ADVANCED CONTROL

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Reference:
Chi-Tsong Chen, “Linear System Theory and Design”, 1999.
I thank my students , Mahhmodi and Samadi, for their help in making slides of this lecture.

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Lecture 7

State feedback and state estimators

Topics to be covered include:

- Pole placement with state feedback.
- Tracking and regulator problem
- Robust tracking and disturbance rejection
- State estimation
- Reduced-Dimensional state estimator
- Feedback from estimated states

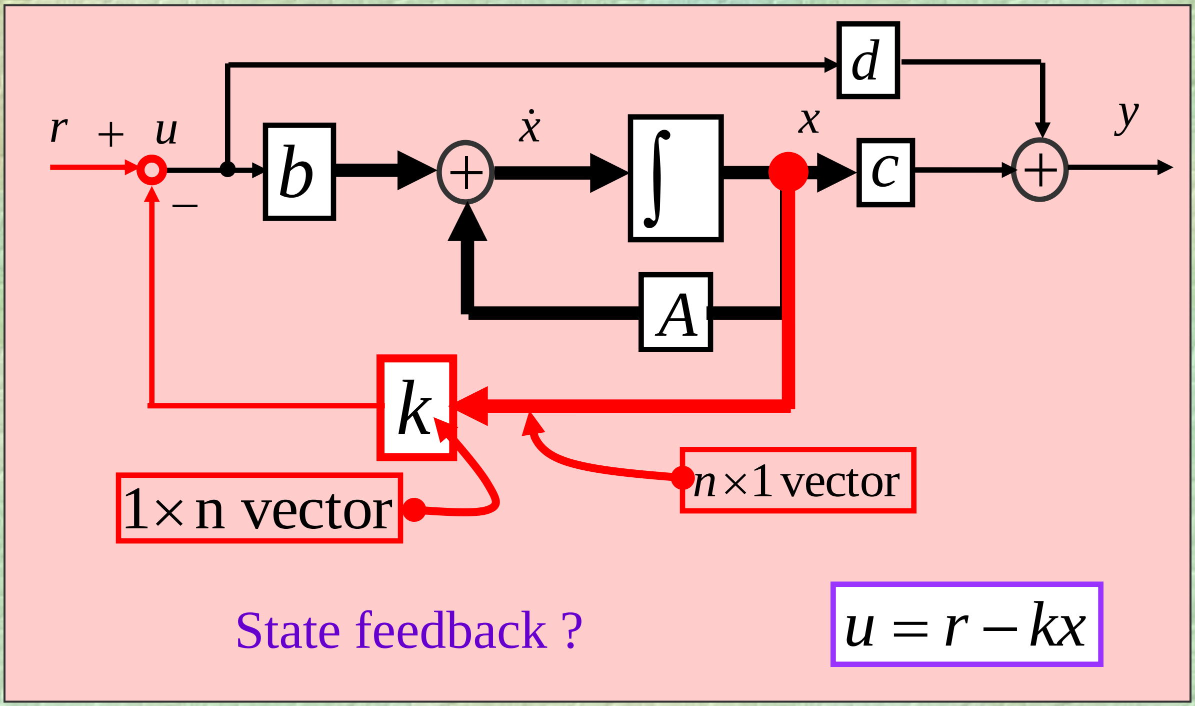
What you will learn after studying this section

- **State feedback idea**
- **Pole placement possibility**
- **Pole placement techniques**
- **Output regulating**
- **Robust output regulating**
- **State estimation**
- **State estimation techniques**
- **Reduced order state estimation**
- **Separation theorem**

Pole placement with state feedback

$$\dot{x} = Ax + bu$$

$$y = cx + du$$



Pole placement with state feedback

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

What are the eigenvalues?

$$\text{roots of } |sI - A| = 0$$

Let u=r-kx where k is an 1×n vector

$$\dot{x} = Ax + b(r - kx)$$
$$y = cx + d(r - kx)$$

→

$$\dot{x} = (A - bk)x + br$$
$$y = (c - dk)x + dr$$

New eigenvalues?

$$\text{roots of } |sI - A + bk| = 0$$

Pole placement with state feedback

Example 1: Consider following controllable and observable system.

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad y = [1 \quad 2]x$$

$u = r - [k_1 \quad k_2]x$ → State feedback

$$\dot{x} = \begin{bmatrix} 1 & 2 \\ 3 - k_1 & 1 - k_2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r \qquad y = [1 \quad 2]x$$

$$C_f = \begin{bmatrix} 0 & 2 \\ 1 & 1 - k_2 \end{bmatrix} \rightarrow |C_f| = -2 \neq 0$$

The state feedback equation is controllable for any k.

$$O_f = \begin{bmatrix} 1 & 2 \\ 7 - 2k_1 & 4 - 2k_2 \end{bmatrix} \rightarrow |O_f| = 4k_1 - 10 - 2k_2$$

Observability depends on k.

Is it a general rule?

Pole placement with state feedback

Theorem 1: The pair ($A-bk,b$) is controllable with a $1\times n$ vector k if and only if the pair (A,b) is controllable.

Proof: We must show that the controllability of the two systems is equivalent.

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

$$\dot{x} = (A-bk)x + br$$
$$y = (c-dk)x + dr$$

$$C_f = \begin{bmatrix} b & (A-bk)b & (A-bk)^2 b & \dots & (A-bk)^{n-1} b \end{bmatrix}$$

Suppose n=4

$$C_f = \underbrace{\begin{bmatrix} b & Ab & A^2 b & A^3 b \end{bmatrix}}_{C} \begin{bmatrix} 1 & -kb & -k(A-bk)b & -k(A-bk)^2 b \\ 0 & 1 & -kb & -k(A-bk)b \\ 0 & 0 & 1 & -kb \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\rho(C_f) = \rho(C)$$

Pole placement with state feedback

$$\dot{x} = Ax + bu$$
$$y = cx + du \qquad (I)$$

$$u = r - kx$$

state feedback

$$\dot{x} = (A - bk)x + br$$
$$y = (c - dk)x + dr$$

Is it possible to assign the eigenvalues arbitrarily?

Theorem 2: If n-dimensional equation (I) is controllable, then by state feedback u=r-kx, which k is a 1×n constant vector with real elements, we can arbitrarily assign eigenvalues of A-bk. Note that eigenvalues must consider as complex conjugate if complex.

Pole placement with state feedback

Example 2: Is it possible to assign the eigenvalues of following system in arbitrary places?

$$\dot{x} = \begin{bmatrix} 0 & 0 & -8 \\ 1 & 0 & -14 \\ 0 & 1 & -7 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} u \quad \Rightarrow \quad C = \begin{bmatrix} b & Ab & A^2 b \end{bmatrix} = \begin{bmatrix} 1 & 0 & -16 \\ 2 & 1 & -28 \\ 0 & 2 & -13 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 1 & 0 & -16 \\ 2 & 1 & -28 \\ 0 & 2 & -13 \end{vmatrix} = -21 \quad \Rightarrow \quad \text{System is controllable}$$

So …..

Pole placement with state feedback

Example 3: Is it possible to assign the eigenvalues of following system in arbitrary places?

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$A_{cl} = A - BK = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 1-k_1 & 1-k_2 \\ 0 & 2 \end{bmatrix}$$

$$\det(sI - A_{cl}) = (s - 1 + k_1)(s - 2) = 0$$

s=2 is a fixed mode.

s=2 is not controllable.

Pole placement with state feedback

Example 4: Consider following system.

$$\dot{x}=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & -3\end{bmatrix}x+\begin{bmatrix}1\\ 2\\ 0\end{bmatrix}u \quad \Rightarrow \quad \text{System is not controllable}$$

I) Is it possible to assign eigenvalues on arbitrary location?

II) Is it possible to assign eigenvalues on -1, -2, -3?

III) Is it possible to assign eigenvalues on $-1\pm j2$, -3?

IV) Is it possible to assign eigenvalues on -1-j2,-1-j3 -3?

Pole placement with state feedback

1- Direct method.

2- Use of similarity transformation.

3- Use of Lyapunov Equation

Pole placement with state feedback

1- Direct method

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

Let $u = r - [k_1 \;\; k_2 \; ... \; k_n]x$

Find : $|sI - A + bk|$ = Desired characteristic equation

Polynomial of Degree n — Polynomial of Degree n

Then determine $k_1, k_2, \dots, k_n$ from above equation

Pole placement with state feedback

2- Use of similarity transformation **(Proof of Theorem 2)**

$$\dot{x} = Ax + bu \qquad w = Px$$

$$q = [0 \quad 0 \cdots 1]C^{-1} \qquad P = \begin{bmatrix} q \\ qA \\ \vdots \\ qA^{n-1} \end{bmatrix}$$

$$\dot{w} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} \end{bmatrix} w + \begin{bmatrix} 0 \\ 0 \\ .. \\ 0 \\ 1 \end{bmatrix} u$$

Characteristic equation of system is : $s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots + a_1 s + a_0$

$$\hat{A} - \hat{b}\hat{k} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0 - \hat{k}_0 & -a_1 - \hat{k}_1 & -a_2 - \hat{k}_2 & \dots & -a_{n-1} - \hat{k}_{n-1} \end{bmatrix}$$

$$-b_0 \qquad -b_1 \qquad -b_2 \qquad -b_{n-1}$$

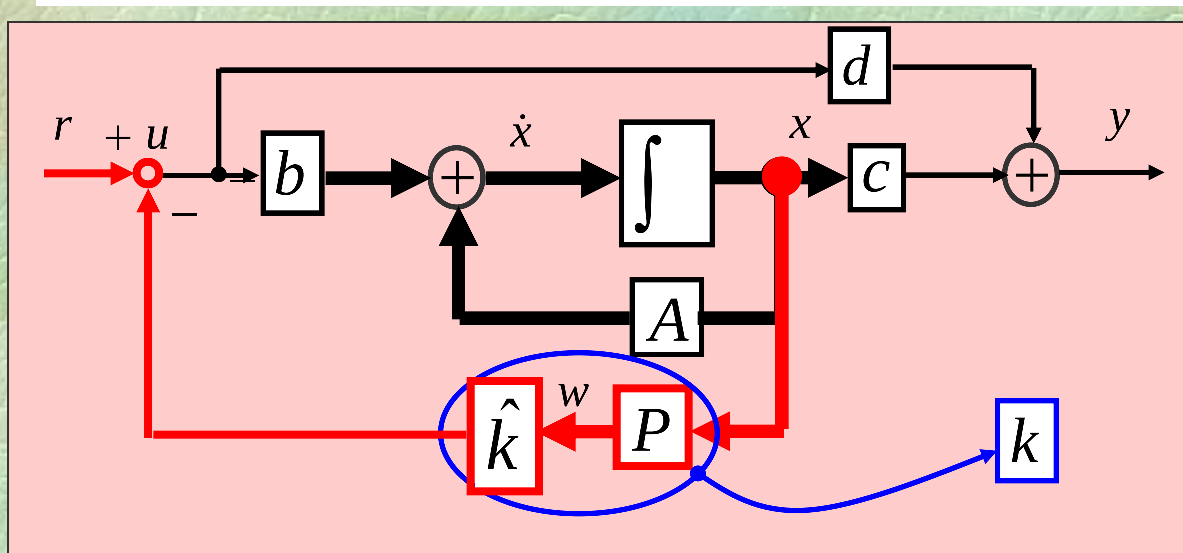
Desired characteristic equation is : $s^n + b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \dots + b_1 s + b_0$

Pole placement with state feedback

$$\hat{A}-\hat{b}\hat{k}=\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_0-\hat{k}_0 & -a_1-\hat{k}_1 & -a_2-\hat{k}_2 & \dots & -a_{n-1}-\hat{k}_{n-1} \end{bmatrix}$$

$$-b_0 \qquad -b_1 \qquad -b_2 \qquad -b_{n-1}$$

$$\hat{k}=\begin{bmatrix} b_0-a_0 & b_1-a_1 & b_2-a_2 & \dots & b_{n-1}-a_{n-1} \end{bmatrix}$$



$$u=r-\hat{k}w=r-\hat{k}Px=r-kx \quad \longrightarrow \quad k=\hat{k}P$$

Pole placement with state feedback

Example 5: Assign the eigenvalues of the following system to −2±j, if possible.

$$\dot{x} = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u \qquad C = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \qquad |C| = -1 \neq 0$$

System is controllable

So it is possible to assign the poles on -2±j .

First method.

$$A - bk = \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} [k_0 \quad k_1] = \begin{bmatrix} 1-k_0 & -1-k_1 \\ -k_0 & -1-k_1 \end{bmatrix}$$

$$|sI - A + bk| = \begin{vmatrix} s-1+k_0 & 1+k_1 \\ k_0 & s+1+k_1 \end{vmatrix} = s^2 + (k_0 + k_1)s - 1 - k_1$$

$$s^2 + (k_0 + k_1)s - 1 - k_1 = (s+2+j)(s+2-j) = s^2 + 4s + 5 \rightarrow k = [10 \quad -6]$$

Pole placement with state feedback

Example 5: Assign the eigenvalues of the following system to −2±j, if possible.

$$\dot{x}=\begin{bmatrix}1 & -1\\ 0 & -1\end{bmatrix}x+\begin{bmatrix}1\\ 1\end{bmatrix}u \qquad C=\begin{bmatrix}1 & 0\\ 1 & -1\end{bmatrix} \qquad |C|=-1\neq 0$$

System is controllable

So it is possible to assign the poles on -2±j .

Second method.

$$P=\begin{bmatrix}q\\ qA\end{bmatrix} \qquad q=[0 \quad 1]C^{-1} \qquad P=\begin{bmatrix}1 & -1\\ 1 & 0\end{bmatrix}$$

System characteristic equation: s^2+0s-1

Desired characteristic equation: $(s+2+j)(s+2-j)=s^2+4s+5$

$$\hat{k}=[b_0-a_0 \quad b_1-a_1]=[5-(-1) \quad 4-0]=[6 \quad 4] \rightarrow k=\hat{k}P=[10 \quad -6]$$

MATLAB

place(A,b,[-2+i -2-i])

Tracking and regulator problems

Is the system track input response?

Steady state error to unit input?

Consider following system:

$$g(s)=\frac{y(s)}{r(s)}=\frac{\beta_1 s^3+\beta_2 s^2+\beta_3 s+\beta_4}{s^4+\alpha_1 s^3+\alpha_2 s^2+\alpha_3 s+\alpha_4}$$

Step response is:

$$If \;\; u(t)=unit \;\; step \;\rightarrow\; y(t)\big|_{t\to\infty}=g(0)=\frac{\beta_4}{\alpha_4}$$

Solving the problem by:

$$u(t)=pr(t)-kx(t)$$

Tracking and regulator problems

$$u(t) = pr(t) - kx(t)$$

Transfer function is:

$$g_f(s) = \frac{y_f(s)}{r(s)} = p\frac{\beta_1 s^3 + \beta_2 s^2 + \beta_3 s + \beta_4}{s^4 + \alpha_1 s^3 + \alpha_2 s^2 + \alpha_3 s + \alpha_4} = pg(s)$$

And now

$$If \quad u(t) = unit \;\; step \;\rightarrow\; y(t)\Big|_{t\rightarrow\infty} = p\frac{\beta_4}{\alpha_4} = pg(0)$$

$$p = \frac{1}{g(0)} = \frac{\alpha_4}{\beta_4}$$

Tracking and regulator problems

Summary:

If the pair (A,b) is controllable, there exists a state feedback $u = r - kx$ where the eigenvalues of the new system matrix $A - bk$ can be assigned arbitrarily. Additionally, if $c(si - A)^{-1}b$ has no zeros at the origin, then in addition to state feedback, there exists a feedforward gain p such that the resultant system follows any step input asymptotically.

Tracking and regulator problems

Example 6: Assign the eigenvalues of the following system such that the damping ratio of new poles is 0.707 if possible.

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

System is controllable

The new poles must be on the line ξ=0.707, so we assign them at 2±2j.

$$k = \begin{bmatrix} -7 & -21 \end{bmatrix}$$

$$u = r - \begin{bmatrix} -7 & -21 \end{bmatrix} x$$

$$\dot{x}(t) = \begin{bmatrix} -6 & -20 \\ 1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

Tracking and regulator problems

System analysis:

$$\dot{x}(t)=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}x(t)+\begin{bmatrix}-1\\ 0\end{bmatrix}u \quad \xrightarrow{u=r-[-7\quad -21]x} \quad \dot{x}(t)=\begin{bmatrix}-6 & -20\\ 1 & 2\end{bmatrix}x(t)+\begin{bmatrix}-1\\ 0\end{bmatrix}r$$

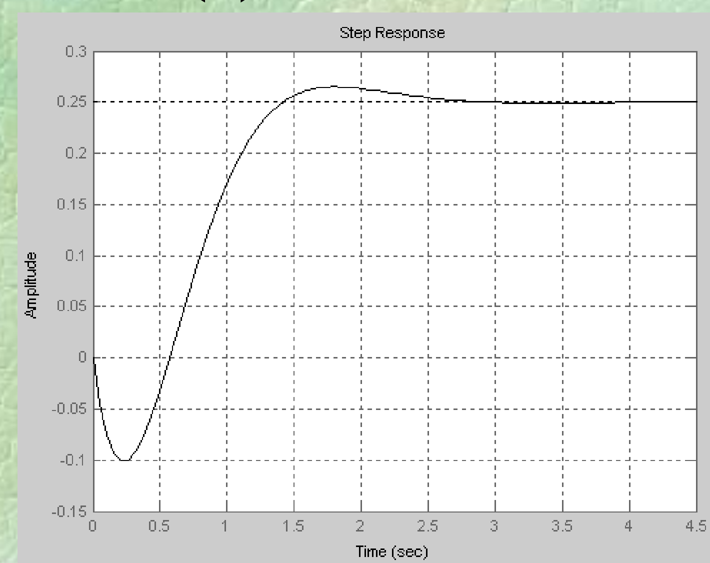
$$y=[1\quad 0]x(t) \qquad\qquad y=[1\quad 0]x(t)$$

$$\frac{y(s)}{r(s)}=\frac{-s+2}{s^2-3s+1} \qquad\qquad \frac{y(s)}{r(s)}=\frac{-s+2}{s^2+4s+8}$$

Draw backs ?

- SS error

$$\left.\frac{y(s)}{r(s)}\right|_{s=0}=\frac{2}{8}\neq 1$$



Tracking and regulator problems

Solving the ss problem:

$$\dot{x}(t)=\begin{bmatrix}1 & 1\\ 1 & 2\end{bmatrix}x(t)+\begin{bmatrix}-1\\ 0\end{bmatrix}u$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}x(t)$$

$$u=r-\begin{bmatrix}-7 & -21\end{bmatrix}x$$

$$\dot{x}(t)=\begin{bmatrix}-6 & -20\\ 1 & 2\end{bmatrix}x(t)+\begin{bmatrix}-1\\ 0\end{bmatrix}r$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}x(t)$$

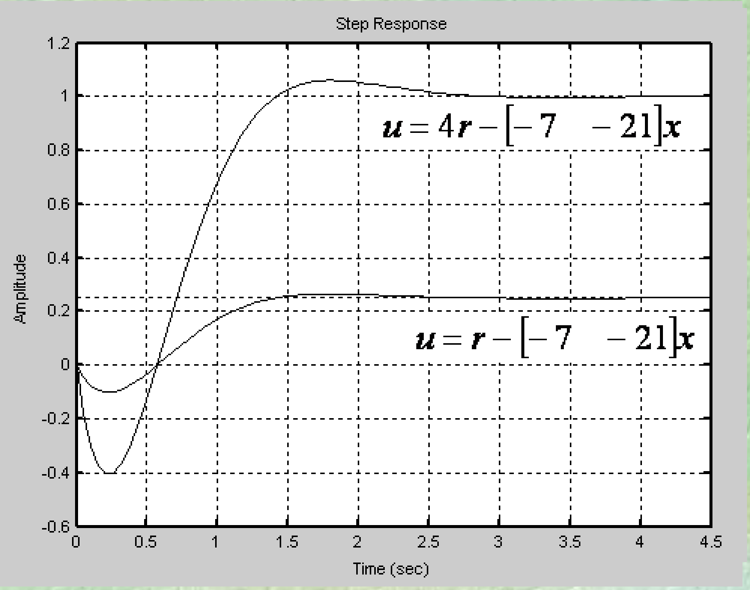
$$\frac{y(s)}{r(s)}=\frac{-s+2}{s^2+4s+8}$$

$$u=pr-\begin{bmatrix}-7 & -21\end{bmatrix}x$$

$$\dot{x}(t)=\begin{bmatrix}-6 & -20\\ 1 & 2\end{bmatrix}x(t)+\begin{bmatrix}-4\\ 0\end{bmatrix}r$$

$$y=\begin{bmatrix}1 & 0\end{bmatrix}x(t)$$

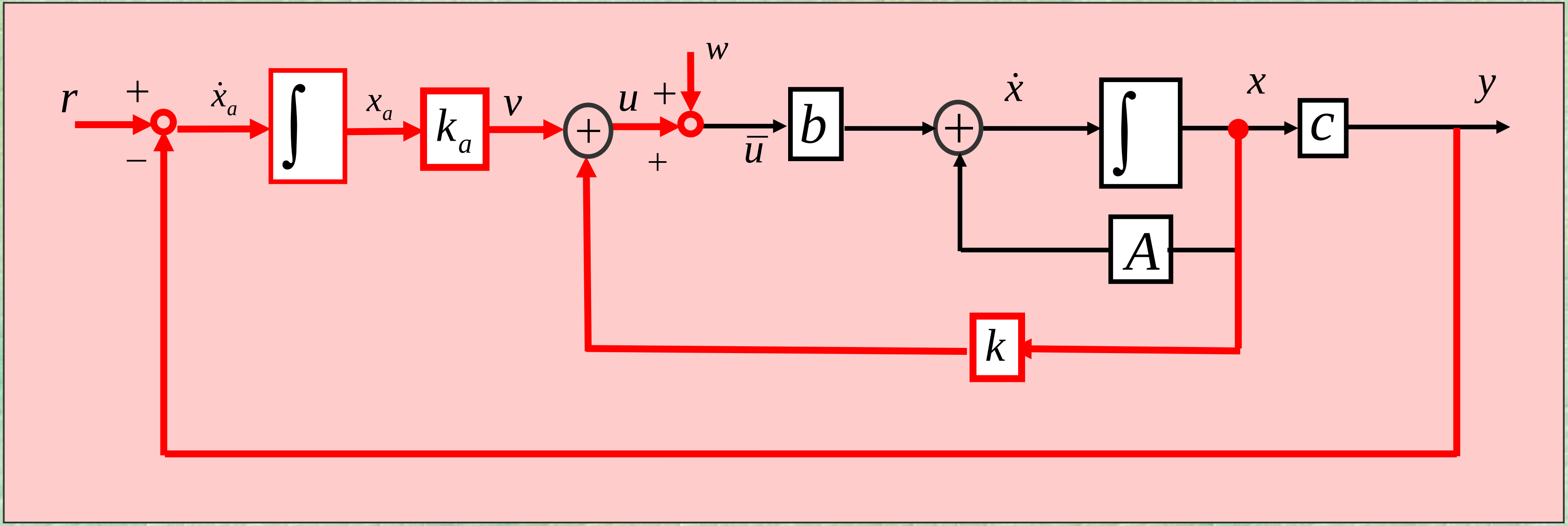
$$\frac{y(s)}{r(s)}=\frac{4(-s+2)}{s^2+4s+8}$$



Robust Tracking and Disturbance Rejection

$$\dot{x} = Ax + bu + bw$$

$$y = cx$$



$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A & 0 \\ -c & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r + \begin{bmatrix} b \\ 0 \end{bmatrix} w$$

$$\dot{x}_a = r - y = r - cx$$

$$y = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \end{bmatrix}$$

Robust Tracking and Disturbance Rejection

$$\dot{x} = Ax + bu + bw \qquad u = [\mathbf{k} \quad k_a]\begin{bmatrix} x \\ x_a \end{bmatrix}$$
$$y = cx$$

state feedback

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \end{bmatrix} = \begin{bmatrix} A + bk & bk_a \\ -c & 0 \end{bmatrix}\begin{bmatrix} x \\ x_a \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r + \begin{bmatrix} b \\ 0 \end{bmatrix} w$$

$$y = [c \quad 0]\begin{bmatrix} x \\ x_a \end{bmatrix} \qquad (1)$$

Theorem 3: If the pair (A,b) is controllable and the transfer function $c(sI - A)^{-1}b$ has no zeros at the origin, then the eigenvalues of system (1) can be assigned to arbitrary position.

Robust Tracking and Disturbance Rejection

Example 7: The equation for the longitudinal motion of an aircraft is:

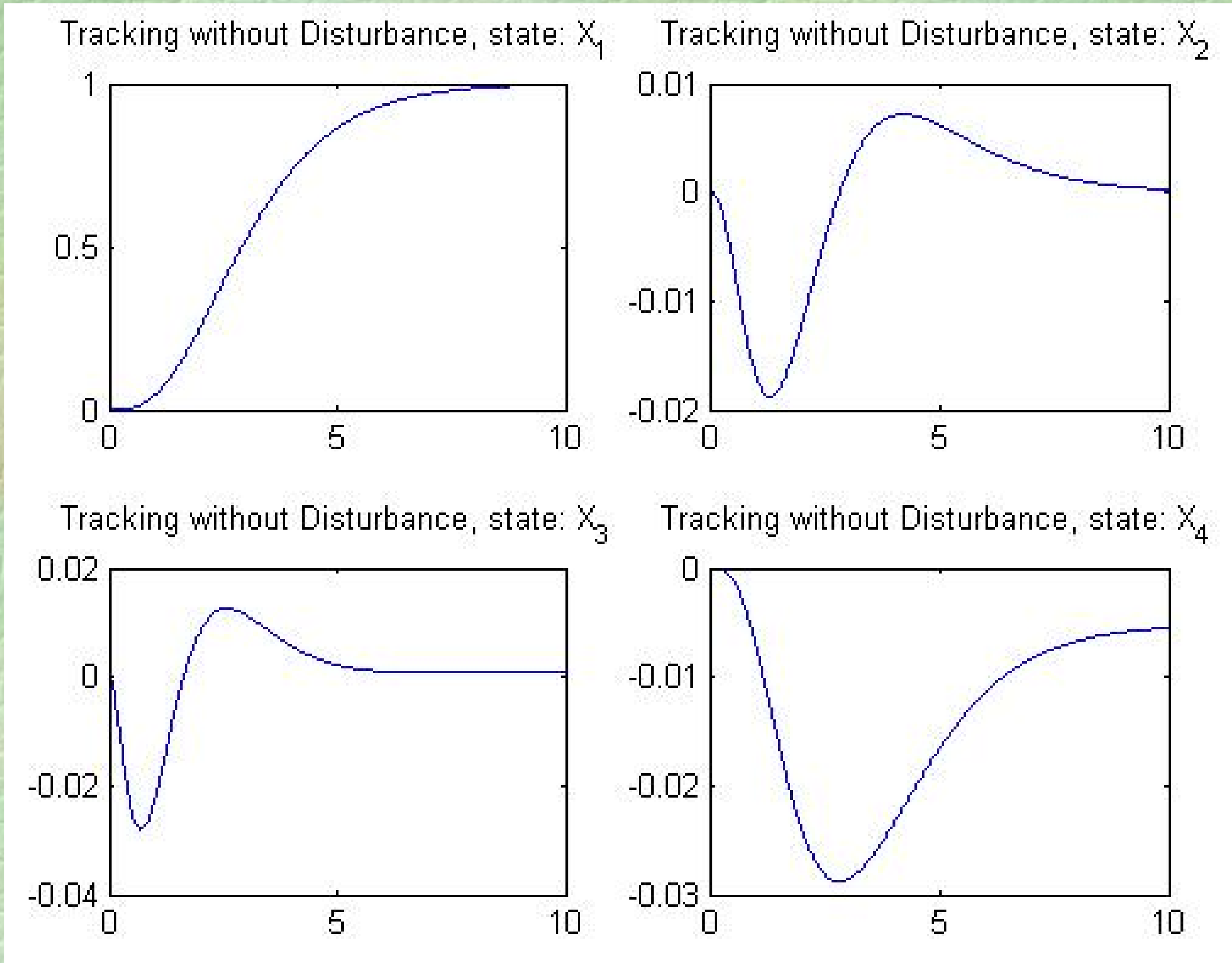
$$\dot{x}(t)=\begin{bmatrix} -0.0507 & -3.861 & 0 & -9.81 \\ -0.0017 & -0.5164 & 1 & 0 \\ -0.000129 & 1.4168 & -0.4932 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x(t)+\begin{bmatrix} 0 \\ -0.717 \\ -1.645 \\ 0 \end{bmatrix} u(t)+\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} w$$

$$y(t)=\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x(t)$$

Design a state feedback controller to assign the eigenvalues to -0.9, -1, -1.6, -2, and -2.5

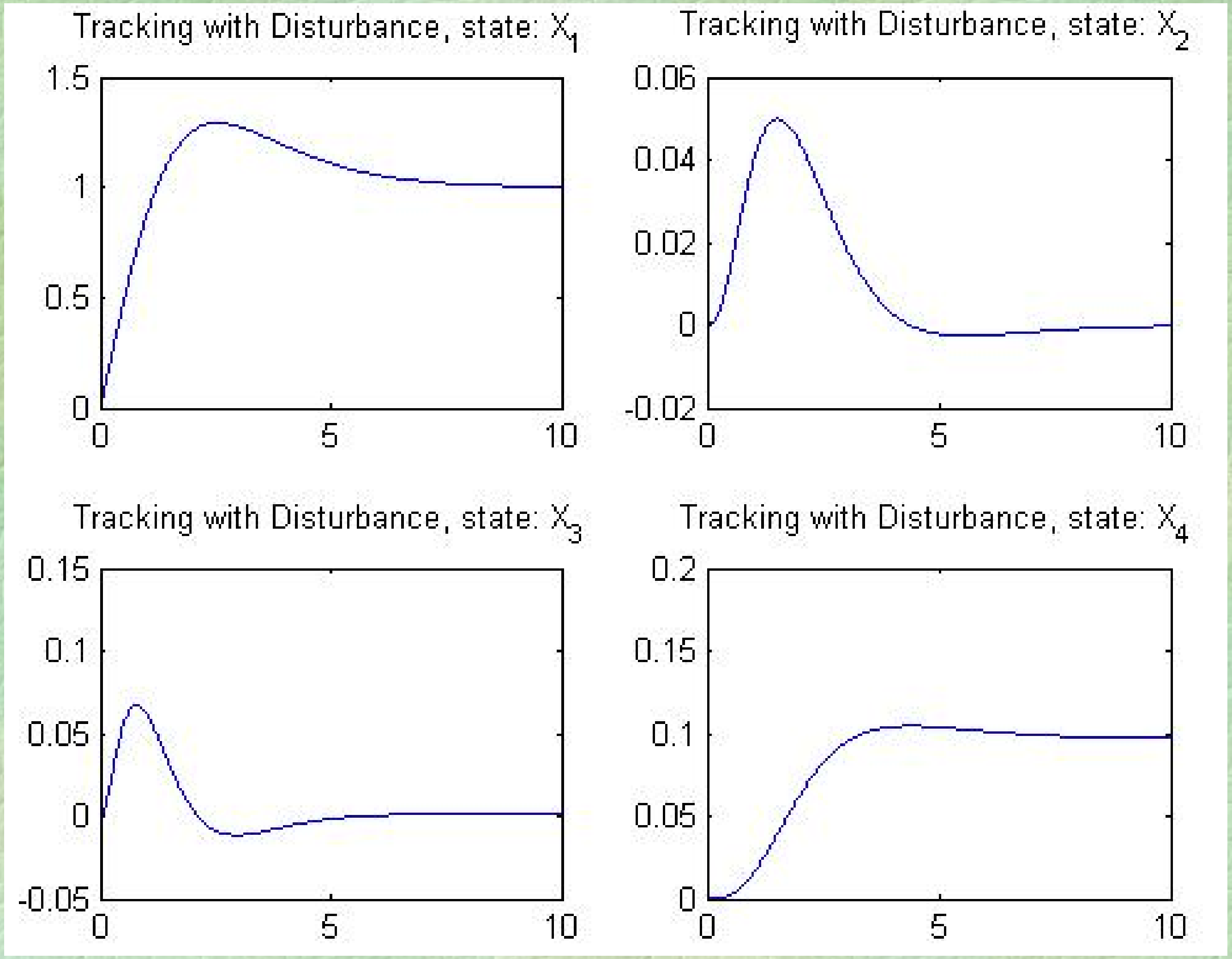
Robust Tracking and Disturbance Rejection

Step response



Robust Tracking and Disturbance Rejection

Step response of input and disturbance:



Stabilization

Stabilizable system?

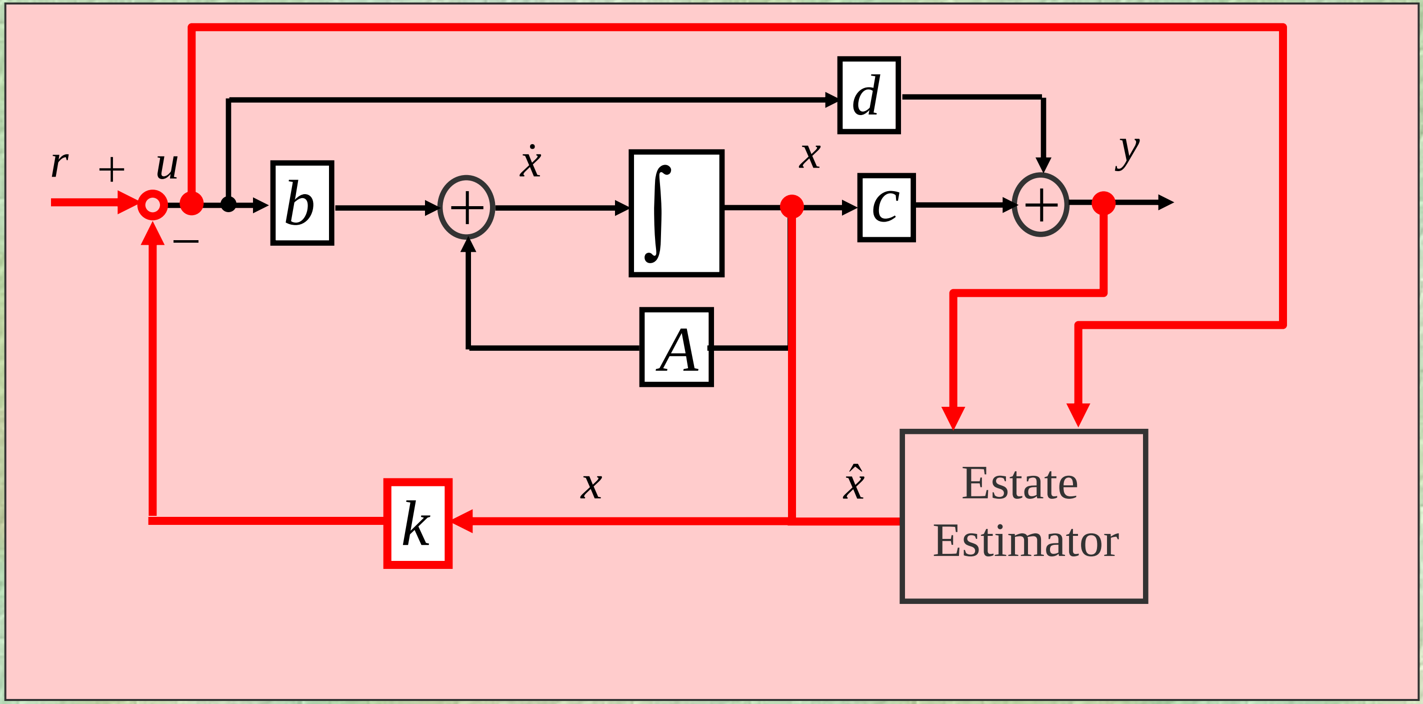
$$P^{-1} = [q_1 \quad q_2 \quad \dots q_{n1} \; \dots . \quad q_n]$$

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{x}}_{\bar{c}} \end{bmatrix} = \begin{bmatrix} \bar{A}_c & \bar{A}_{12} \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} \bar{b}_c \\ 0 \end{bmatrix} u \qquad (I)$$

(I) with $u = r - kx = r - [\bar{k}_1 \;\; \bar{k}_2] \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix}$ →

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{x}}_{\bar{c}} \end{bmatrix} = \begin{bmatrix} \bar{A}_c - \bar{b}_c \bar{k}_1 & \bar{A}_{12} - \bar{b}_c \bar{k}_2 \\ 0 & \bar{A}_{\bar{c}} \end{bmatrix} \begin{bmatrix} \bar{x}_c \\ \bar{x}_{\bar{c}} \end{bmatrix} + \begin{bmatrix} \bar{b}_c \\ 0 \end{bmatrix} r$$

Use of state estimation to use in feedback loop



States are not available!

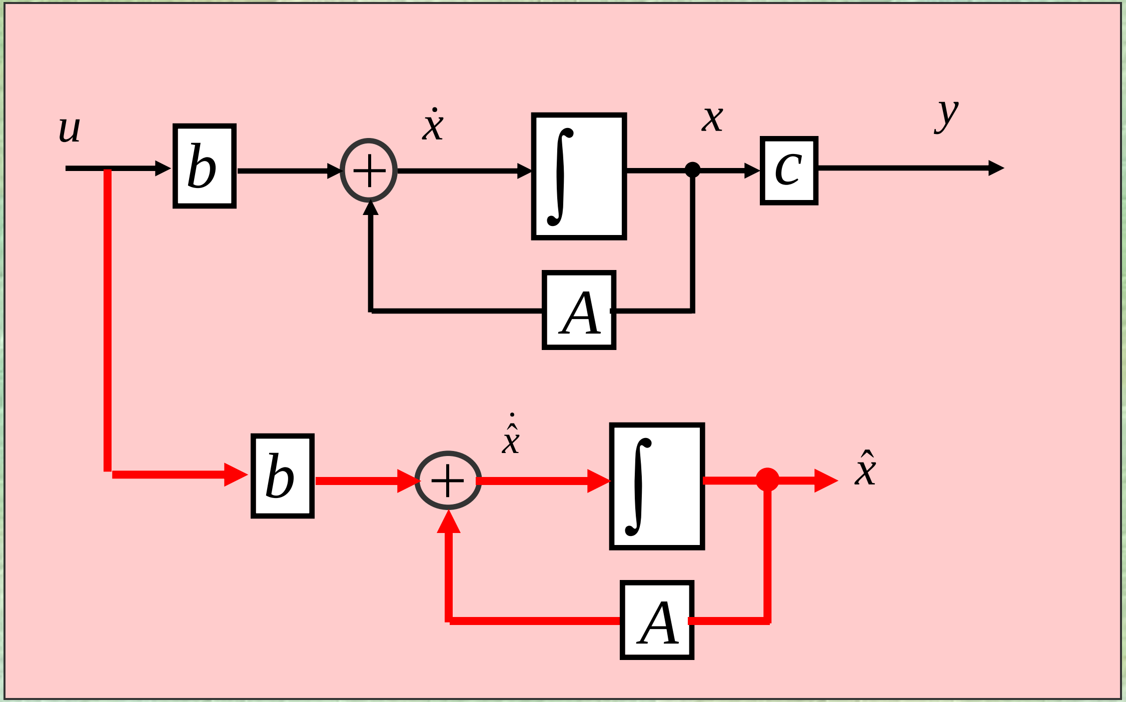
Condition for $\hat{x} \rightarrow x$

Open loop state estimator

$$\dot{x} = Ax + bu$$
$$y = cx$$

$$\dot{\hat{x}} = A\hat{x} + bu$$

$$e(t) = x(t) - \hat{x}(t)$$



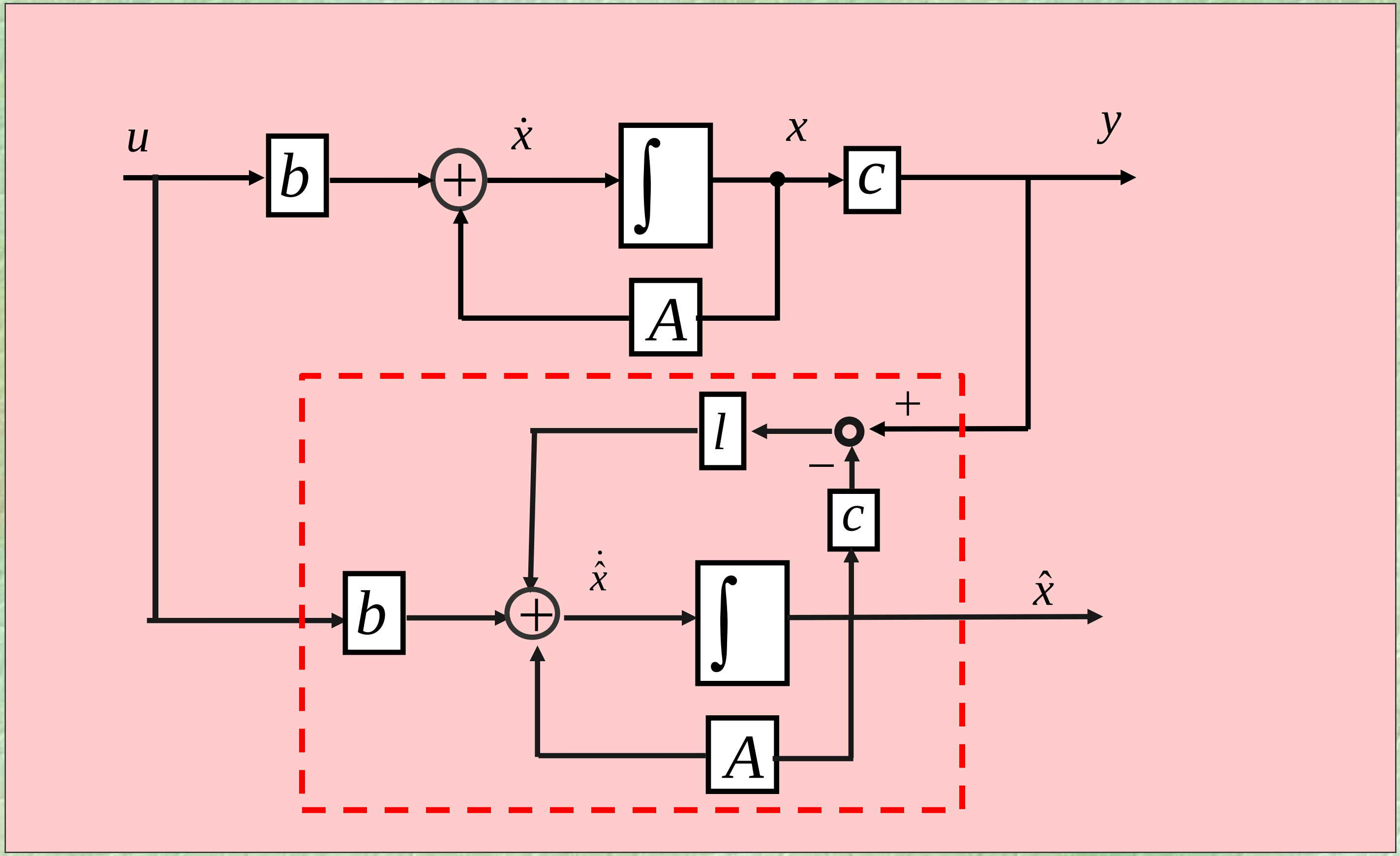
$$\dot{e} = \dot{x} - \dot{\hat{x}}$$
$$= Ax + bu - A\hat{x} - bu$$

$$\dot{e}(t) = Ae(t)$$

$$e(t) = e^{At} e_0$$

Drawbacks?

Close loop state estimator



$$\dot{\hat{x}} = A\hat{x} + bu + l(y - c\hat{x}) \quad \longrightarrow \quad \dot{\hat{x}} = (A - lc)\hat{x} + bu + ly$$

Estimation error

$$e(t) = x(t) - \hat{x}(t)$$

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + bu - (A - lc)\hat{x} - bu - l(cx)$$

$$= (A - lc)x - (A - lc)\hat{x}$$

$$= (A - lc)(x - \hat{x})$$

$$\dot{e}(t) = (A - lc)e(t) \qquad \Rightarrow \qquad e(t) = e^{(A-lc)t}e_0$$

In what situation can we place eigenvalues in arbitrary locations?

State estimator

Theorem 4: Consider the pair (A,c). All eigenvalues of (A-lc) can be assigned by a real vector l in suitable locations if and only if (A,c) is observable.

Proof:

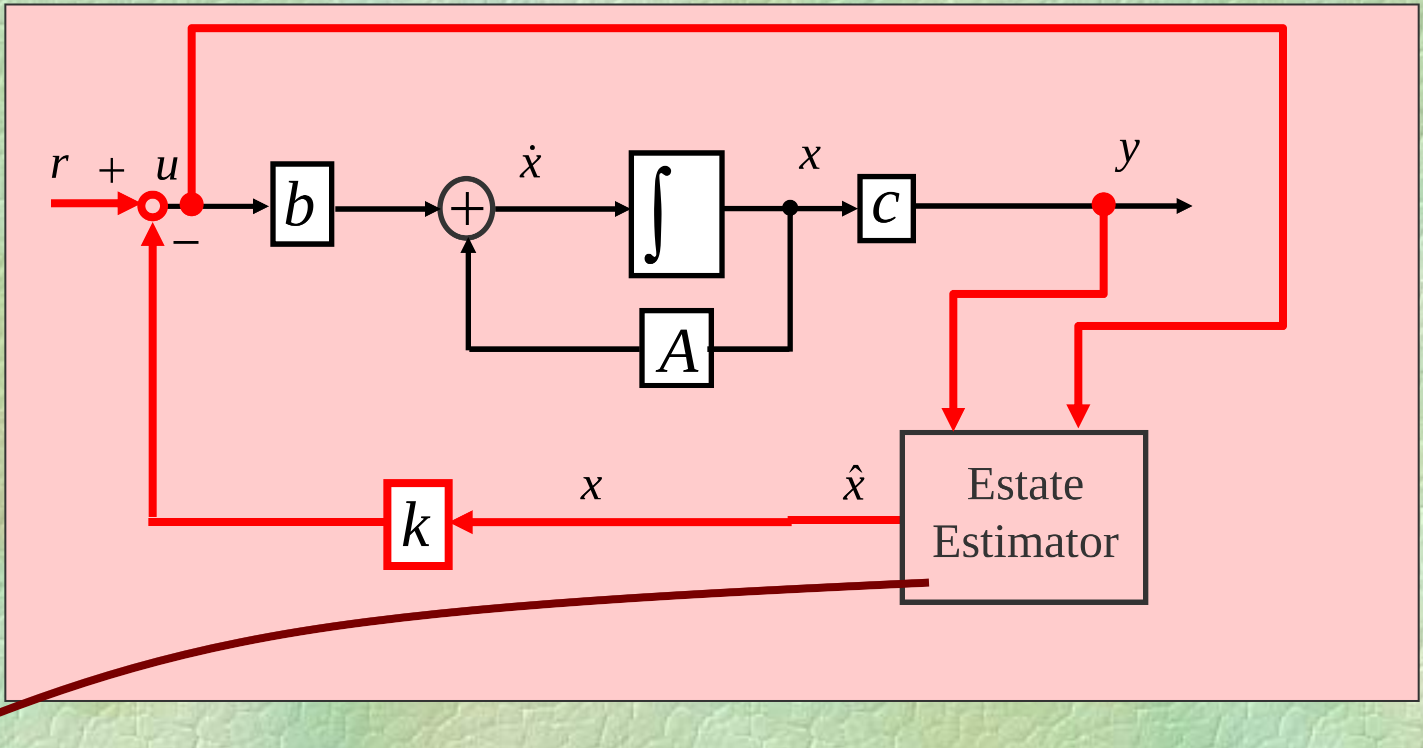
$$(\mathrm{A},\mathrm{c})\ \text{ observable} \Leftrightarrow (\mathrm{A}',\mathrm{c}')\ \text{controllable}$$

$$(\mathrm{A}',\mathrm{c}')\ \text{controllable} \Leftrightarrow \text{All eigenvalue of } (\mathrm{A}'-\mathrm{c}'\mathrm{k})\ \text{can be assigned}$$

$$\text{aribitrarily by selecting a constant vector k}$$

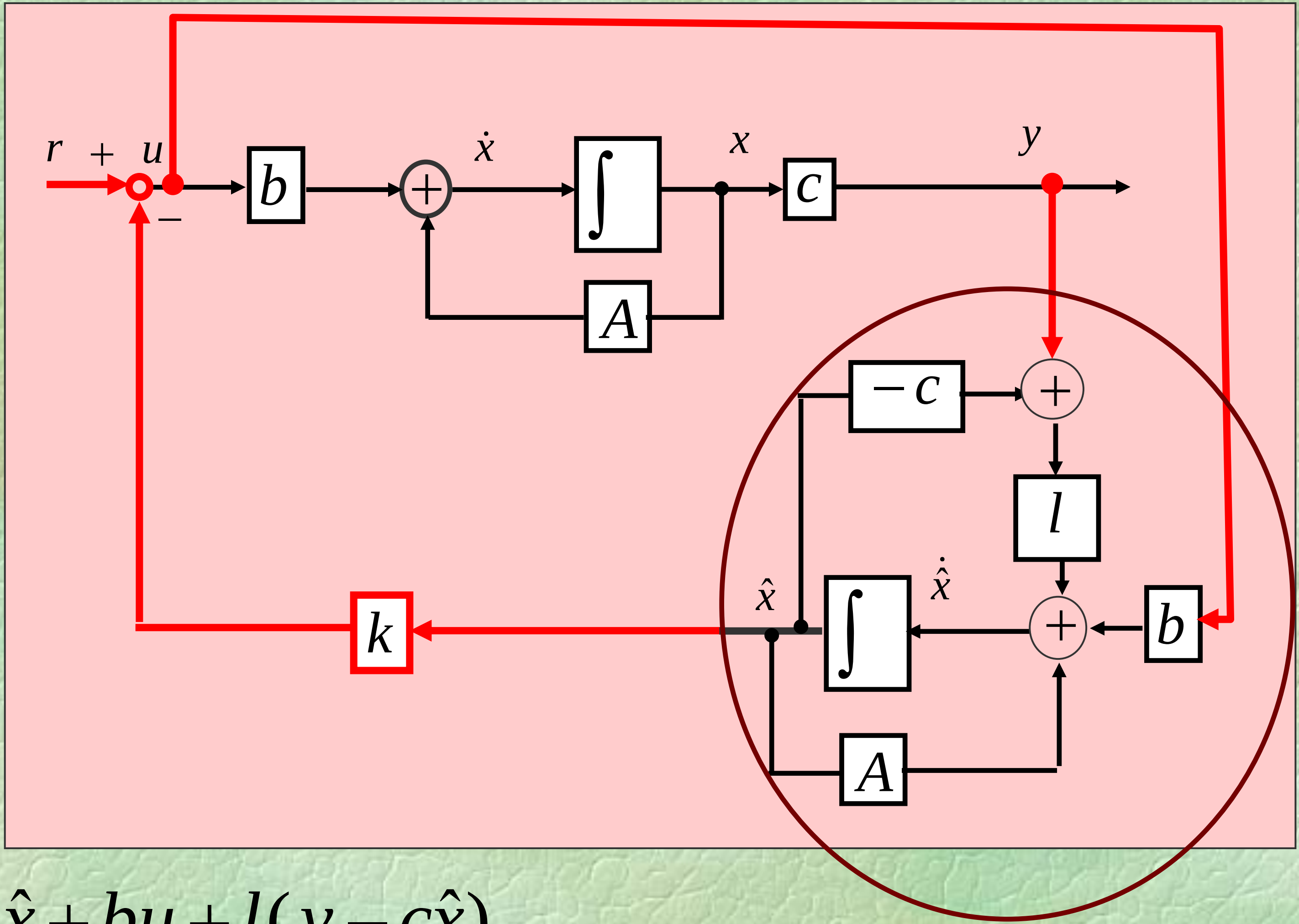
$$(\mathrm{A}'-\mathrm{c}'\mathrm{k})' = \mathrm{A}-\mathrm{ck}' \Rightarrow \mathrm{l} = \mathrm{k}'$$

Use of state estimation to use in feedback loop



$$\dot{\hat{x}} = A\hat{x} + bu + l(cx - c\hat{x})$$

Use of state estimation to use in feedback loop



$$\dot{\hat{x}} = A\hat{x} + bu + l(y - c\hat{x})$$

Estimator

Use of state estimation to use in feedback loop

$$u = r - k\hat{x}$$

state feedback

$$\dot{x} = Ax - bk\hat{x} + br$$

$$\dot{\hat{x}} = (A - lc)\hat{x} + b(r - k\hat{x}) + lcx$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -bk \\ lc & A - lc - bk \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} r$$

$$y = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - bk & bk \\ 0 & A - lc \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} r$$

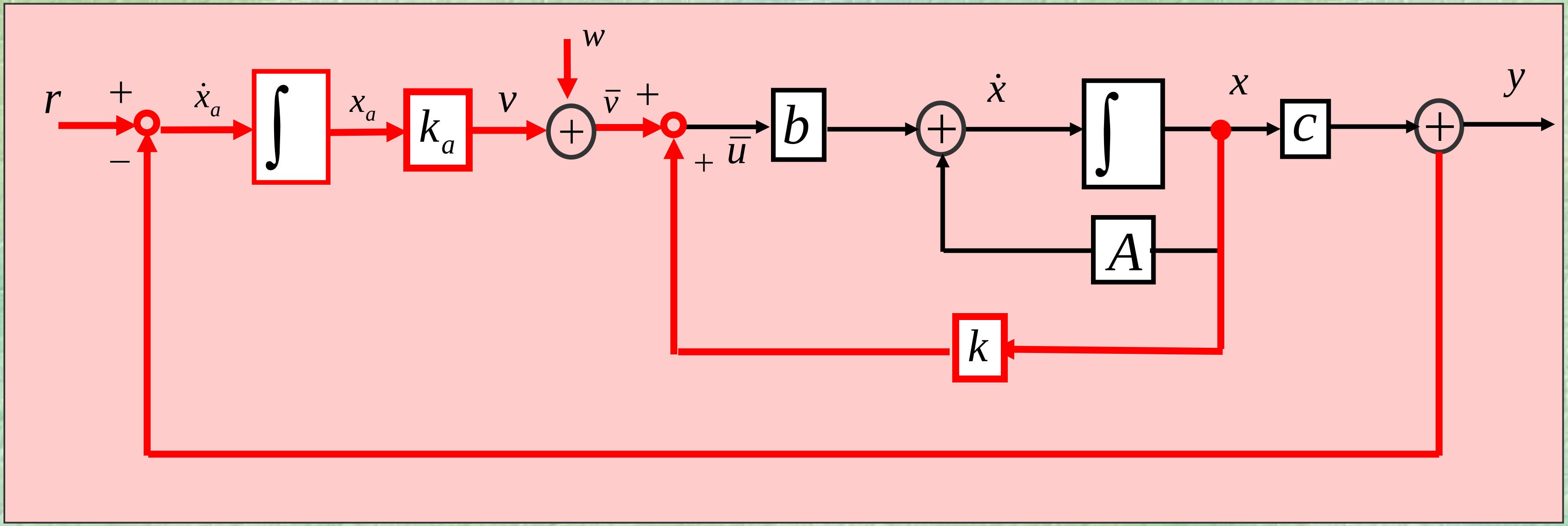
$$y = \begin{bmatrix} c & 0 \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}$$

Separation Property:
Using a state estimator does not have any side effect on the eigenvalues of the state feedback. Additionally, the eigenvalues of the state estimator do not change with this connection.

Exercises

Exercise 1: Prove theorem 3.

Exercise 2: In the following system suppose (A,b) is controllable, and the transfer function $c(sI-A)^{-1}b$ has no zeros at the origin. Show that the output is zero at steady state if there is a step input in disturbance w.



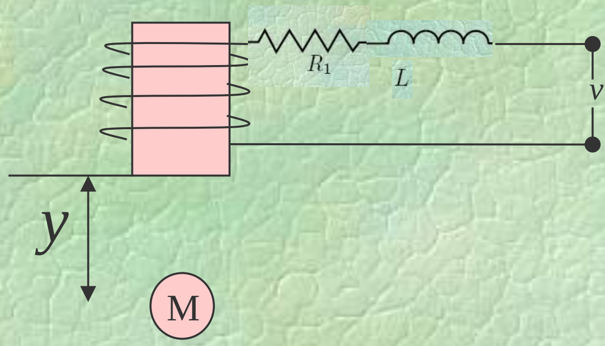
Exercise 3: In the above system suppose (A,b) is controllable, and the transfer function $c(sI-A)^{-1}b$ has no zeros at the origin. Show that the output follows a step in the reference r in steady state

Exercises

Exercise 4: Consider the following state-space model, which corresponds to a suspension system.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 980 & 0 & -208 \\ 0 & 0 & -100 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x$$



a) Place the eigenvalues of the system at -2±2j and −10 using appropriate state feedback.

b) Plot the system response to arbitrary non-zero initial condition.

c) Design a state estimator for the system.

d) Repeat part (b) for new system with same initial condition.

e) Plot the real states and the estimated states in the same figure.

Exercises

Exercise 5: Consider the given state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -24 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ -8 \\ 106 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) Place the eigenvalues of the system at -1±2j and −5 using appropriate state feedback.

b) Plot the system response to arbitrary non-zero initial condition.

c) Design a state estimator for the system and place its eigenvalues on -10, -10, and -10.

d) Repeat part (b) for new system with same initial condition.

e) Plot the real states and the estimated states in the same figure.

f) Design a reduced order state estimator for the system and place its eigenvalues on -10, and -10.

g) Repeat part (b) and (e) for the new system with same initial condition.

Exercises

تمرینها

Exercise 6: Consider the given state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Place the eigenvalues of the system at -1 and -2 using appropriate state feedback and the direct method.

b) Place the eigenvalues of the system at -1 and -2 using appropriate state feedback and the similarity transformation method.

c) Place the eigenvalues of the system at -1 and -2 using appropriate state feedback and the Lyapunov equation method.

Exercise 7: Consider the given state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

Place the eigenvalues of the system at -1±1j and -2.

Exercises

Exercise 8: Consider the given state-space model.

$$\dot{x} = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} x$$

Design the state feedback and feedforward gain so that the system's eigenvalues are placed at −1±j1 and −2, and the system tracks a step input with zero error.

Exercise 9: Consider the given state-space model.

$$\dot{x} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} u$$

a) Is it possible to place the eigenvalues of the system at −1, -1, -2, and -2 using state feedback?

b) Is it possible to place the eigenvalues of the system at −1, -2, -2, and -2 using state feedback?

c) Is it possible to place the eigenvalues of the system at −2, -2, -2, and -2 using state feedback?

d) Is this system stabilizable?

Exercises

تمرینها

Exercise 10: Consider the given state-space model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Design a full order state estimator and reduced order state estimator for the system and chose its eigenvalues from the set {-3, -2±2j}.

Exercise 11: Consider the following transfer function.

$$g(s) = \frac{(s-1)(s+2)}{(s+1)(s-2)(s+3)}$$

Is it possible to convert it to following transfer function through state feedback?

$$g_f(s) = \frac{s-1}{(s+2)(s+3)}$$

Is it the resultant system BIBO stable? Is it asymptotically stable?

Exercises

Exercise 12: Consider the following transfer function.

$$g(s)=\frac{(s-1)(s+2)}{(s+1)(s-2)(s+3)}$$

Is it possible to convert it to following transfer function through state feedback?

$$g_f(s)=\frac{1}{s+3}$$

Is it the resultant system BIBO stable? Is it asymptotically stable?

Exercise 13: Consider the given state-space model(Final 2013).

$$x=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}x+\begin{bmatrix}0\\ 1\end{bmatrix}u$$

a) Place the eigenvalues of the system at -1±2j using appropriate state feedback.

b) Design a state estimator for the system and place its eigenvalues of estimator at -10, and -10.

c) Draw block diagram of resultant system.

d) Derive the eigenvalue of resultant system.

Exercises

Exercise 14: Consider the given state-space model(Final 2013).

$$x = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

a) Is it possible to assign the eigenvalue of the system at arbitrary locations by state feedback.

b) Place the eigenvalues of the system at -2±2j by state feedback if possible.

Answers to selected problems

Answer 6: [4 1]

Answer 8: u=pr-kx, p=0.5, k=[15 47 -8]

Answer 10: Second order state estimation.

$$\dot{z} = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix} z + \begin{bmatrix} 0.6282 \\ -0.3105 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} y$$

$$\hat{x} = \begin{bmatrix} -12 & -27.5 \\ 19 & 32 \end{bmatrix} z$$

First order state estimation

$$\dot{z} = -3z + (13/21)u + y$$

$$\hat{x} = \begin{bmatrix} -4 & 21 \\ 5 & -21 \end{bmatrix} \begin{bmatrix} y \\ z \end{bmatrix}$$

Answer 11: yes, yes, and yes