Engineering Mathematics

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Introduction

What do we mean by engineering mathematics?

Engineering mathematics: is a branch of applied mathematics that applies mathematical theory to complex real-world problems, combing theoretical concepts with practical engineering challenges to solve today's technological issues.

Any Example?

…….

References

- Advanced Engineering Mathematics , E. Kreyszig
- Advanced Engineering Mathematics, C. R. Wylie
- Complex Variables and Applications, J. Brown and R. Churchill

The content of this course

1. Fourier Series and Fourier Integral.

2. Partial Differential Equation and Its Solutions.

3. Complex Analysis. (The theory of functions of a complex variable)

Engineering Mathematics Course Syllabus.

Part One: Fourier Series and Fourier Integral.

- Introduction to Fourier Series
- Determining Fourier Series Coefficients and Related Theorems
- Half-Range Expansions
- Different Representations of Fourier Series
- Applications of Fourier Series in Engineering
- Fourier Integral
- Applications of Fourier Integral in Engineering

Engineering Mathematics Course Syllabus.

Part Two: Partial Differential Equation and Its Solutions.

- Introduction to Partial Differential Equations
- Derivation of Partial Differential Equations
- D'Alembert Solution for One-Dimensional Wave Equations
- Classification of Partial Differential Equations
- Solving Partial Differential Equations by Separation of Variables

Engineering Mathematics Course Syllabus.

Part Three: Complex Analysis. (The theory of functions of a complex variable)

- Fundamentals
- Analytic Functions and Differentiability
- Integration in the Complex Plane
- Complex Series
- Residue Theory and Calculation of Real Integrals

Prerequisite and Grading

Prerequisite

- English Knowledge, Primary Mathematics

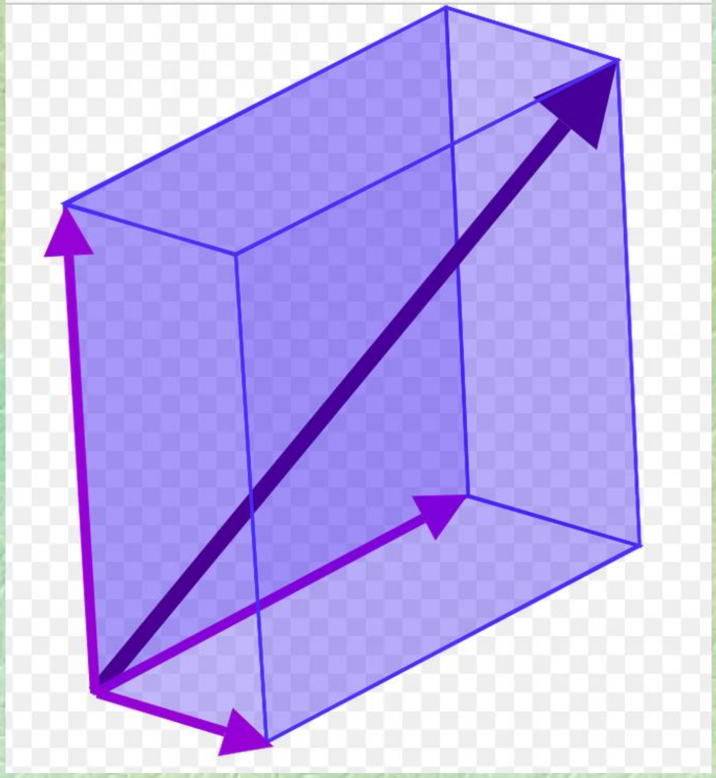
Grading

- Pop quizzes (15% / 3 points or 1.5 points).
- Assignments (10% by your TA / 2 points).
- Make-up Exam (if class time permits, 50% of quizzes / 1.5 points)
- Midterm (25% around 8^{th} week / 5 points).
- Final (50% scheduled by office of the registrar / 10 points).
- Extra credit with Strict and Fair Assessments (10% additional to your grade / 2 extra points)

Vector decomposition

Vector decomposition

Application ?



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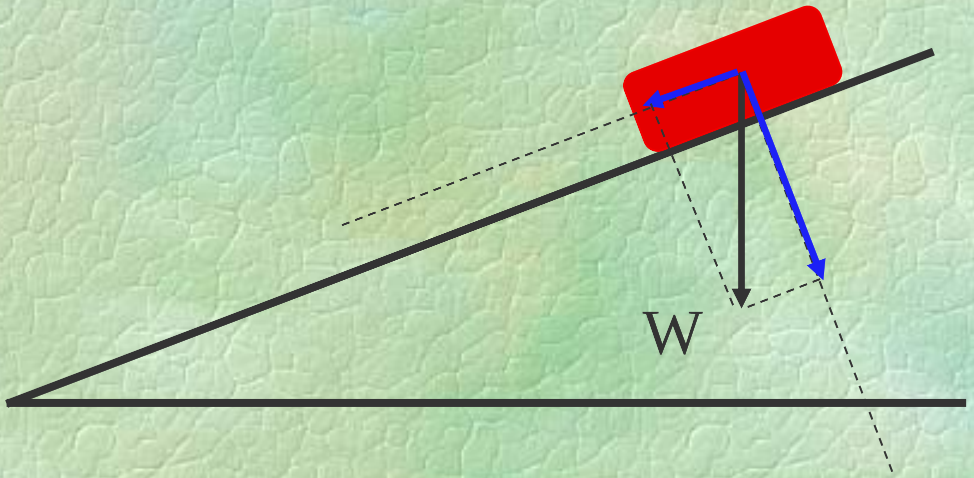
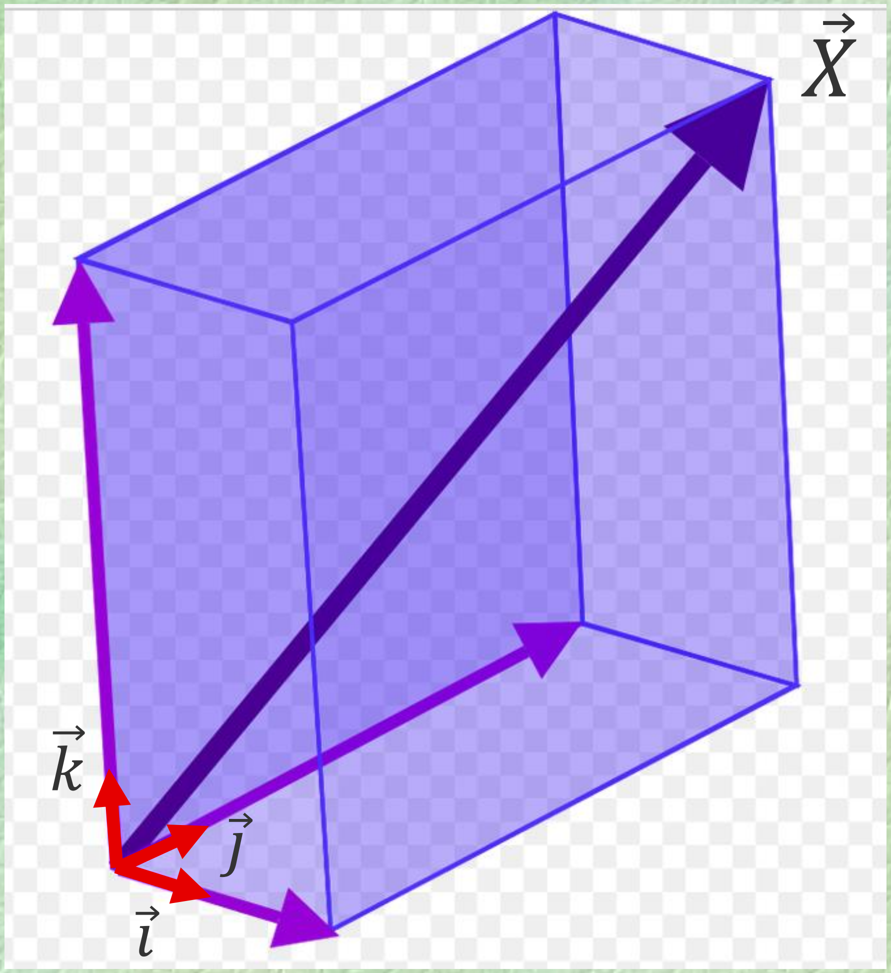


Vector decomposition



$$\vec{X} = a\vec{i} + b\vec{j} + c\vec{k}$$

At which condition we can decompose $\vec{X}$ to $\vec{i}$, $\vec{j}$, and $\vec{k}$?

How can we find a , b , and c ?

Vector space

A vector space (also called a linear space) is a set whose elements, often called vectors, may be added together and multiplied by numbers called scalars. Scalars are often real numbers, but can be complex numbers or, more generally, elements of any field.

Vector space property:

1- Closure Property of addition: $\forall\, x, y \in V, then\ x + y \in V$

2- Closure Property of scalar multiplication:

$$\forall\, \alpha \in F\ \&\ \ y \in V, then\ \alpha y \in V$$

3- Commutative property:

$$\forall\, x, y \in V, then\ \ x + y = y + x$$

4- Associative property:

$$\forall\, x, y, z \in V, then\ \ x + (y + z) = (x + y) + z$$

Vector space

Vector space property(continue):

5- Additive identity property: $\exists\, 0 \in V \ s.t. \forall\, x \in V, x + 0 = x$

6- Additive inverse property:

$$\forall\, x \in V, \exists -x \in V \ s.t. \ x + (-x) = 0$$

7- Vector distribution property:

$$\forall\, \alpha \in F \ \& \ x, y \in V, then\ \alpha(x + y) = \alpha x + \alpha y$$

8- Scalar distribution property:

$$\forall\, \alpha, \beta \in F \ \& \ x \in V, then\ (\alpha + \beta)x = \alpha x + \beta x$$

9- Scalar associative property:

$$\forall\, \alpha, \beta \in F \ \& \ x \in V, then\ \alpha(\beta x) = (\alpha\beta)x$$

10- Scalar identity property: $\exists\, 1 \in F \ s.t. \forall\ x \in V, then\ 1x = x$

Some famous vector spaces

$R^n\ over\ the\ field\ of\ R$ $\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} x_1, x_2, \dots, x_2 \in R$

$C^n\ over\ the\ field\ of\ C$ $\begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix} x_1, x_2, \dots, x_2 \in C$

$A\ periodic\ function\ with\ period\ of\ T\ over\ the\ field\ of\ R$

Basis in vector space

A set of vectors in a vector space V is called a **basis,** if every element of V may be written in a **unique** way as a finite linear combination of that basis.

$$\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix} and \begin{bmatrix}0\\0\\1\end{bmatrix} are\ a\ set\ of\ basis\ for\ R^3$$

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$$\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix} and \begin{bmatrix}1\\1\\0\end{bmatrix} are\ not\ a\ set\ of\ basis\ for\ R^3$$

Why?

$$\begin{bmatrix}1\\0\\0\end{bmatrix},\begin{bmatrix}0\\1\\0\end{bmatrix},\begin{bmatrix}1\\1\\0\end{bmatrix} and \begin{bmatrix}0\\0\\1\end{bmatrix} are\ not\ a\ set\ of\ basis\ for\ R^3$$

Why?

Inner product in vector space

$$\forall x, y \in R^n \qquad < x, y > = \sum_{i=1}^{n} x_i\, y_i$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix} \textit{ and } \begin{bmatrix}0\\0\\1\end{bmatrix} \textit{ are a set of orthogonal basis for } R^3$$

$$\forall f(x), g(x) \in \textit{Periodic functions}$$

$$< f(x), g(x) > = \int_d^{d+T} f(x) g(x) dx$$

Vector decomposition

$\boldsymbol{Example}$: $Present\ \ x = \begin{bmatrix} 2 \\ 4 \\ \sqrt{2} \end{bmatrix}\ with\ respect\ to\ following\ basis$:

$$b_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\ and\ b_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

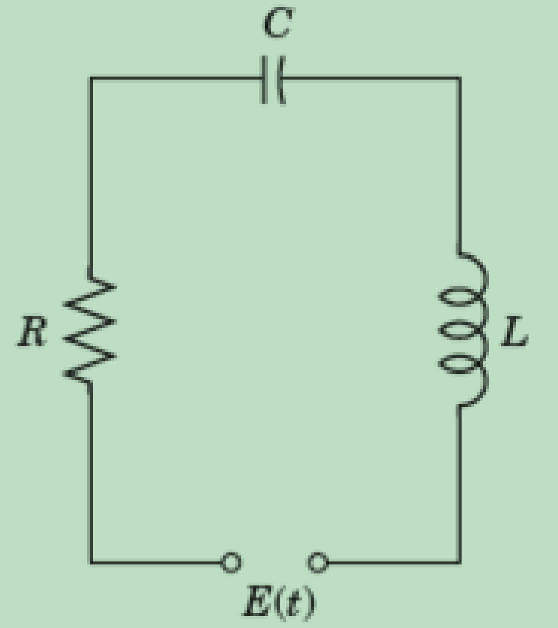
Vector decomposition

$\boldsymbol{Example}$: $Present\ \ x = \begin{bmatrix} 2 \\ 4 \\ \sqrt{2} \end{bmatrix}$ $with\ respect\ to\ following\ basis$:

$$b_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, b_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} and\ b_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Response of an electric circuit?

Consider the following system. Given the input E(t), it is desired to calculate I(t).

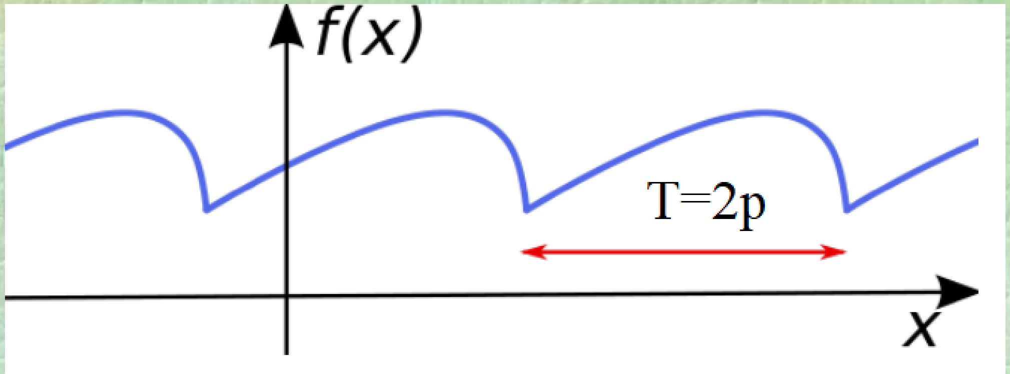


Electrical analog of the system
(*RLC-circuit*)

$$LI'' + RI' + \frac{1}{C} I = E'(t)$$

Periodic function decomposition (Fourier series)

Periodic function:



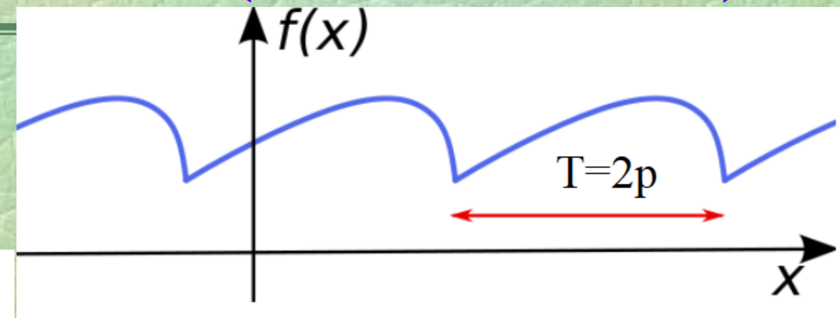
What is the basis for this space?

$$0.5 \qquad \cos 1\frac{2\pi}{T}x \qquad \cos 2\frac{2\pi}{T}x \qquad \cos 3\frac{2\pi}{T}x \qquad \dots$$

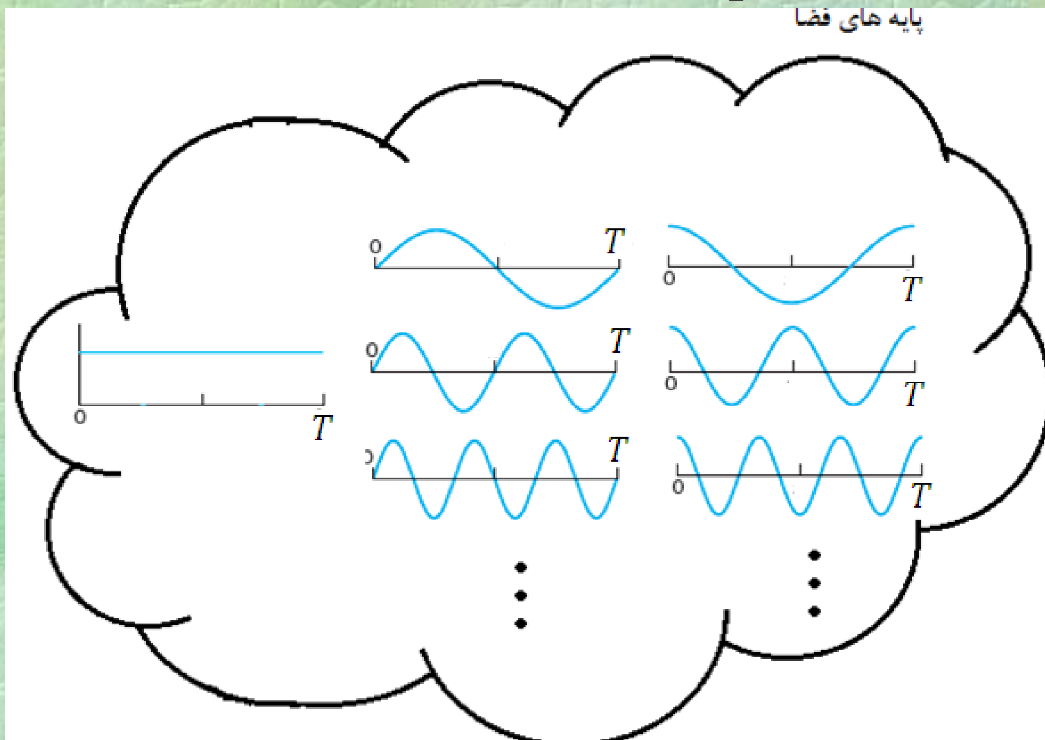
$$\sin 1\frac{2\pi}{T}x \qquad \sin 2\frac{2\pi}{T}x \qquad \sin 3\frac{2\pi}{T}x \qquad \dots$$

Periodic function decomposition (Fourier series)

Periodic function:



What is the basis for this space?



0.5 $\quad cos1\frac{2\pi}{T}x \quad cos2\frac{2\pi}{T}x$

$sin1\frac{2\pi}{T}x$ $\quad \ldots$

$sin2\frac{2\pi}{T}x$ $\quad \ldots$

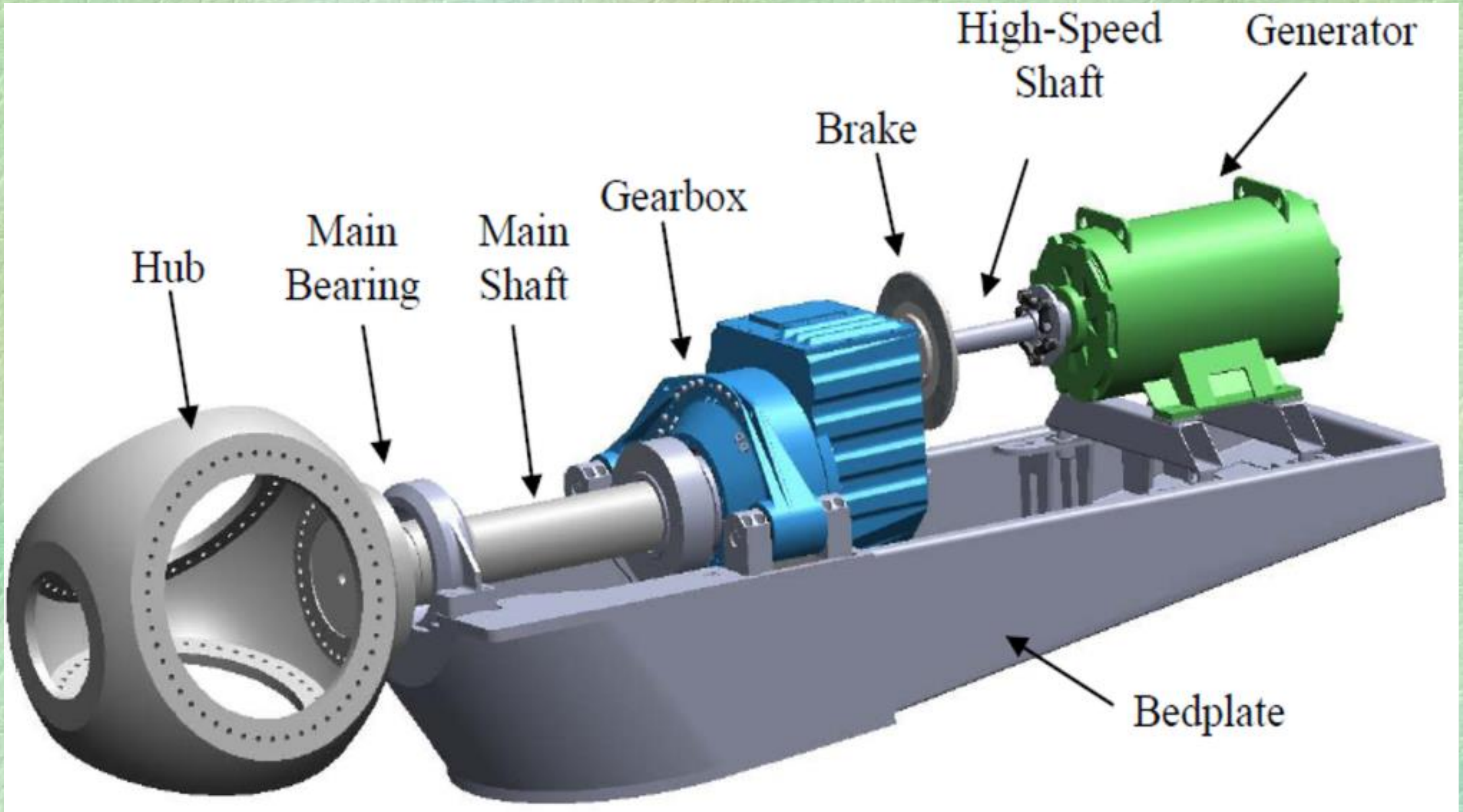
Note1: It is not finite dimensional.

Note2: What is the representation of f(x)?

Lightning

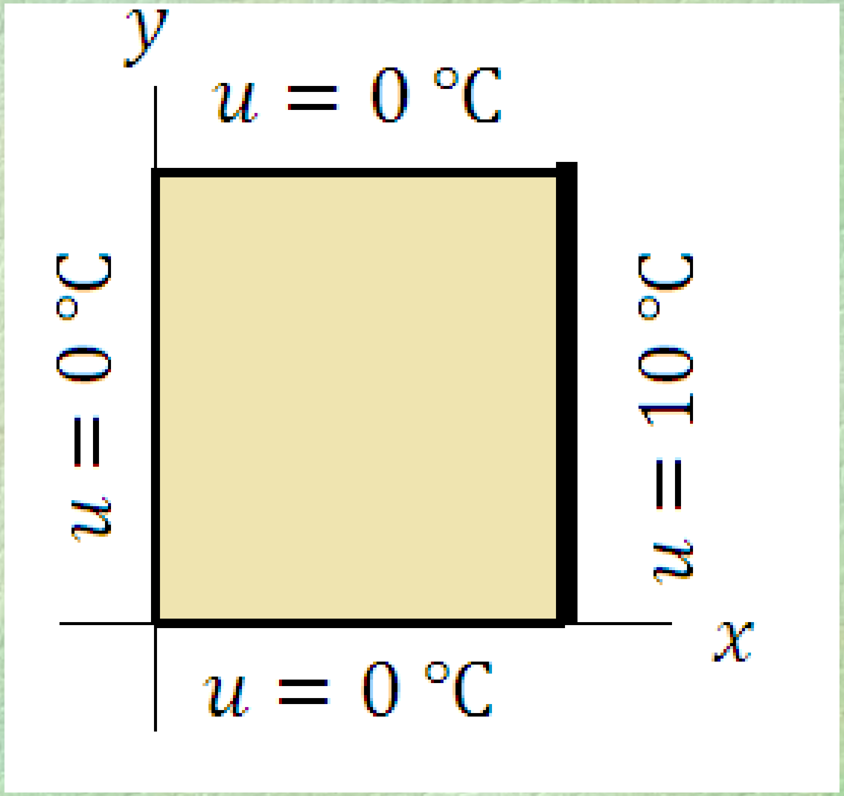
Partial differential equation

Shaft rotation



Partial differential equation

Temperature analysis



Partial differential equation

Integral computation

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} cosmxdx$$

$$\int_{0}^{2\pi} \frac{d\theta}{1+asin\theta} \qquad -1 < a < 1$$

Complex analysis