Multivariable Control Systems

Ali Karimpour
Professor
Ferdowsi University of Mashhad

Lecture 3

References are appeared in the last slide.

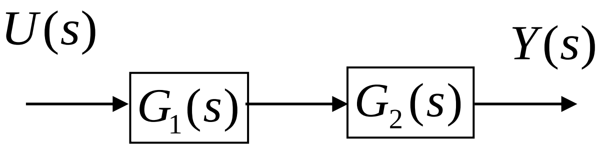
Introduction to Multivariable Control

Topics to be covered include:

- Multivariable Connections
- Multivariable System Representation
    - Polynomial Matrix Description & Rosenbrock’s System Matrix
    - General Control Problem Formulation
    - Matrix Fraction Description (MFD)

Multivariable Connections

- Cascade (series) interconnection of transfer matrices

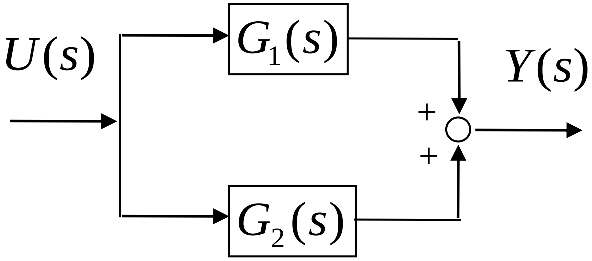


$$Y(s) = G_2(s)G_1(s)U(s) = G(s)U(s)$$

$$\boxed{G(s) = G_2(s)G_1(s)} \neq G_1(s)G_2(s)$$

Generally

- Parallel interconnection of transfer matrices

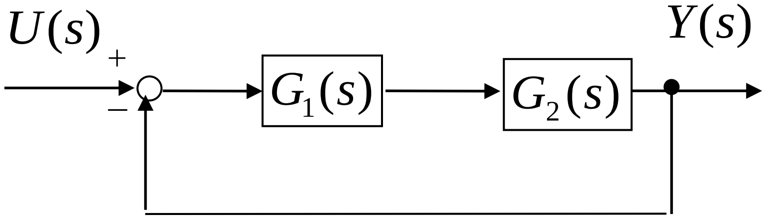


$$Y(s) = (G_1(s) + G_2(s))U(s) = G(s)U(s)$$

$$\boxed{G(s) = G_1(s) + G_2(s)}$$

Multivariable Connections

- Feedback interconnection of transfer matrices



$$Y(s) = G_2(s)G_1(s)\big(U(s) - Y(s)\big)$$

$$Y(s) = (I + G_2(s)G_1(s))^{-1}G_2(s)G_1(s)U(s) = G(s)U(s)$$

$$\boxed{G(s) = (I + G_2(s)G_1(s))^{-1}G_2(s)G_1(s)}$$

A useful relation in multivariable is push-through rule

$$(I + G_2(s)G_1(s))^{-1}G_2(s) = G_2(s)(I + G_1(s)G_2(s))^{-1}$$

Exercise 3-1: Proof the push-through rule

Multivariable Connections

MIMO rule: To derive the output of a MIMO system,

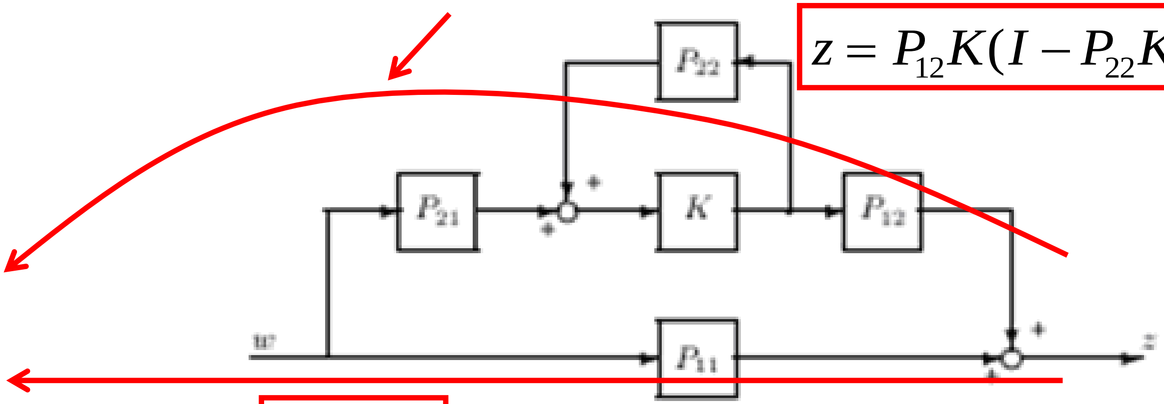
Start from the output and write down the blocks as you meet them when moving backward (against the signal flow) towards the input.

If you exit from a feedback loop then include a term $(I-L)^{-1}$ or $(I+L)^{-1}$ according to the feedback sign where L is the transfer function around that loop (evaluated against the signal flow starting at the point of exit from the loop).

Parallel branches should be treated independently and their contributions added together.

Multivariable Connections

Example 3-1: **Derive the transfer function of the system shown in figure**



$$z = P_{12}K(I - P_{22}K)^{-1}P_{21}\omega$$

$$z = P_{11}\omega$$

$$z = \left(P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}\right)\omega$$

Introduction to Multivariable Control

Topics to be covered include:

- Multivariable Connections
- Multivariable System Representation
  - Polynomial Matrix Description & Rosenbrock’s System Matrix
  - General Control Problem Formulation
  - Matrix Fraction Description (MFD)

Polynomial Matrix Description

General form of a polynomial matrix description

System Variables

System Inputs

$$P\left(\frac{d}{dt}\right)\xi(t)=Q\left(\frac{d}{dt}\right)u(t)$$

$$y(t)=R\left(\frac{d}{dt}\right)\xi(t)+W\left(\frac{d}{dt}\right)u(t)$$

System Outputs

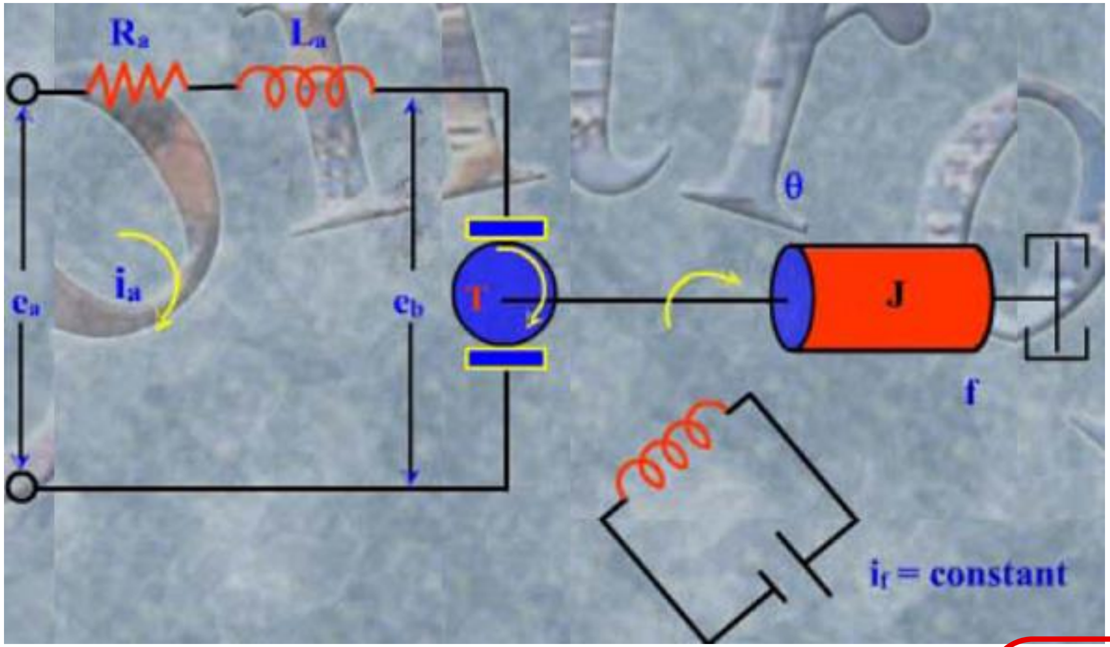
$$P(s)\xi(s)=Q(s)U(s)$$
$$Y(s)=R(s)\xi(s)+W(s)U(s)$$

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix}\begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix}=\begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

Rosenbrock’s system matrix

Polynomial Matrix Description

Example 3-2: A position control system.



$$J\frac{d^2\theta}{dt^2}+f\frac{d\theta}{dt}=T \ =Ki_a$$

$$L_a\frac{di_a}{dt}+R_ai_a+e_b=e_a$$

$$L_a\frac{di_a}{dt}+R_ai_a+K_b\frac{d\theta}{dt}=e_a$$

$$P\left(\frac{d}{dt}\right)\xi(t)=Q\left(\frac{d}{dt}\right)u(t)$$

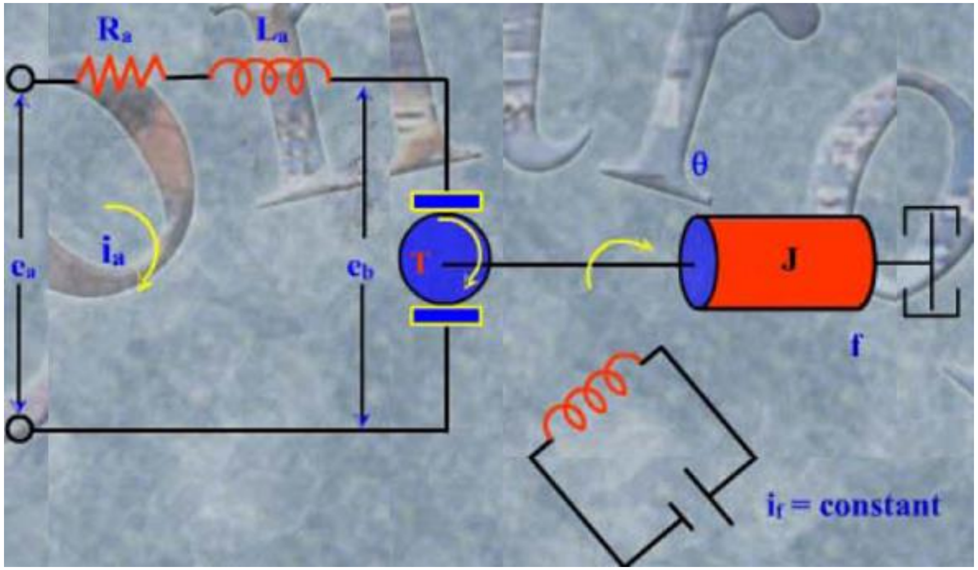
$$y(t)=R\left(\frac{d}{dt}\right)\xi(t)+W\left(\frac{d}{dt}\right)u(t)$$

$$\boxed{P\left(\frac{d}{dt}\right)}\ \boxed{\begin{bmatrix} J\frac{d^2}{dt^2}+f\frac{d}{dt} & -K \\ K_b\frac{d}{dt} & L_a\frac{d}{dt}+R_a \end{bmatrix}} \overset{\boxed{\xi(t)}}{\begin{bmatrix}\theta(t)\\ i_a(t)\end{bmatrix}} = \begin{bmatrix}0\\1\end{bmatrix}\underset{\boxed{u(t)}}{e_a(t)}$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta(t)\\ i_a(t)\end{bmatrix}$$

Polynomial Matrix Description

Example 3-2(Continue): A position control system.



Polynomial matrix description

$$\boxed{\xi(t)}$$

$$\begin{bmatrix} J\dfrac{d^2}{dt^2}+f\dfrac{d}{dt} & -K \\ K_b\dfrac{d}{dt} & L_a\dfrac{d}{dt}+R_a \end{bmatrix}\begin{bmatrix}\theta(t)\\ i_a(t)\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}e_a(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\theta(t)\\ i_a(t)\end{bmatrix}$$

$$\boxed{u(t)}$$

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix}\begin{bmatrix}\xi(s)\\ -U(s)\end{bmatrix}=\begin{bmatrix}0\\ -Y(s)\end{bmatrix}$$

$$\left[\begin{array}{cc:c} Js^2+fs & -K & 0 \\ K_bs & L_as+R_a & 1 \\ \hdashline -1 & 0 & 0 \end{array}\right]\begin{bmatrix}\Theta(s)\\ I_a(s)\\ -E_a(s)\end{bmatrix}=\begin{bmatrix}0\\0\\-Y(s)\end{bmatrix}$$

Rosenbrock’s system matrix

Polynomial Matrix Description

Transfer function matrix from Rosenbrock's system matrix.

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix} \begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

Rosenbrock's system matrix

Suppose P is nonsingular.

$$Y(s) = \left(R(s)P^{-1}(s)Q(s) + W(s) \right) U(s)$$

$$G(s) = R(s)P^{-1}(s)Q(s) + W(s)$$

Polynomial Matrix Description

System order

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix}\begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix}=\begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

System order is the number of independent initial condition that is necessary to describe the system.

System order in Rosenbrock’s system matrix is equal to degree of det(P(s)).

For previous example:

$$\left[\begin{array}{cc:c} Js^2+fs & -K & 0 \\ K_b s & L_a s+R_a & 1 \\ \hdashline -1 & 0 & 0 \end{array}\right]\begin{bmatrix} \Theta(s) \\ I_a(s) \\ -E_a(s) \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ -Y(s) \end{bmatrix}$$

$$|P(s)|=(Js^2+fs)(L_a s+R_a)+KK_b s$$

System order is 3
and P(s) is 2×2

Polynomial Matrix Description

State Space Model and Rosenbrock's system matrix

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix} \begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

$$\begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix} \begin{bmatrix} X(s) \\ -U(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

$$G(s) = C(sI - A)^{-1}B + D$$

Remark1: (sI-A) is n×n and also system order is n. But generally dimension of *P*(s) in Rosenbrok's system matrix is not the same as system order.(See previous example)

Remark2: G(s) is strictly proper if D=0 otherwise it is proper.

Polynomial Matrix Description

Example(Two important remarks)

$$(s+1)^2\xi(s)=s^3U(s)$$
$$Y(s)=\xi(s)+(2-s)U(s)$$

$$\left[\begin{array}{c|c} (s+1)^2 & s^3 \\ \hline -1 & 2-s \end{array}\right]\begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix}=\begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

$$G(s)=R(s)P^{-1}(s)Q(s)+W(s)=\frac{s^3}{(s+1)^2}+2-s=\frac{3s+2}{(s+1)^2}$$

Remark1: G(s) is strictly proper but W(s)=2-s !

Remark 2: Another form of Resenbrock’s system matrix.

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & (s+1)^2 & s^3 \\ \hline 0 & -1 & 2-s \end{array}\right]\begin{bmatrix} \eta(s) \\ \xi(s) \\ -U(s) \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ -Y(s) \end{bmatrix}$$

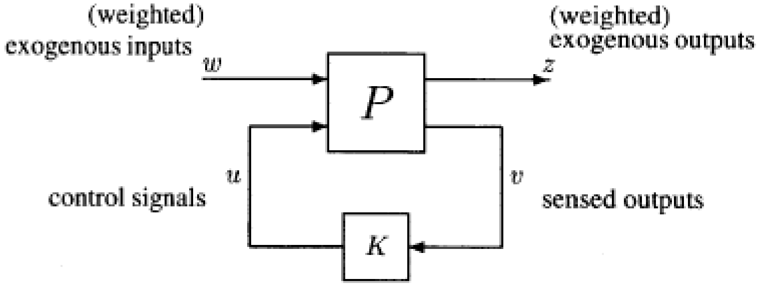
Introduction to Multivariable Control

Topics to be covered include:

- Multivariable Connections
- Multivariable System Representation
    - Polynomial Matrix Description & Rosenbrock’s System Matrix
    - General Control Problem Formulation
    - Matrix Fraction Description (MFD)

General Control Problem Formulation

System without uncertainty



w exogenous inputs: Inputs that are not used to control the system. i.e. references, disturbances, noises

z exogenous outputs: Outputs that we want to control them and push them to zero.

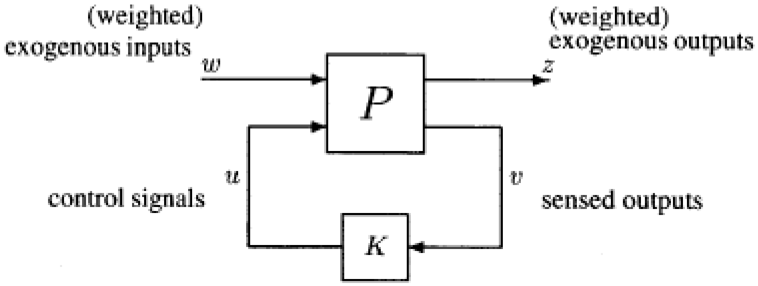
u control signals: Signals that produced by controller to control the system.

v sensed outputs: Signals that used by controller.

Problem description: Derive K(s) such that closed loop system be stable and z be as small as possible for bounded w.

General Control Problem Formulation

Problem description: Derive K(s) such that closed loop system be stable and z be as small as possible for bounded w.



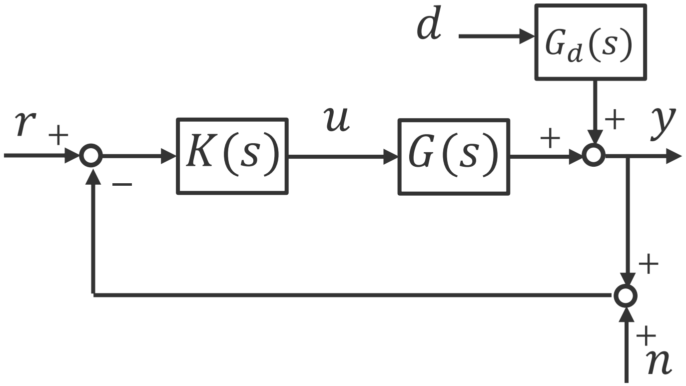
$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$u = Kv$$

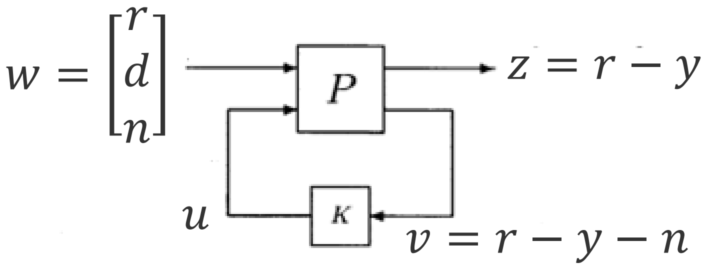
$$z = \left(P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}\right)w = Nw \qquad N = F_l(P,K)$$

Problem description: Make N stable and as small as possible.

General Control Problem Formulation



Example 3-3: Change following system to general control problem formulation.



$$\begin{bmatrix} z \\ v \end{bmatrix} = \left[\begin{array}{ccc|c} I & -G_d(s) & 0 & -G(s) \\ \hline I & -G_d(s) & -I & -G(s) \end{array}\right] \begin{bmatrix} r \\ d \\ n \\ u \end{bmatrix}$$

$$z = \left(P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}\right)w = Nw$$

$$z = \left(\begin{bmatrix} I & -G_d(s) & 0 \end{bmatrix} - G(s)K(s)(I + G(s)K(s))^{-1}\begin{bmatrix} I & -G_d(s) & -I \end{bmatrix}\right)\begin{bmatrix} r \\ d \\ n \end{bmatrix}$$

$$z = \left(\underbrace{(I + G(s)K(s))^{-1}}_{S(s)} \quad \underbrace{-G_d(s)(I + G(s)K(s))^{-1}}_{S(s)} \quad \underbrace{G(s)K(s)(I + G(s)K(s))^{-1}}_{T(s)}\right)\begin{bmatrix} r \\ d \\ n \end{bmatrix}$$

Introduction to Multivariable Control

Topics to be covered include:

- Multivariable Connections
- Multivariable System Representation
    - Polynomial Matrix Description & Rosenbrock's System Matrix
    - General Control Problem Formulation
    - Matrix Fraction Description (MFD)

Matrix Fraction Description (MFD)

Let $g(s)=\dfrac{s+1}{s^2+5s+6}$ is a SISO transfer function

We can write $$g(s)=\underbrace{(s+1)}_{\text{polynomial}}\underbrace{(s^2+5s+6)}_{\text{polynomial}}{}^{-1}$$

This is a Right Matrix Fraction Description (RMFD)

We can also write $$g(s)=\underbrace{(s^2+5s+6)}_{\text{polynomial}}{}^{-1}\underbrace{(s+1)}_{\text{polynomial}}$$

This is a Left Matrix Fraction Description (LMFD)

Remark: Are LMFD and RMFD the same for any system?

Matrix Fraction Description (MFD)

$$\text{Let } g(s)=\frac{s+1}{s^2+5s+6} \text{ is an SISO transfer function}$$

$$\text{We can write } g(s)=(s+1)\left(s^2+5s+6\right)^{-1}$$

polynomial polynomial

This is a Right Matrix Fraction Description (RMFD)

$$\text{We can also write } g(s)=\big((s+1)(s+a)\big)\big((s+a)(s^2+5s+6)\big)^{-1}$$

polynomial polynomial

This is a also a Right Matrix Fraction Description (RMFD)

Remark: Uniqueness of MFD?

Matrix Fraction Description (MFD)

Matrix Fraction Description for Transfer Function Matrix

$$\mathrm{G}(s)=\frac{1}{d(s)}N(s)$$

G is a p×q matrix

$$\mathrm{G}(s)=\left(d(s)I_p\right)^{-1}N(s)=D_L^{-1}(s)\boxed{N_L(s)}$$

Left Matrix Fraction Description (LMFD)

polynomial matrix

polynomial matrix

$$\mathrm{G}(s)=N(s)\left(d(s)I_q\right)^{-1}=\boxed{N_R(s)}D_R^{-1}(s)$$

Right Matrix Fraction Description (RMFD)

polynomial matrix

polynomial matrix

Matrix Fraction Description (MFD)

Matrix Fraction Description for Transfer Matrix

$$\mathrm{G}(s)=\frac{1}{d(s)}N(s)$$

Suppose G is a p×q matrix so

$$\mathrm{G}(s)=\left(d(s)I_p\right)^{-1}N(s)=D_L^{-1}(s)N_L(s)$$

Left Matrix Fraction Description
(LMFD)

Degree of denominator matrix is defined as: $\deg \mathrm{D_L}(s)=\deg\det \mathrm{D_L}(s)=rp$

$$\mathrm{G}(s)=N(s)\left(d(s)I_q\right)^{-1}=N_R(s)D_R^{-1}(s)$$

Right Matrix Fraction Description
(RMFD)

Degree of denominator matrix is defined as: $\deg \mathrm{D_R}(s)=\deg\det \mathrm{D_R}(s)=rq$

Matrix Fraction Description (MFD)

We can show that the MFD is not unique, because, for any nonsingular

$m\times m$ matrix $\Omega(s)$ we can write G(s) as:

$$G(s)=N_R(s)\left(\Omega(s)\Omega(s)^{-1}\right)D_R^{-1}(s)=\left(N_R(s)\Omega(s)\right)\left(D_R(s)\Omega(s)\right)^{-1}$$

$\Omega(s)$ is said to be a right common factor.

When the only right common factors of $N_R(s)$ and $D_R(s)$ is unimodular matrix, then, we say that the RMFD $\left(N_R(s),D_R(s)\right)$ is irreducible.

Matrix Fraction Description (MFD)

Example 3-4 Consider

If $D_L(s)$ and $N_L(s)$ are not irreducible, find irreducible one.

$$G(s)=D_L^{-1}(s)N_L(s)=\begin{bmatrix}1 & -1\\ -1 & s+1\end{bmatrix}^{-1}\begin{bmatrix}1 & -1\\ 1 & s+1\end{bmatrix}$$

Checking irreducibility(left common factor):

Form:

$$\begin{bmatrix}N_L(s) & D_L(s)\end{bmatrix}=\begin{bmatrix}1 & -1 & 1 & -1\\ 1 & s+1 & -1 & s+1\end{bmatrix}$$

Do preliminary transformation(on columns) to make right part zero:

1) Add -C_2 on C_4 $\begin{bmatrix}1 & -1 & 1 & 0\\ 1 & s+1 & -1 & 0\end{bmatrix}$

2) Add -C_1 on C_3 $\begin{bmatrix}1 & -1 & 0 & 0\\ 1 & s+1 & -2 & 0\end{bmatrix}$

3) Add C_1 on C_2 $\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & s+2 & -2 & 0\end{bmatrix}$

4) Add 0.5(s+2)C_3 on C_2 $\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & 0 & -2 & 0\end{bmatrix}$

5) Change C_3 and C_2 $\begin{bmatrix}1 & 0 & 0 & 0\\ 1 & -2 & 0 & 0\end{bmatrix}$

Common factor is: $Q(s)=\begin{bmatrix}1 & 0\\ 1 & -2\end{bmatrix}$

Since Q(s) is unimodular so $D_L(s)$ and $N_L(s)$ are irreducible.

Matrix Fraction Description (MFD)

Example 3-5 Consider

If $D_L(s)$ and $N_L(s)$ are not irreducible, find irreducible one.

$$G(s)=D_L^{-1}(s)N_L(s)=\begin{bmatrix} -s & s^2+s \\ s-2 & s+2 \end{bmatrix}^{-1}\begin{bmatrix} s & s^2+s \\ s+2 & s+2 \end{bmatrix}$$

Checking irreducibility(left common factor):

Form:

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix}=\begin{bmatrix} s & s^2+s & -s & s^2+s \\ s+2 & s+2 & s-2 & s+2 \end{bmatrix}$$

Do preliminary transformation(on columns) to make right part zero:

1) Add $-C_2$ on C_4

$$\begin{bmatrix} s & s^2+s & -s & 0 \\ s+2 & s+2 & s-2 & 0 \end{bmatrix}$$

2) Add C_1 on C_3

$$\begin{bmatrix} s & s^2+s & 0 & 0 \\ s+2 & s+2 & 2s & 0 \end{bmatrix}$$

3) Add $-(s+1)C_1$ on C_2

$$\begin{bmatrix} s & 0 & 0 & 0 \\ s+2 & -s(s+2) & 2s & 0 \end{bmatrix}$$

4) Add $0.5(s+2)C_3$ on C_2

$$\begin{bmatrix} s & 0 & 0 & 0 \\ s+2 & 0 & 2s & 0 \end{bmatrix}$$

Matrix Fraction Description (MFD)

Example 3-5 Consider

If $D_L(s)$ and $N_L(s)$ are not irreducible, find irreducible one.

$$G(s)=D_L^{-1}(s)N_L(s)=\begin{bmatrix}-s & s^2+s\\ s-2 & s+2\end{bmatrix}^{-1}\begin{bmatrix}s & s^2+s\\ s+2 & s+2\end{bmatrix}$$

Checking irreducibility (left common factor):

$$\begin{bmatrix}N_L(s) & D_L(s)\end{bmatrix}=\begin{bmatrix}s & s^2+s & -s & s^2+s\\ s+2 & s+2 & s-2 & s+2\end{bmatrix}$$

1) Add $-C_2$ on C_4 **2) Add C_1 on C_3** **3) Add $-(s+1)C_1$ on C_2** **4) Add $0.5(s+2)C_3$ on C_2**

$$\begin{bmatrix}s & 0 & 0 & 0\\ s+2 & 0 & 2s & 0\end{bmatrix}$$

5) Change C_3 and C_2

$$\begin{bmatrix}s & 0 & 0 & 0\\ s+2 & 2s & 0 & 0\end{bmatrix}$$

$$\text{Common factor is}: \ Q(s)=\begin{bmatrix}s & 0\\ s+2 & 2s\end{bmatrix}$$

$$Q(s)\hat{D}_L(s)=D_L(s) \qquad Q(s)\hat{N}_L(s)=N_L(s)$$

$$\hat{D}_L(s)=\begin{bmatrix}-1 & s+1\\ 1 & -\dfrac{s+2}{2}\end{bmatrix} \qquad \hat{N}_L(s)=\begin{bmatrix}1 & s+1\\ 0 & -\dfrac{s+2}{2}\end{bmatrix}$$

$$G(s)=D_L^{-1}(s)N_L(s)=\hat{D}_L^{-1}(s)\,\hat{N}_L(s)$$

Why this procedure leads to ….?

Matrix Fraction Description (MFD)

Why previous procedure leads to greatest left common factor:

Let:

$$G(s) = D_L^{-1}(s)N_L(s) \quad G \; is \;\; p \times q$$

By suitable preliminary transformation :

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} \;\rightarrow\; \begin{bmatrix} Q(s) & 0 \end{bmatrix}$$

We must show that Q(s) is the greatest left common devisor.

$$\Rightarrow \;\; \begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} U(s) = \begin{bmatrix} Q(s) & 0 \end{bmatrix}$$

Since U is unimodular so its inverse is also unimodular thus

$$\Rightarrow \;\; \begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} = \begin{bmatrix} Q(s) & 0 \end{bmatrix} U^{-1}(s)$$

$$U^{-1}(s) = V(s) = \begin{bmatrix} V_{11}(s) & V_{12}(s) \\ V_{21}(s) & V_{22}(s) \end{bmatrix}$$

So we have

$$N_L(s) = Q(s)V_{11}(s) \quad D_L(s) = Q(s)V_{12}(s)$$

Clearly Q(s) is a left common divisor but why greatest common divisor?

Matrix Fraction Description (MFD)

Why previous procedure leads to greatest left common factor:

$$N_L(s)=Q(s)V_{11}(s)\quad D_L(s)=Q(s)V_{12}(s)$$

Clearly Q(s) is a left common divisor but why greatest common divisor?

Now let:

$$U(s)=\begin{bmatrix} U_{11}(s) & U_{12}(s) \\ U_{21}(s) & U_{22}(s) \end{bmatrix}$$

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} U(s)=\begin{bmatrix} Q(s) & 0 \end{bmatrix} \Rightarrow \quad N_L(s)U_{11}(s)+D_L(s)U_{21}(s)=Q(s)$$

Let W(s) is another left common divisor so:

$$W(s)\overline{N}_L(s)U_{11}(s)+W(s)\overline{D}_L(s)U_{21}(s)=Q(s)$$

So W(s) is a left common divisor of Q(s), → Q(s) is gcd.

Matrix Fraction Description (MFD)

An important theorem.

Theorem 1: $D_L(s)$ and $N_L(s)$ are left coprime (irreducible) if and only if there exist two polynomial matrix $X_L(s)$ and $Y_L(s)$ such that following equation satisfied.

$$N_L(s)X_L(s)+D_L(s)Y_L(s)=I$$

This equation is called simple Bezout identity.

Example 3-6: Let $N_L(s)=s+2$ and $D_L(s)=s^2+5s+6$, are they left coprime?

One cannot derive X(s) and Y(s) s.t. $N_L(s)X(s)+D_L(s)Y(s)=1$

Example 3-7: Let $N_L(s)=s+1$ and $D_L(s)=s^2+5s+6$, are they left coprime?

Example 3-8: Let $N(s)=2s$ and $D_L(s)=2s^2+10s+2$, are they left coprime?

Let $X(s)=-s-2$ and $Y(s)=0.5$ s.t. $N_L(s)X(s)+D_L(s)Y(s)=1$

Matrix Fraction Description (MFD)

Left MFD

$$G(s)=D_L^{-1}(s)N_L(s) \quad G \;\; is \;\; p\times q$$

$$\downarrow \qquad\quad \downarrow$$

$$p\times p \qquad p\times q$$

Are they coprime?

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} \rightarrow \begin{bmatrix} Q(s) & 0 \end{bmatrix}$$

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} U(s)=\begin{bmatrix} Q(s) & 0 \end{bmatrix}$$

If Q(s) is unimodular

$$\begin{bmatrix} N_L(s) & D_L(s) \end{bmatrix} U_l(S)=\begin{bmatrix} I & 0 \end{bmatrix}$$

Bezout identity:

$$N_L(s)X_L(s)+D_L(s)Y_L(s)=I$$

Right MFD

$$G(s)=N_R(s)D_R^{-1}(s) \quad G \;\; is \;\; p\times q$$

$$\downarrow \qquad\quad \downarrow$$

$$p\times q \qquad q\times q$$

Are they coprime?

$$\begin{bmatrix} N_R(s) \\ D_R(s) \end{bmatrix} \rightarrow \begin{bmatrix} Q(s) \\ 0 \end{bmatrix}$$

$$V(s)\begin{bmatrix} N_R(s) \\ D_R(s) \end{bmatrix} = \begin{bmatrix} Q(s) \\ 0 \end{bmatrix}$$

If Q(s) is unimodular

$$V_r(s)\begin{bmatrix} N_R(s) \\ D_R(s) \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

Bezout identity:

$$X_R(s)N_R(s)+Y_R(s)D_R(s)=I$$

In the reminder of course …………

Coprime Factorizations over Stable Transfer Functions

Now let *P* be a proper real-rational matrix. A right-coprime factorization (rcf) of *P* is a factorization of the form

$$\boxed{P = NM^{-1}}$$

where *N* and *M* are right-coprime in the set of stable transfer matrices.

Similarly, a left-coprime factorization (lcf) of *P* has the form

$$\boxed{P = \tilde{M}^{-1}\tilde{N}}$$

Exercises

Exercise 3-1: Proof the push-through rule.

Exercise 3-2: Derive Rosenbrock’s system matrix for following system. What is the order of system?

$$\frac{d\xi_1}{dt}+\frac{d^3\xi_2}{dt^3}=-\xi_1$$

$$\frac{d\xi_2}{dt}=-\xi_1+u$$

$$y=\xi_1$$

Exercise 3-3: Derive Rosenbrock’s system matrix for following system. What is the order of system?

$$\frac{d\xi_1}{dt}+\frac{d^3\xi_2}{dt^3}+\frac{d^2\xi_3}{dt^2}=\xi_1+\xi_2$$

$$\frac{d^2\xi_2}{dt^2}=\xi_2+u_1$$

$$\frac{d\xi_3}{dt}=\xi_3+u_2$$

$$y_1=\xi_1$$

$$y_2=\xi_2$$

Exercises

Exercise 3-4: a)Derive two different order MFD for following system.

b) Check the irreducibility of derived MFD in part “a”

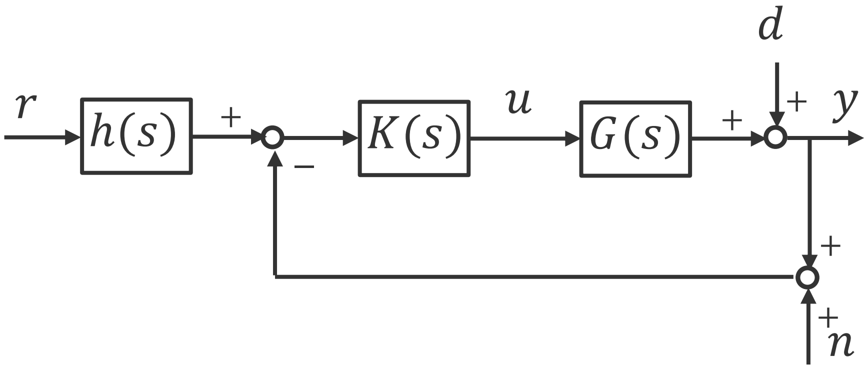
c) Derive an irreducible MFD for the system.

$$G(s)=\begin{bmatrix} \dfrac{s+2}{(s+1)^2} & \dfrac{s}{(s+2)^2} \\ \dfrac{-s}{s+2} & \dfrac{s}{(s+2)^2} \end{bmatrix}$$

Exercise 3-6: Derive an irreducible RMFD for following system.

$$G(s)=\begin{bmatrix} \dfrac{s+2}{s} & 0 \\ \dfrac{2}{s} & 1 \end{bmatrix}$$

Exercise 3-7: Derive general control problem formulation for following system and derive N.(we need y track r)



References

References

- Multivariable Feedback Design, J M Maciejowski, Wesley,1989.
- Multivariable Feedback Control, S. Skogestad, I. Postlethwaite, Wiley,2005.
- تحلیل و طراحی سیستم های چند متغیره، دکتر علی خاکی صدیق
- کنترل مقاوم H_∞ ، دکتر حمید رضا تقی راد، محمد فتحی و فرینا زمانی اسگویی

Web References

- http://karimpor.profcms.um.ac.ir/index.php/courses/9319