

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Equilibrium of a Rigid Body

Sample Problem 5.3 - Solution

Free Body Diagram Apply Equations of Equilibrium (3 - unknowns, 3 eqs.)

find $(R_B)_y$ using moment equation about A

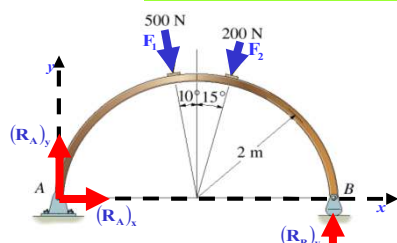
$$\sum M_A = 0$$

$$(R_B)_y(4) - F_2 \cos 15^\circ(2) - F_1 \cos 10^\circ(2) = 0$$

$$(R_B)_y = \frac{1}{4}[(200)\cos 15^\circ(2) + (500)\cos 10^\circ(2)]$$

$$(R_B)_y = 342.8 \text{ N}$$

R_{Ax} acts opposite of the assumed direction



find reactions $(R_A)_x$ and $(R_A)_y$ using force equilibrium

$$\sum F_x = 0$$

$$+R_{Ax} + F_1 \sin 10^\circ - F_2 \sin 15^\circ = 0$$

$$R_{Ax} = +F_2 \sin 15^\circ - F_1 \sin 10^\circ$$

$$R_{Ax} = (200)\sin 15^\circ - (500)\sin 10^\circ$$

$$R_{Ax} = -35.1 \text{ N}$$

$$\sum F_y = 0$$

$$R_{Ay} - F_1 \cos 10^\circ - F_2 \cos 15^\circ + R_{By} = 0$$

$$R_{Ay} = F_1 \cos 10^\circ + F_2 \cos 15^\circ - R_{By}$$

$$R_{Ay} = (500)\cos 10^\circ + (200)\cos 15^\circ - 342.8$$

$$R_{Ay} = 342.8 \text{ N}$$

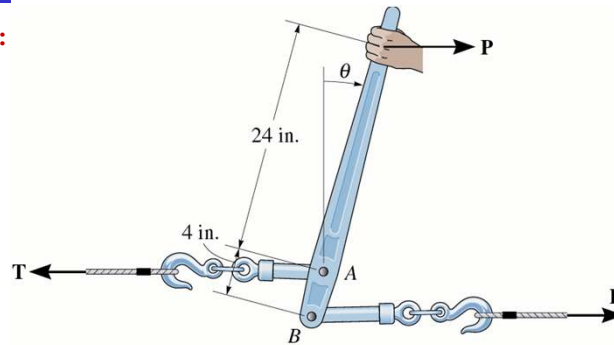
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Equilibrium of a Rigid Body

Sample Problem 5.4

A horizontal force $F=300$ lb is applied to the cable when the level is at $\theta=0^\circ$. If the force increases linearly to $F=650$ lb when $\theta=60^\circ$, determine the horizontal force P that must be applied to the handle as a function of θ . Also, determine the force T applied to the cable at A . Plot the forces for $0^\circ \leq \theta \leq 60^\circ$.

Solution:

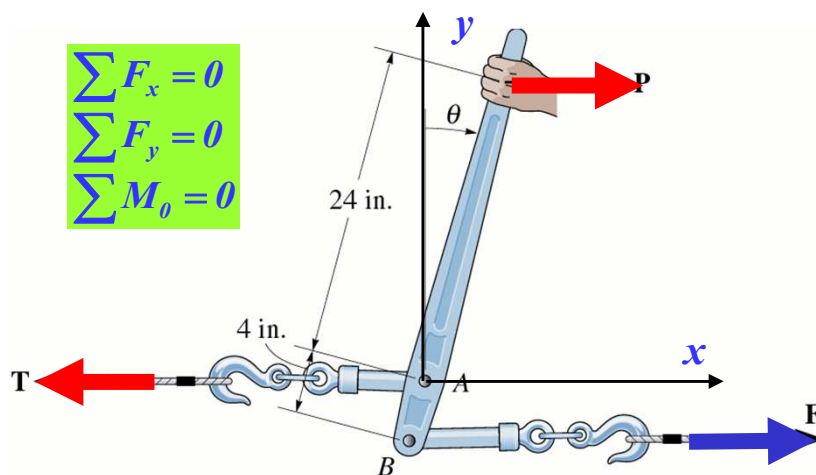


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Equilibrium of a Rigid Body

Sample Problem 5.4

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_o &= 0\end{aligned}$$



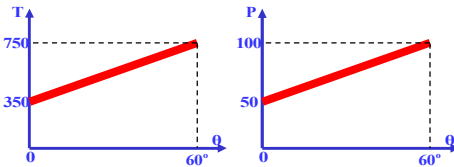
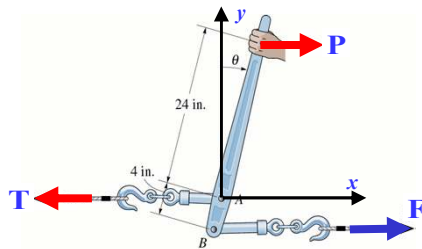
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Equilibrium of a Rigid Body

Sample Problem 5.4 - Solution

Free Body Diagram

find P using moment equation about A



$$\sum M_A = 0$$

$$-P(24 \cos 15^\circ) + F(4 \cos \theta) = 0$$

$$P = .1667 F$$

when $\theta = 0^\circ$ $F = 300$ lb

$\theta = 60^\circ$ $F = 650$ lb

since the change is linear

$$F = \frac{650-300}{60-0} \theta + 300 \text{ \{lb\}}$$

$$F = 5.833\theta + 300 \text{ \{lb\}}$$

then

$$P = .1667 F = .1667(5.833\theta + 300) \text{ \{lb\}}$$

$$P = .972\theta + 50 \text{ \{lb\}}$$

sum forces in x to find T

$$\sum F_x = 0$$

$$-T + P + F = 0$$

$$T = P + F$$

$$T = (.972\theta + 50) + (5.833\theta + 300)$$

$$T = 6.81\theta + 350$$

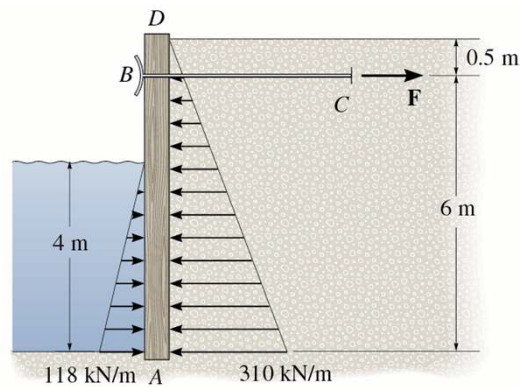
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Equilibrium of a Rigid Body

Sample Problem 5.5

The bulkhead is subjected to water and soil backfill pressures. Assuming that AD is pinned to the ground at A, Determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulkhead has a mass of 800 kg.

Solution:

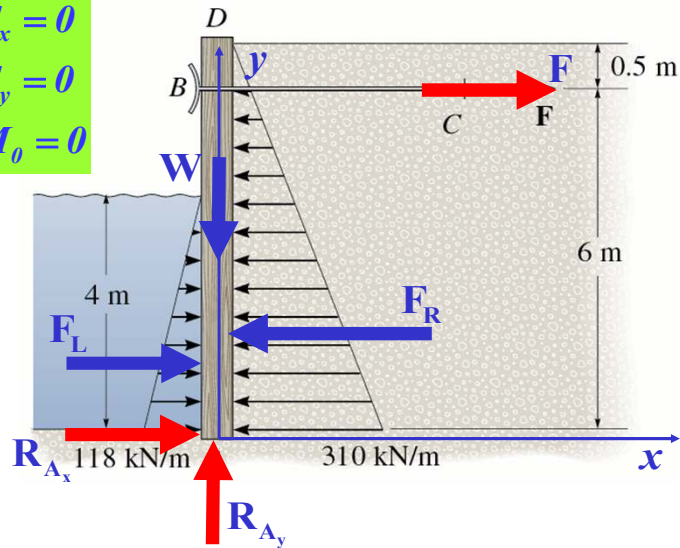


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Equilibrium of a Rigid Body

Sample Problem 5.5

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_O &= 0\end{aligned}$$

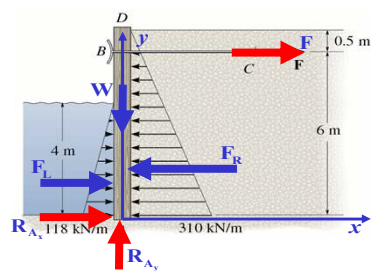


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Equilibrium of a Rigid Body

Sample Problem 5.6 - Solution

Free Body Diagram



find W , F_R , and F_L

$$W = mg = (9.81)(800) = 7.848 \text{ kN}$$

$$F_R = \frac{1}{2}(310)(6) = 930 \text{ kN}$$

$$F_L = \frac{1}{2}(118)(4) = 236 \text{ kN}$$

use moments about A to find F

$$\sum M_A = 0$$

$$F_R \left(\frac{1}{3} \right) (6) - F_L \left(\frac{1}{3} \right) (4) - F(6) = 0$$

$$(930)(2) - (236)(1.333) - F(6) = 0$$

$$F = 311.4 \text{ kN}$$

sum forces in x and y to find reaction forces

$$\sum F_x = 0$$

$$R_{Ax} + F_L - F_R + F = 0$$

$$R_{Ax} = F_R - F_L - F$$

$$R_{Ax} = 930 - 236 - 311.4$$

$$R_{Ax} = 460 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} - W = 0$$

$$R_{Ay} = W = 7.848 \text{ kN}$$

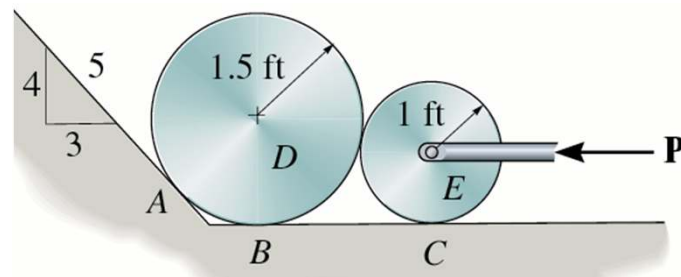
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Equilibrium of a Rigid Body

Sample Problem 5.6

Disks D and E have the weight of 200 and 100 lb, respectively. If a horizontal force of $P=200$ lb is applied to the center of the disk E, determine the normal reactions at the points of contact with the ground at A, B, C.

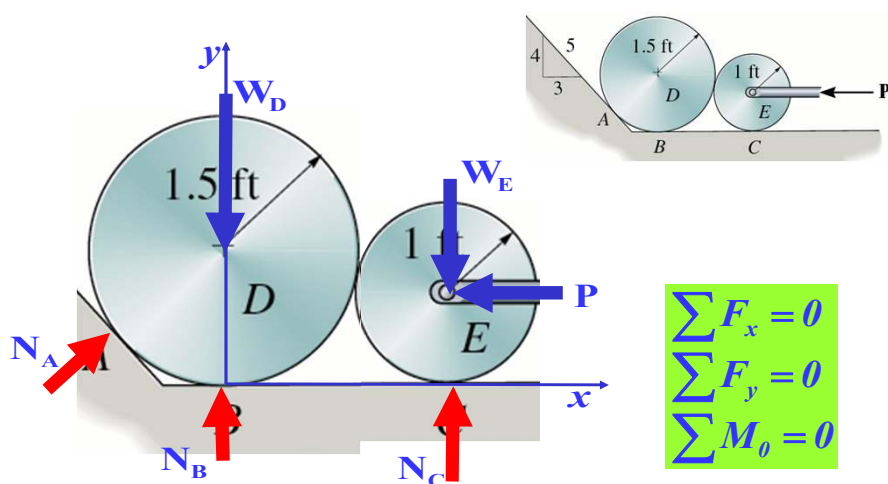
Solution:



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Equilibrium of a Rigid Body

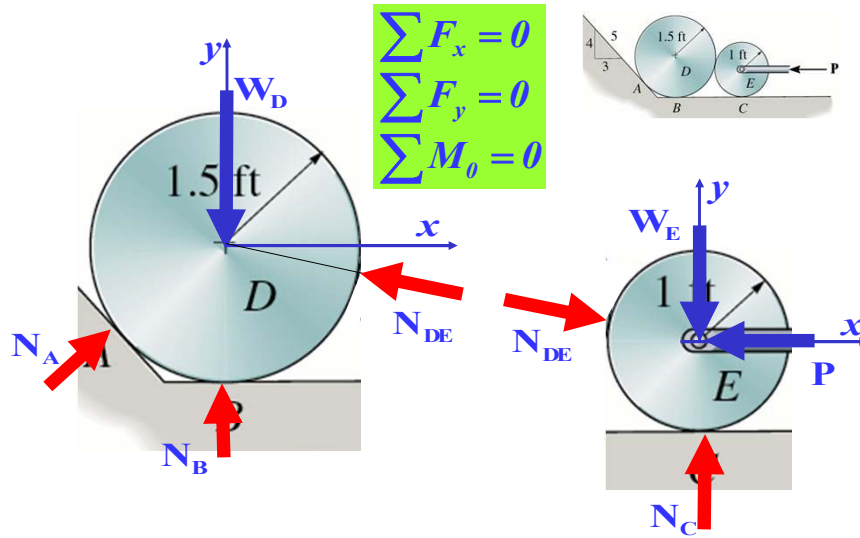
Sample Problem 5.6



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Equilibrium of a Rigid Body

Sample Problem 5.6



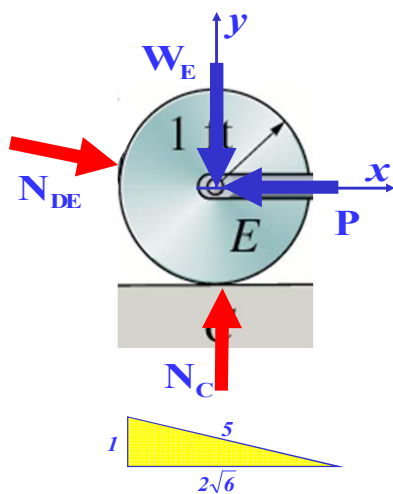
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Equilibrium of a Rigid Body

Sample Problem 5.6 - Solution

Free Body Diagram

sum forces in x and y to find reaction forces disk E



$$\sum F_x = 0$$

$$P - N_{DE} \left(\frac{2\sqrt{6}}{5} \right) = 0$$

$$N_{DE} = \left(\frac{5}{2\sqrt{6}} \right) P$$

$$\sum F_y = 0$$

$$N_C - W_E - N_{DE} \left(\frac{1}{5} \right) = 0$$

$$N_C = W_E + N_{DE} \left(\frac{1}{5} \right)$$

$$N_C = W_E + \left(\frac{5}{2\sqrt{6}} \right) \left(\frac{1}{5} \right) P$$

$$N_C = 100 + \left(\frac{1}{2\sqrt{6}} \right) (200)$$

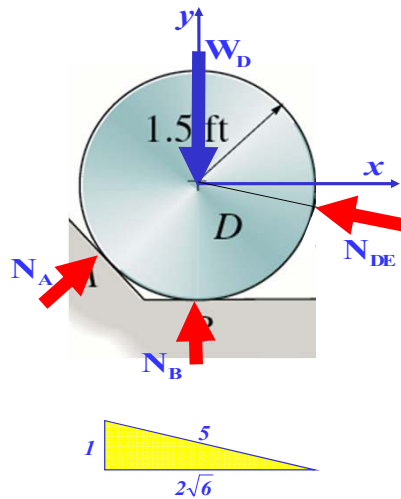
$$N_C = 141 \text{ lb}$$

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Equilibrium of a Rigid Body

Sample Problem 5.6 - Solution

Free Body Diagram



sum forces in x and y to find reaction forces disk D

$$\sum F_x = 0$$

$$N_A \left(\frac{4}{5} \right) - N_{DE} \left(\frac{2\sqrt{6}}{5} \right) = 0$$

$$N_A = \left(\frac{2\sqrt{6}}{4} \right) N_{DE} = \left(\frac{\sqrt{6}}{2} \right) \left(\frac{5}{2\sqrt{6}} \right) P$$

$$N_A = \left(\frac{5}{4} \right) P = 250 \text{ lb}$$

$$\sum F_y = 0$$

$$N_A \frac{3}{5} - N_B - W_D + N_{DE} \left(\frac{1}{5} \right) = 0$$

$$N_B = W_D - N_A \frac{3}{5} - \left(\frac{5}{2\sqrt{6}} \right) P \left(\frac{1}{5} \right)$$

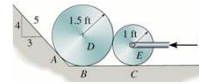
$$N_B = W_D - \left(\frac{5}{4} \right) P \left(\frac{3}{5} \right) - \left(\frac{1}{2\sqrt{6}} \right) P$$

$$N_B = W_D - \left[\left(\frac{3}{4} \right) + \left(\frac{1}{2\sqrt{6}} \right) \right] P$$

$$N_B = 200 - \left[\left(\frac{3}{4} \right) + \left(\frac{1}{2\sqrt{6}} \right) \right] (200)$$

$$N_B = 9.18 \text{ lb}$$

$$N_{DE} = 204.1 \text{ lb}$$



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Equilibrium of a Rigid Body

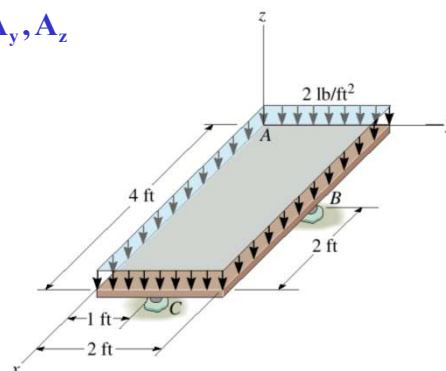
Sample Problem 5.7

Determine the x,y,z components of reaction at the ball supports B and C and the ball and socket A not shown for the uniformly loaded plate.

Given : ω , and geometry

Find : A_x, A_y, A_z

Solution :



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Equilibrium of a Rigid Body

Sample Problem 5.7

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0$$

$$\sum M_y = 0$$

$$\sum M_z = 0$$

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Equilibrium of a Rigid Body

Sample Problem 5.7 - Solution

$W = \omega A = (2)(2)(4) = 16 \text{ lb}$

Forces :

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = A_z + B_z + C_z - W = 0$$

$$A_z + B_z + C_z = W$$

Moments :

$$\sum M_{A_x} = B_z(2) + C_z(1) - W(1) = 0$$

$$2B_z + C_z = W$$

$$\sum M_{A_y} = -B_z(2) - C_z(4) + W(2) = 0$$

$$2B_z + 4C_z = 2W$$

$$B_z + 2C_z = W$$

$$\sum M_{A_z} = 0$$

Solving for A_z, B_z, C_z

$$A_z = B_z = C_z = 5.31$$

Free Body Diagram

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Equilibrium of a Rigid Body

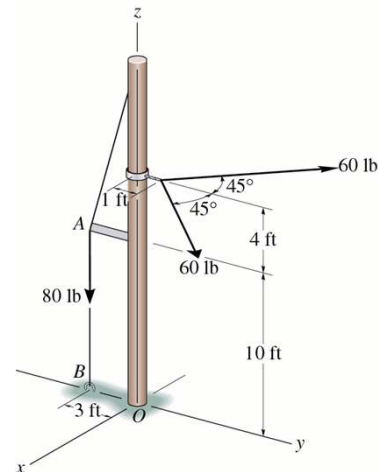
Sample Problem 5.8

The pole for a power line is subjected to the two cable forces of 60 lb., each force lying in a plane parallel to the x-y plane. If the tension in the guy wire AB is 80 lb, determine the x,y,z components of reaction at the fixed base of the pole, O.

Given: T_1, T_2, T_{AB} , powerline pole

Find: $R_x, R_y, R_z, M_{R_x}, M_{R_y}, M_{R_z}$

Solution:

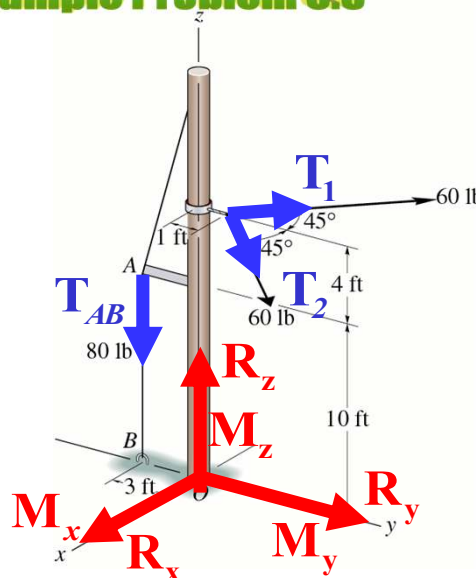


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Equilibrium of a Rigid Body

Sample Problem 5.8

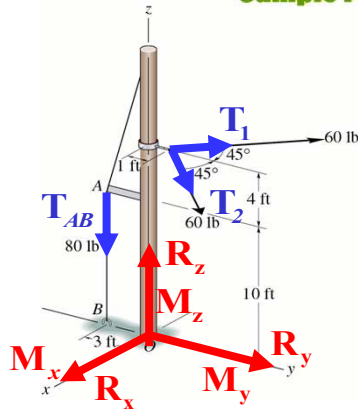
$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \\ \sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0\end{aligned}$$



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Equilibrium of a Rigid Body

Sample Problem 5.8 - Solution



Free Body Diagram

Solving force equations

$$\sum F_x = 0 \quad (\text{with } T_1 = T_2 = 60 \text{ lb})$$

$$T_1 \sin 45^\circ - T_2 \sin 45^\circ + R_x = 0$$

$$R_x = 0$$

$$\sum F_y = 0$$

$$T_1 \cos 45^\circ + T_2 \cos 45^\circ + R_y = 0$$

$$R_y = -2T_1 \cos 45^\circ = -(2)(60)\left(\frac{1}{\sqrt{2}}\right)$$

$$R_y = -84.9 \text{ lb}$$

$$\sum F_z = 0$$

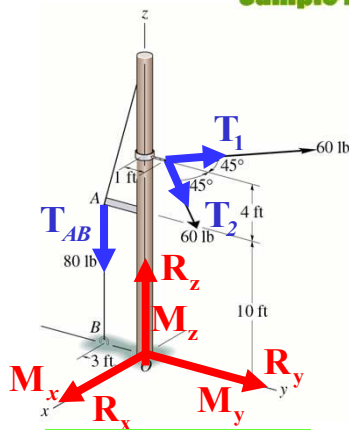
$$-T_{AB} + R_z = 0$$

$$R_z = 80 \text{ lb}$$

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Equilibrium of a Rigid Body

Sample Problem 5.8 - Solution



Free Body Diagram

Solving moment equations

$$\sum M_{O_x} = 0$$

$$T_{AB}(3) - T \cos 45^\circ (14) - T \cos 45^\circ (14) + M_{R_x} = 0$$

$$M_{R_x} = 2T \cos 45^\circ (14) - T_{AB}(3)$$

$$M_{R_x} = 2(60) \cos 45^\circ (14) - (80)(3)$$

$$M_{R_x} = 948 \text{ lb-ft}$$

$$\sum M_{O_y} = 0$$

$$T \sin 45^\circ (14) - T \sin 45^\circ (14) + M_{R_y} = 0$$

$$M_{R_y} = 0$$

$$\sum M_{O_z} = 0$$

$$T \sin 45^\circ (1) - T \sin 45^\circ (1) + M_{R_z} = 0$$

$$M_{R_z} = 0$$

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Equilibrium of a Rigid Body

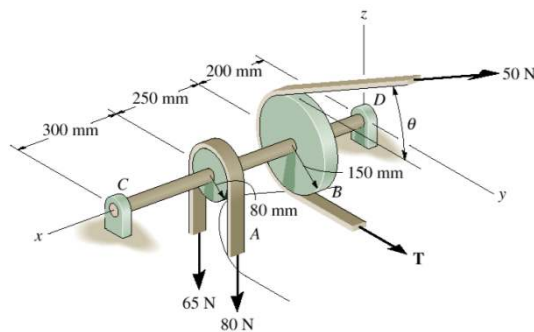
Sample Problem 5.9

Both pulleys are fixed to the shaft and as the shaft turns with a constant angular velocity, the power of the pulley A is transmitted to pulley B. Determine the horizontal tension T in the belt of the pulley B and the x, y, z components of reaction at the journal bearing C and the thrust bearing D if $\theta = 45^\circ$. The bearings are in proper alignment and exert only force reactions on the shaft.

Given: Belt-pulley, T_u, P_L, P_R

Find: $T, C_x, C_y, C_z, D_x, D_y, D_z$

Solution:

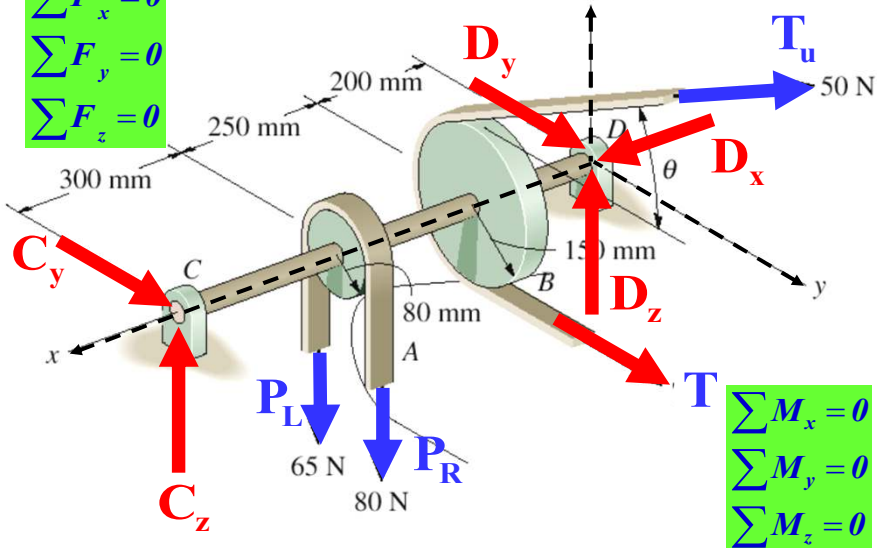


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Equilibrium of a Rigid Body

Sample Problem 5.9

$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0\end{aligned}$$



$$\begin{aligned}\sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0\end{aligned}$$

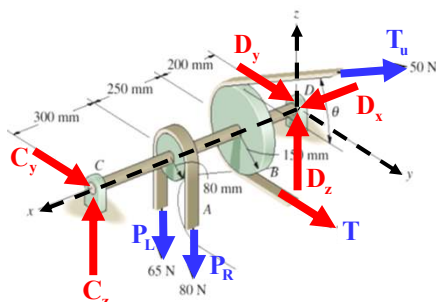
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Equilibrium of a Rigid Body

Sample Problem 5.9 - Solution

Solving force equations

$$\begin{aligned}\sum F_x &= 0 \quad (\text{properly aligned}) \\ D_x &= 0 \\ \sum F_y &= 0 & \sum F_z &= 0 \\ D_y + C_y + T_u \cos 45^\circ + T &= 0 & D_z + C_z + T_u \sin 45^\circ - P_L - P_R &= 0 \\ D_y + C_y + T &= -T_u \cos 45^\circ & D_z + C_z &= P_L + P_R - T_u \sin 45^\circ\end{aligned}$$



Free Body Diagram

Solving moment equations

$$\begin{aligned}\sum M_{D_x} &= 0 \\ P_L(.08) - P_R(.08) - T_u(.15) + T(.15) &= 0 \\ T &= [P_R(.08) + T_u(.15) - P_L(.08)] / (.15) \\ T &= 58 \text{ N} \\ \sum M_{D_y} &= 0 \\ -C_z(.75) + (P_L + P_R)(.45) - T_u \sin 45^\circ(.2) &= 0 \\ C_z &= [(P_L + P_R)(.45) - T_u \sin 45^\circ(.2)] / (.75) \\ C_z &= 77.6 \text{ N} \\ \sum M_{D_z} &= 0 \\ C_y(.75) + (T + T_u \cos 45^\circ)(.2) &= 0 \\ C_y &= -[(T + T_u \cos 45^\circ)(.2)] / (.75) \\ C_y &= -24.9 \text{ N}\end{aligned}$$

Solving for D_x and D_y

$$\begin{aligned}D_z &= P_L + P_R - T_u \sin 45^\circ - C_z \\ D_z &= 32.05 \text{ N} \\ D_y &= -T_u \cos 45^\circ - C_y - T \\ D_y &= -68.45 \text{ N}\end{aligned}$$

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Equilibrium of a Rigid Body

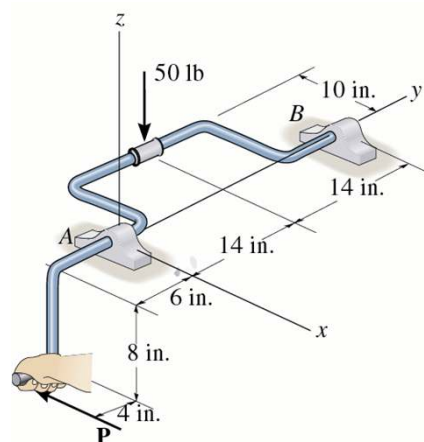
Sample Problem 5.10

A vertical force of 50 lb acts on the crankshaft. Determine the horizontal equilibrium force P that must be applied to the handle and the x,y,z components of reaction at the journal bearing A and thrust bearing B. The bearing are properly aligned and exert only force reactions on the shaft.

Given: T, crankshaft

Find: A_x, A_y, A_z, B_x, B_y, B_z

Solution:

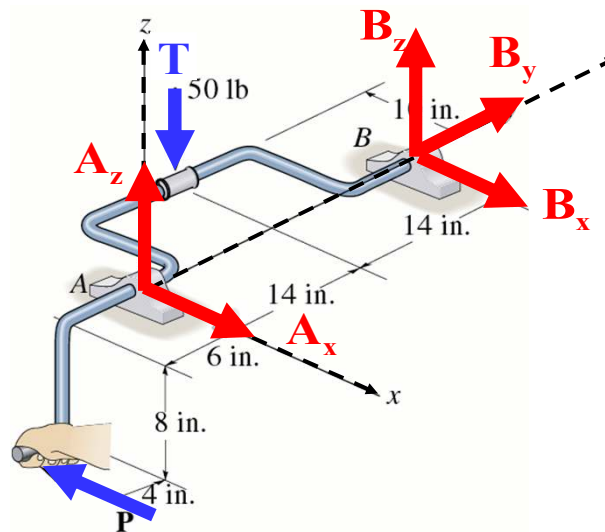


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Equilibrium of a Rigid Body

Sample Problem 5.10

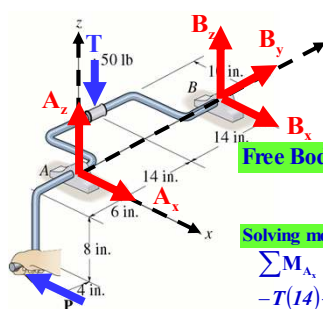
$$\begin{aligned}\sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \\ \sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0\end{aligned}$$



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Equilibrium of a Rigid Body

Sample Problem 5.10 - Solution



Solving force equations

$$\begin{aligned}\sum F_x &= 0 \\ R_{Ax} + R_{Bx} - P &= 0 \\ R_{Ax} &= P - R_{Bx} \\ \sum F_y &= 0 \quad (\text{properly aligned}) \\ R_{By} &= 0 \\ \sum F_z &= 0 \\ R_{Az} + R_{Bz} - T &= 0 \\ R_{Az} + R_{Bz} &= T = 50\end{aligned}$$

Free Body Diagram

Solving moment equations

$$\begin{aligned}\sum M_{A_x} &= 0 \\ -T(14) + R_{B_z}(28) &= 0 \\ R_{B_z} &= T(14)/(28) = 25 \text{ lb} \\ \sum M_{A_y} &= 0 \\ -T(10) + P(8) &= 0 \\ P &= T(10)/(8) = 62.5 \text{ lb} \\ \sum M_{A_z} &= 0 \\ -P(10) - R_{B_x}(28) &= 0 \\ R_{B_x} &= P(10)/(28) = -22.3 \text{ lb}\end{aligned}$$

Solving R_{A_x} and R_{A_z}

$$\begin{aligned}R_{A_x} &= P - R_{B_x} = 62.0 - (-22.3) \\ R_{A_x} &= 84.8 \text{ lb} \\ R_{A_z} &= T - R_{B_z} = 50.0 - 25 \\ R_{A_z} &= 25 \text{ lb}\end{aligned}$$

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Equilibrium of a Rigid Body

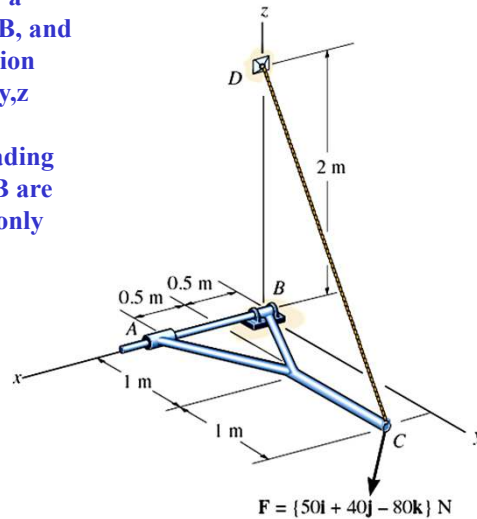
Sample Problem 5.11

The forked rod is supported by a collar at A, a thrust bearing at B, and a cable CD. Determine the tension with in the cable CD and the x, y, z components of reaction at the supports A and B due to the loading shown. The supports at A and B are in proper alignment and exert only force reactions on the rod.

Given: F, forked rod

Find: $A_x, A_y, A_z, B_x, B_y, B_z$

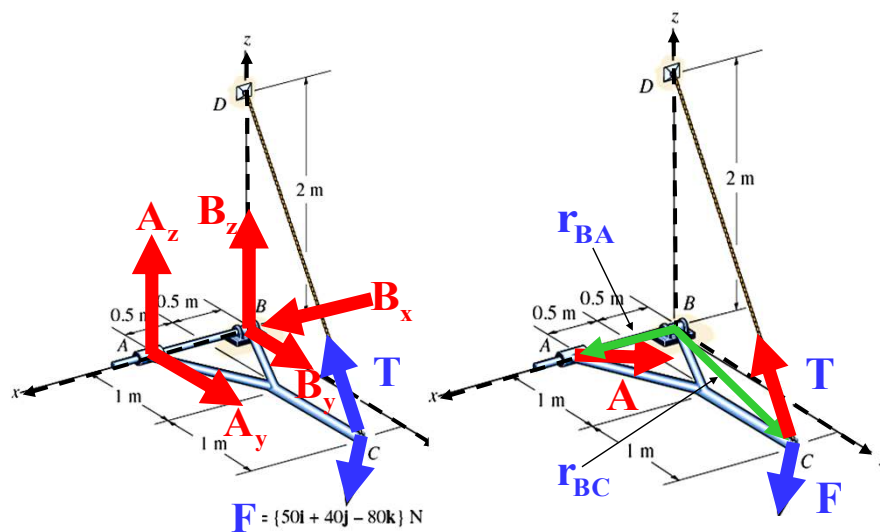
Solution:



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Equilibrium of a Rigid Body

Sample Problem 5.11



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Equilibrium of a Rigid Body

Sample Problem 5.11 - Solution

define force and position vectors

$$\sum \mathbf{F} = \mathbf{0}$$

$$\mathbf{F} = \{50\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}$$

$$\mathbf{T} = |\mathbf{T}|\mathbf{u}_T$$

$$\mathbf{u}_T = \frac{(0 - .5)\mathbf{i} + (0 - 2)\mathbf{j} + (2 - 0)\mathbf{k}}{\sqrt{(0 - .5)^2 + (0 - 2)^2 + (2 - 0)^2}}$$

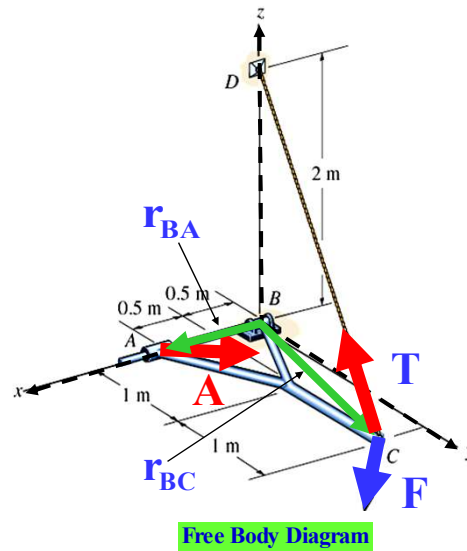
$$\mathbf{u}_T = -.174\mathbf{i} - .696\mathbf{j} + .696\mathbf{k}$$

$$\mathbf{T} = (-.174|\mathbf{T}| - .696|\mathbf{T}| + .696|\mathbf{T}|)\mathbf{T}$$

$$\mathbf{r}_{BC} = .5\mathbf{i} + 2\mathbf{j} \quad \mathbf{r}_{BA} = \mathbf{i}$$

$$\mathbf{R}_A = R_{A_x}\mathbf{j} + R_{A_z}\mathbf{k}$$

$$\mathbf{R}_B = R_{B_x}\mathbf{i} + R_{B_y}\mathbf{j} + R_{B_z}\mathbf{k}$$



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Equilibrium of a Rigid Body

Sample Problem 5.11 - Solution

equilibrium equations in vector form

$$\sum \mathbf{F} = \mathbf{0}$$

$$\sum \mathbf{F} = \mathbf{0}$$

$$\mathbf{R}_A + \mathbf{R}_B + \mathbf{F} + \mathbf{T} = \mathbf{0}$$

$$\{R_{A_x}\mathbf{j} + R_{A_z}\mathbf{k}\} + \{R_{B_x}\mathbf{i} + R_{B_y}\mathbf{j} + R_{B_z}\mathbf{k}\} + \{50\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\} + \{(-.174\mathbf{i} - .696\mathbf{j} + .696\mathbf{k})|\mathbf{T}|\} = \mathbf{0}$$

$$\{R_{B_x} + 50 - .174|\mathbf{T}|\}\mathbf{i} + \{R_{A_y} + R_{B_y} + 40 - .696|\mathbf{T}|\}\mathbf{j} + \{R_{A_z} + R_{B_z} - 80 + .696|\mathbf{T}|\}\mathbf{k} = \mathbf{0}$$

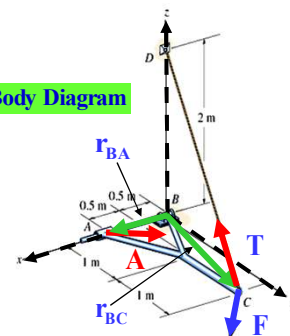
component equations are

$$R_{B_x} + 50 - .174|\mathbf{T}| = 0$$

$$R_{A_y} + R_{B_y} + 40 - .696|\mathbf{T}| = 0$$

$$R_{A_z} + R_{B_z} - 80 + .696|\mathbf{T}| = 0$$

Free Body Diagram



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Equilibrium of a Rigid Body

Sample Problem 5.11 - Solution

moment equilibrium equations in vector form

$$\sum \mathbf{M}_B = \sum \mathbf{r} \times \mathbf{F} = \mathbf{0}$$

$$(\mathbf{r}_{BA} \times \mathbf{R}_A) + (\mathbf{r}_{BC} \times \mathbf{F}) + (\mathbf{r}_{BD} \times \mathbf{T}) = \mathbf{0}$$

$$(\mathbf{r}_{BA} \times \mathbf{R}_A) + \mathbf{r}_{BC} \times (\mathbf{F} + \mathbf{T}) = \mathbf{0}$$

$$(\mathbf{i}) \times \{R_{A_x} \mathbf{j} + R_{A_z} \mathbf{k}\} + (.5\mathbf{i} + 2\mathbf{j}) \times \{(50 - .174|T|)\mathbf{i} + (40 - .696|T|)\mathbf{j} + (.696|T| - 80)\mathbf{k}\} = \mathbf{0}$$

$$\{(1.393|T| - 160)\mathbf{i} - (.348|T| - 40 + R_{A_z})\mathbf{j} + (R_{A_y} - 80)\mathbf{k}\}$$

the component moment equations are :

$$1.393|T| - 160 = 0$$

$$.348|T| - 40 + R_{A_z} = 0$$

$$R_{A_y} - 80 = 0$$

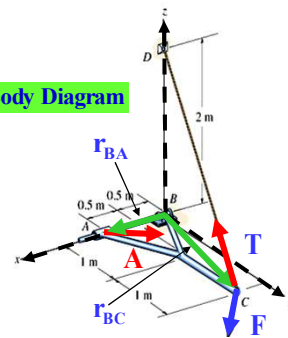
solving six equations for six unknowns :

$$T = 115 \text{ N} \quad R_{B_x} = -30 \text{ N}$$

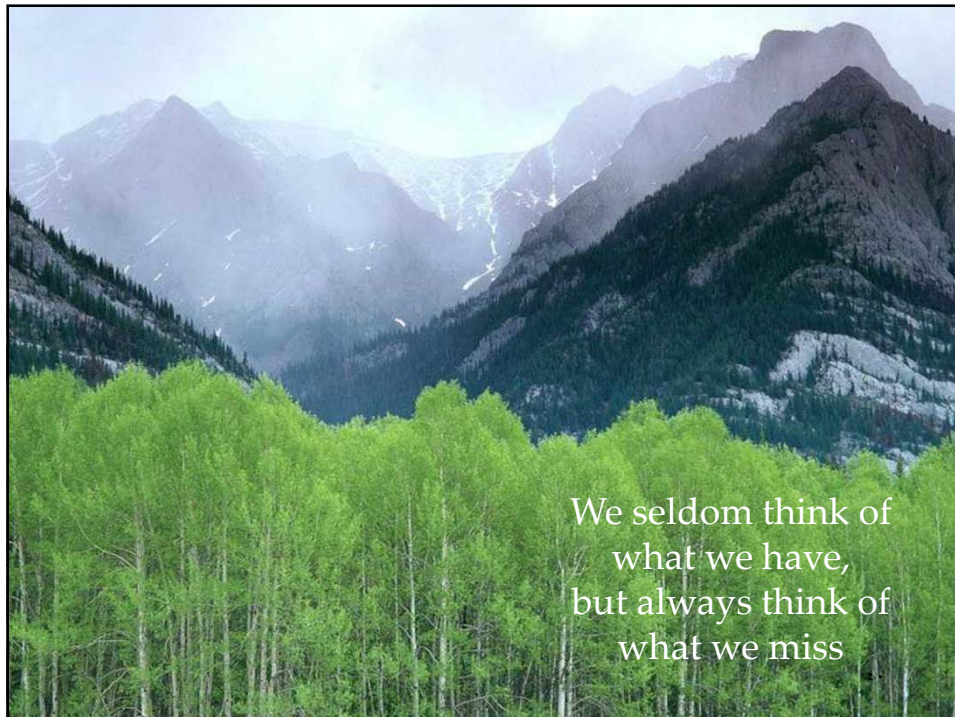
$$R_{A_y} = 80 \text{ N} \quad R_{B_y} = -40 \text{ N}$$

$$R_{A_z} = 0 \quad R_{B_z} = 0$$

Free Body Diagram



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We seldom think of
what we have,
but always think of
what we miss