LINEAR CONTROL SYSTEMS

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Lecture 8 - Part III

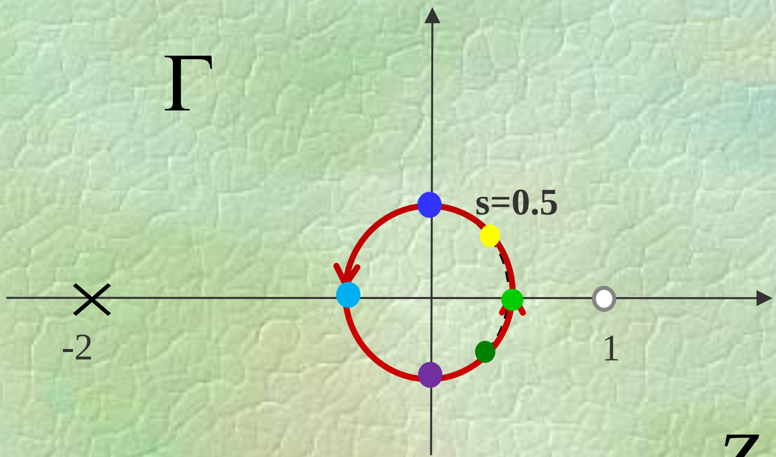
Nyquist Stability Criteria

Topics to be covered include:

- Nyquist stability criteria.
- Minimum phase systems.
- Simplified Nyquist stability criterion.

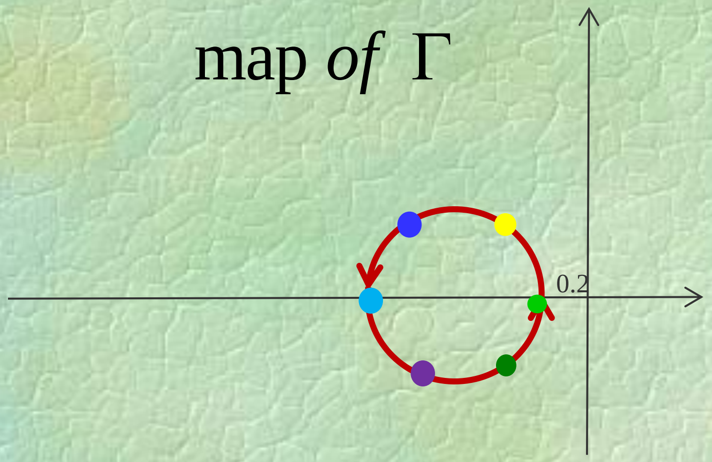
Mapping

Γ



$$f(s) = \frac{s-1}{s+2}$$

map *of* Γ



$$Z_0 - P_0 = N_0$$

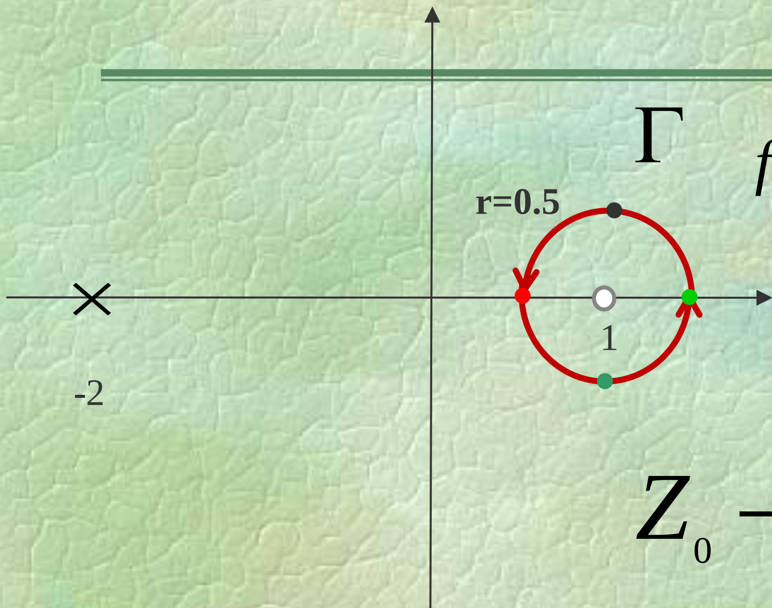
$Z_0 = \textit{number of zeros of } f(s) \textit{ that are encircled by } \Gamma$

$P_0 = \textit{number of poles of } f(s) \textit{ that are encircled by } \Gamma$

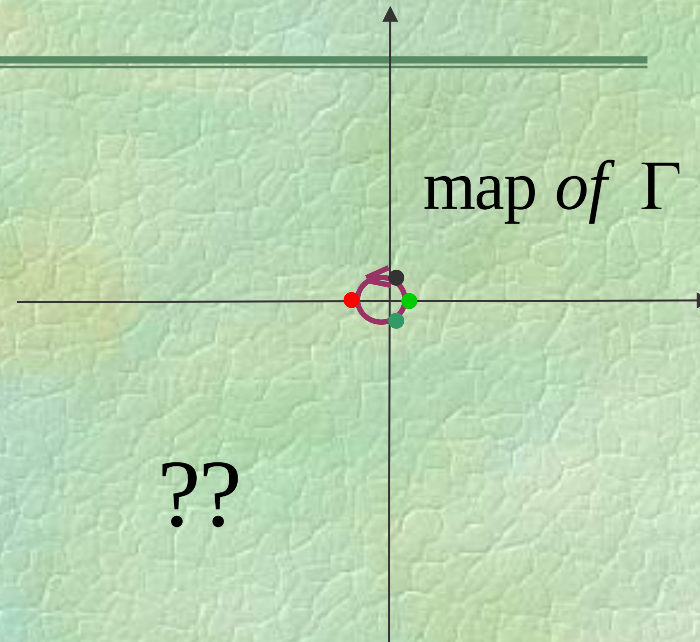
$N_0 = \textit{number of encirclements of the origin made by } f(s)$

$$0 - 0 = 0$$

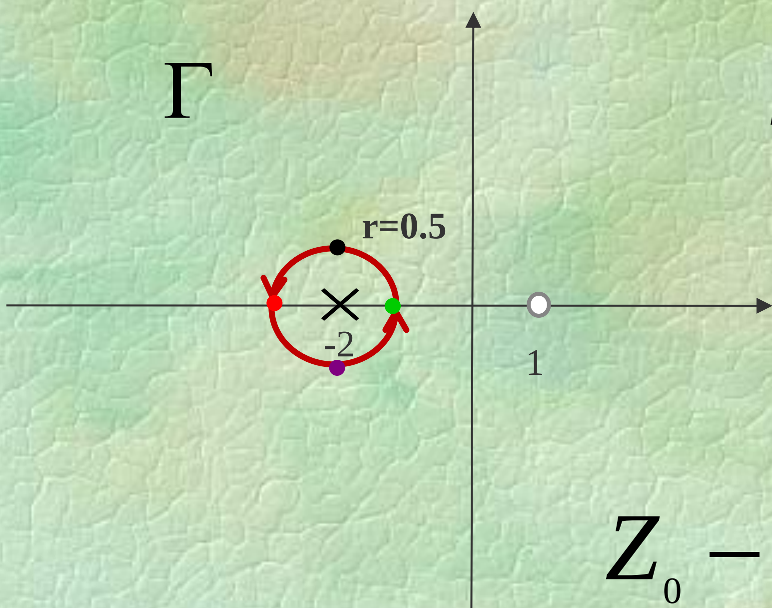
Mapping



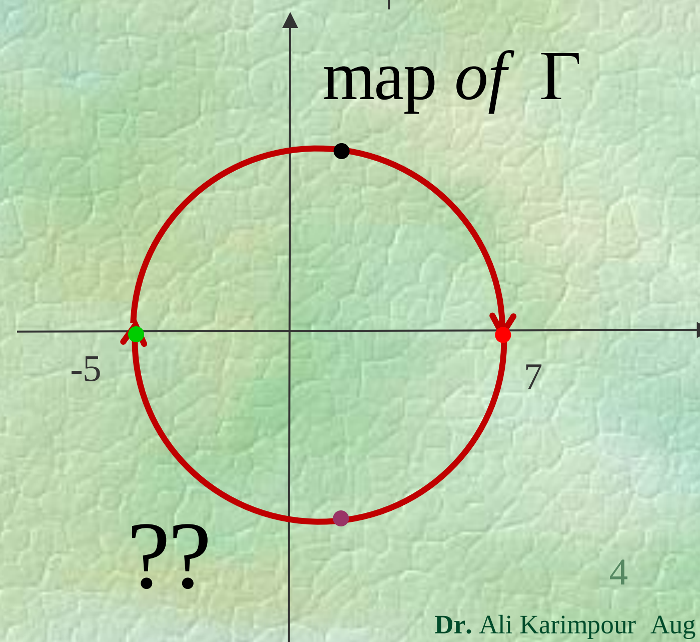
$$f(s)=\frac{s-1}{s+2}$$



$$Z_0 - P_0 = N_0$$??

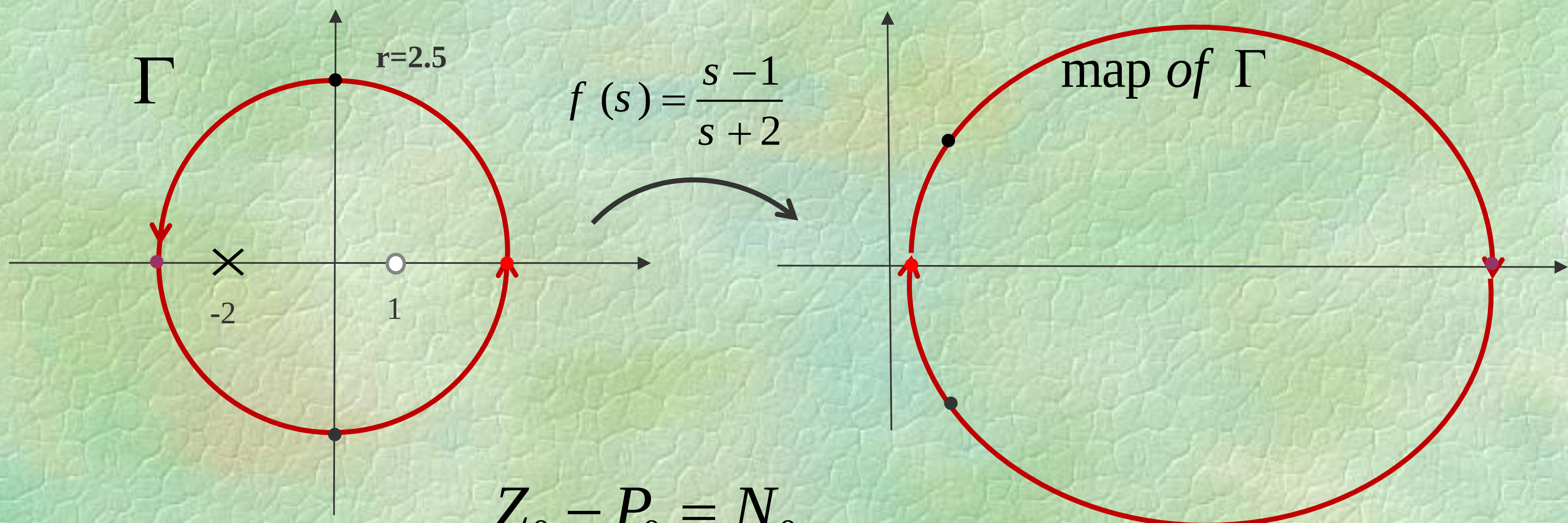


$$f(s)=\frac{s-1}{s+2}$$



$$Z_0 - P_0 = N_0$$??

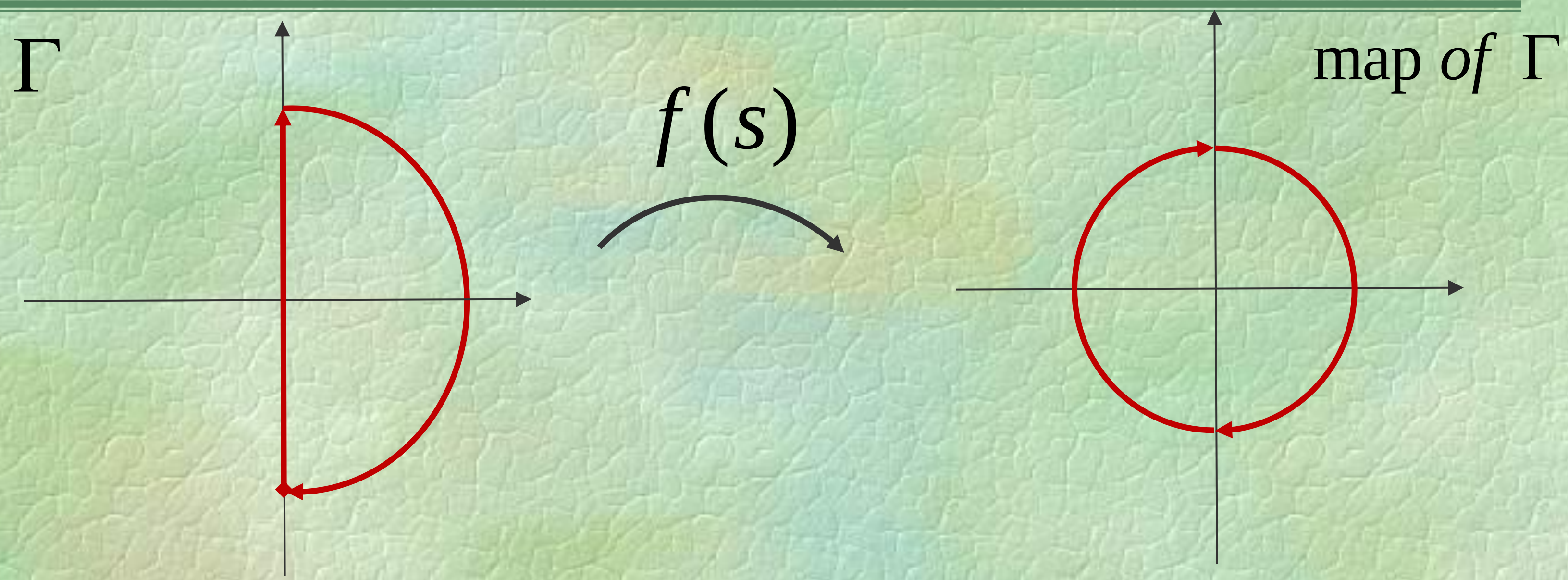
Mapping



$$Z_0 - P_0 = N_0$$

$$1-1=0$$

Nyquist fundamental



Nyquist path

$$Z_0 - P_0 = N_0$$

Z_0 : Number of RHP zeros of f(s)

P_0 : Number of RHP poles of f(s)

Is it our interest? ??

So what is our interest?

Number of RHP zeros of 1+kf(s) : Z_{-1} Why?

Number of RHP poles of 1+kf(s) : P_{-1}

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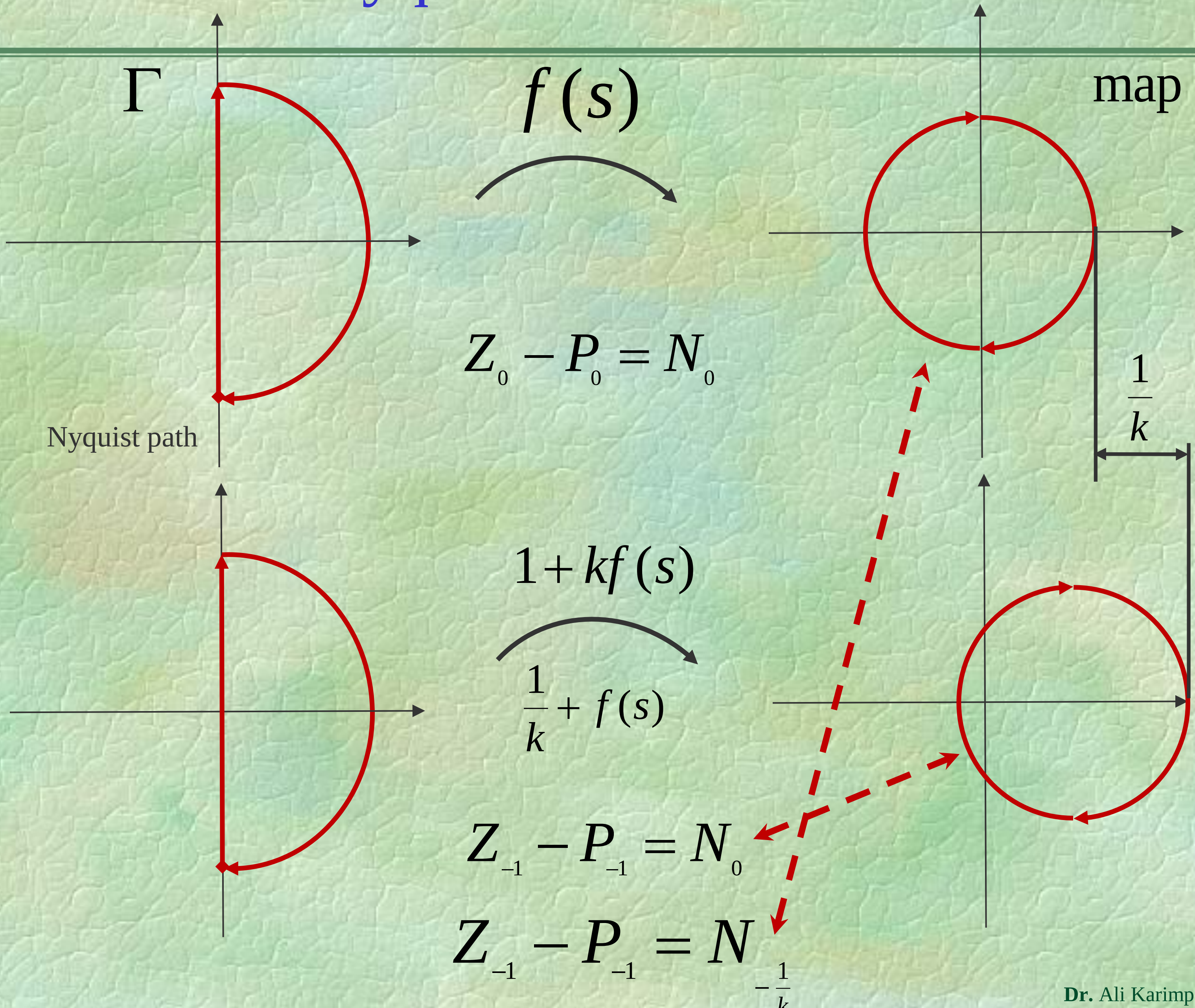
Nyquist fundamental



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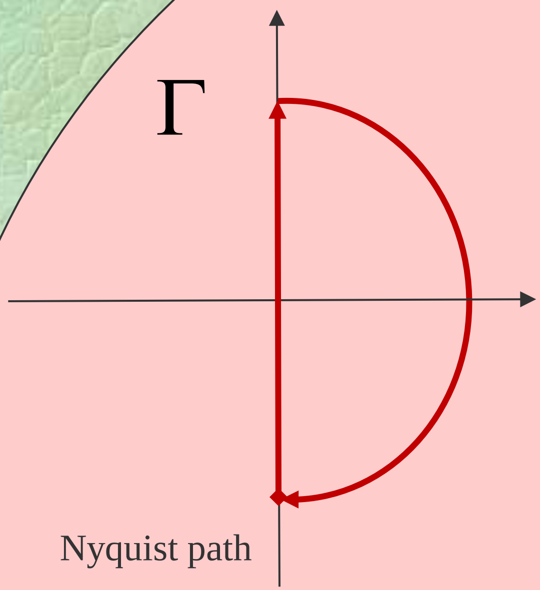
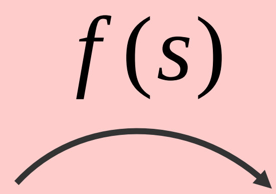
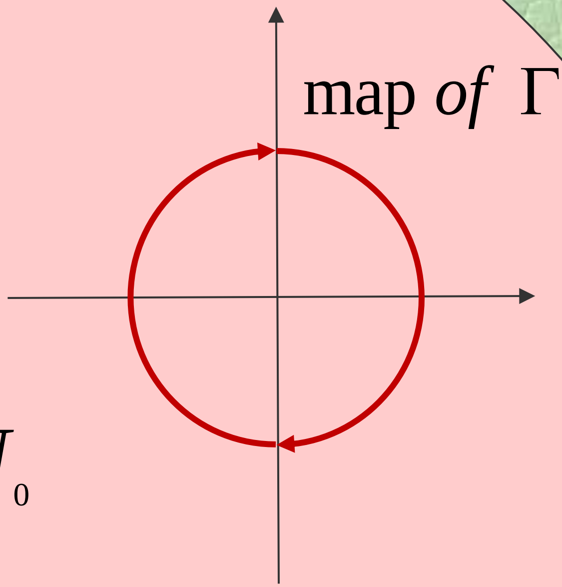
Nyquist fundamental







$$Z_0 - P_0 = N_0$$

$$Z_{-1} - P_{-1} = N_{-\frac{1}{k}}$$

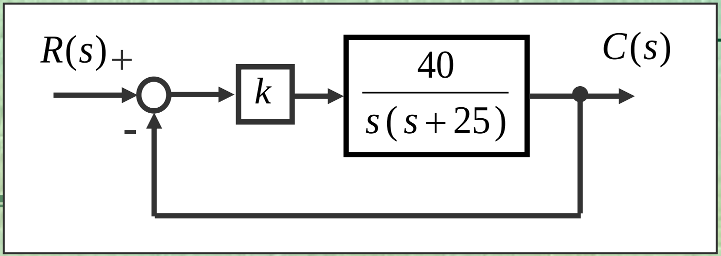
Z_0 : Number of RHP zeros of f(s)

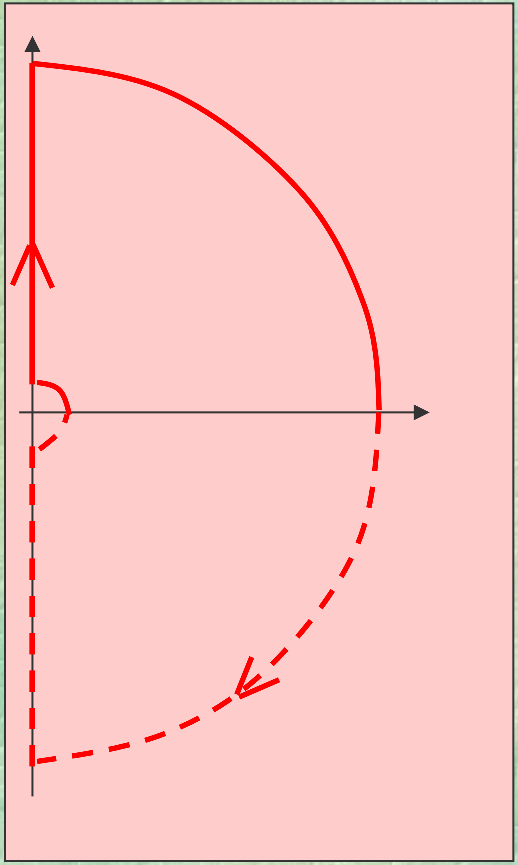
P_0 : Number of RHP poles of f(s)

Z_{-1} : Number of RHP zeros of 1+kf(s)

P_{-1} : Number of RHP poles of 1+kf(s)

Example 1: Check the stability of the following system by Nyquist method.



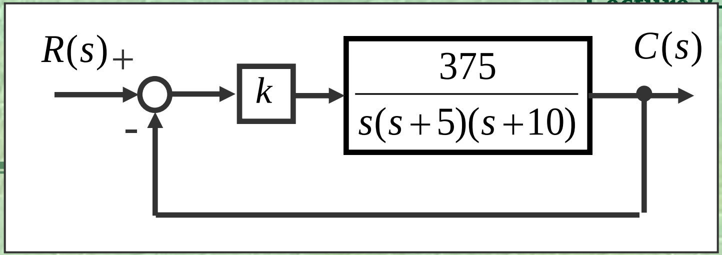


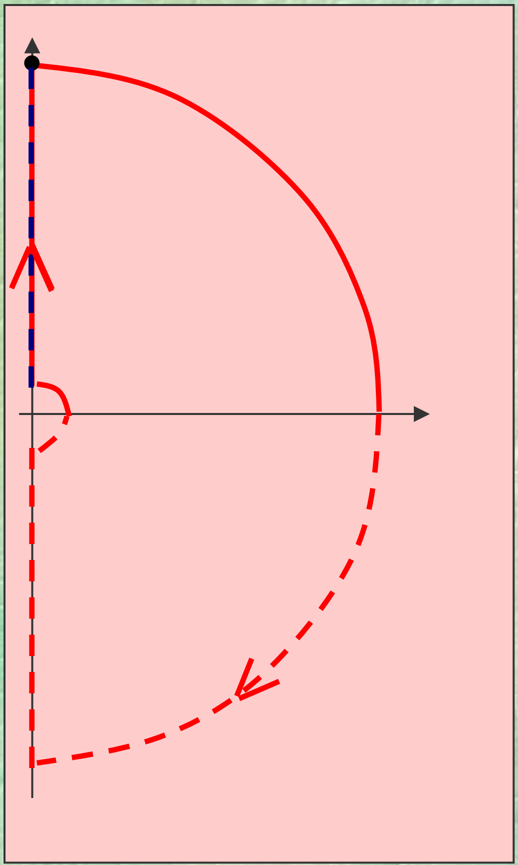
$$f(s) = \frac{40}{s(s+25)}$$

Stability condition ??

$$Z_0 - P_0 = N_0$$

Example 2: Check the stability of following system by Nyquist method.





$$f(s) = \frac{375}{s(s+5)(s+10)}$$

Stable for $0 < k < 2$

Unstable for $k > 2$ (2 RHP)

Unstable for $k < 0$ (1 RHP)

$$Z_0 - P_0 = N_0 \quad ??$$

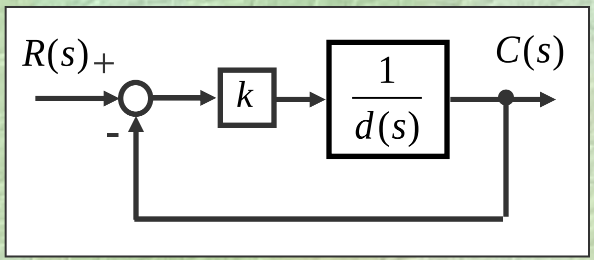
....

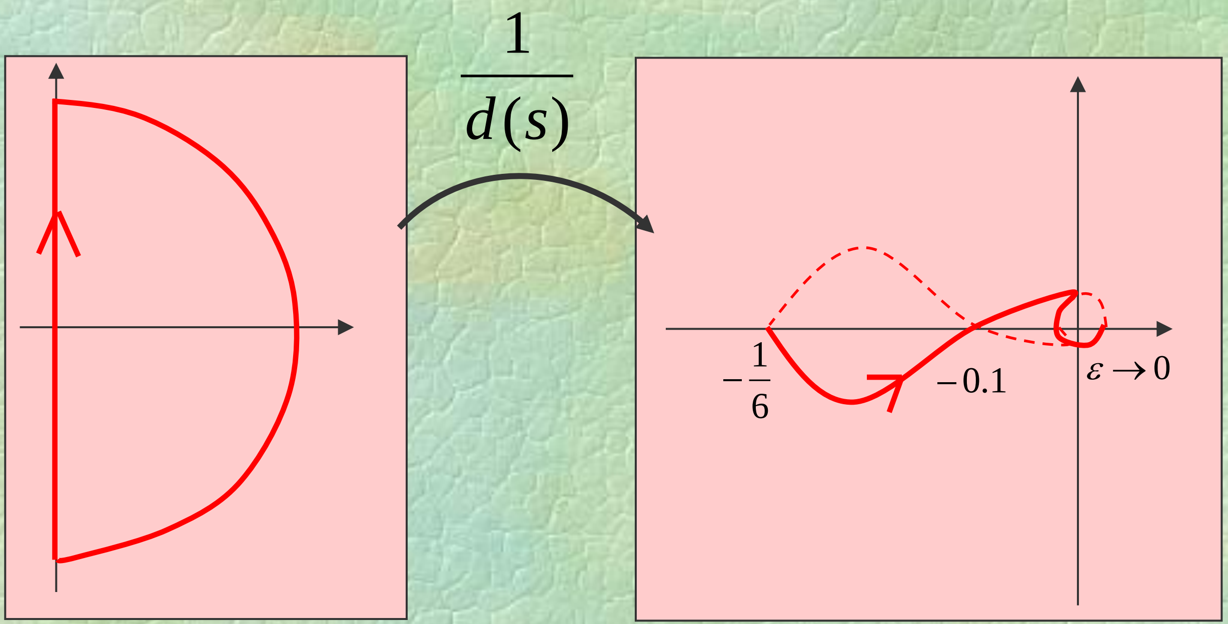
MATLAB

nyquist

Example 3: Check the stability of following system from the given Nyquist plot.

d(s) is a polynomial.





$$Z_0 - P_0 = N_0 \qquad 0 - P_0 = -1 \qquad P_0 = 1 = P_{-1}$$

$0 < k < 6$ System is unstable(1 RHP zero)

$6 < k < 10$ System is stable

$k > 10$ System is unstable(2 RHP zero)

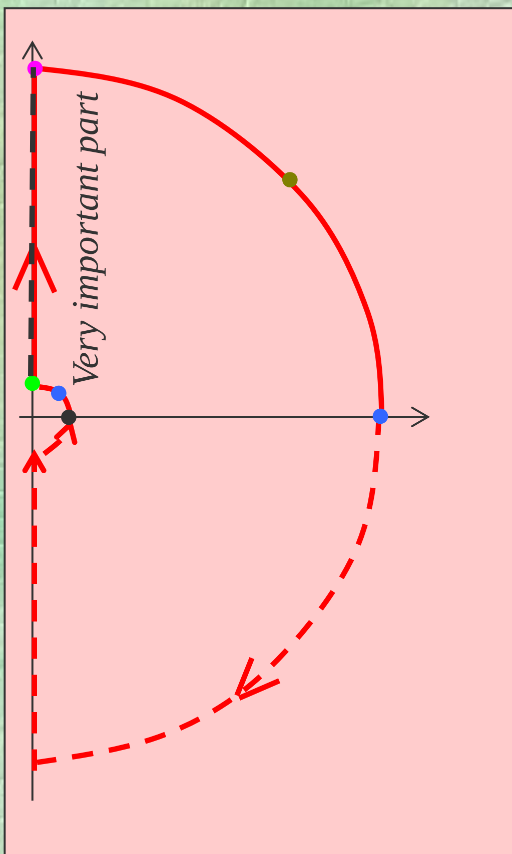
$k < 0$ System is unstable(1 RHP zero)

……….

$$Z_{-1} - P_{-1} = N_{-\frac{1}{k}}$$

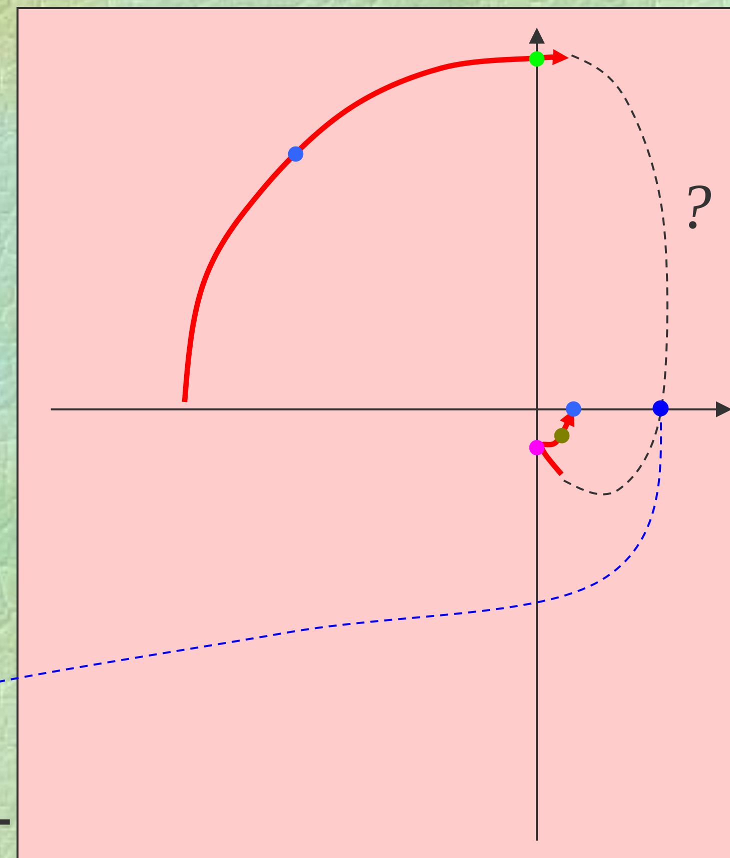
Example 4: Discuss about the RHP roots of following system.

$$1+k\frac{2(s-1)}{s(s+1)}=0$$



$$f(s)=\frac{2(s-1)}{s(s+1)}$$

$$f(j\omega)=\frac{2(j\omega-1)}{j\omega(j\omega+1)}$$

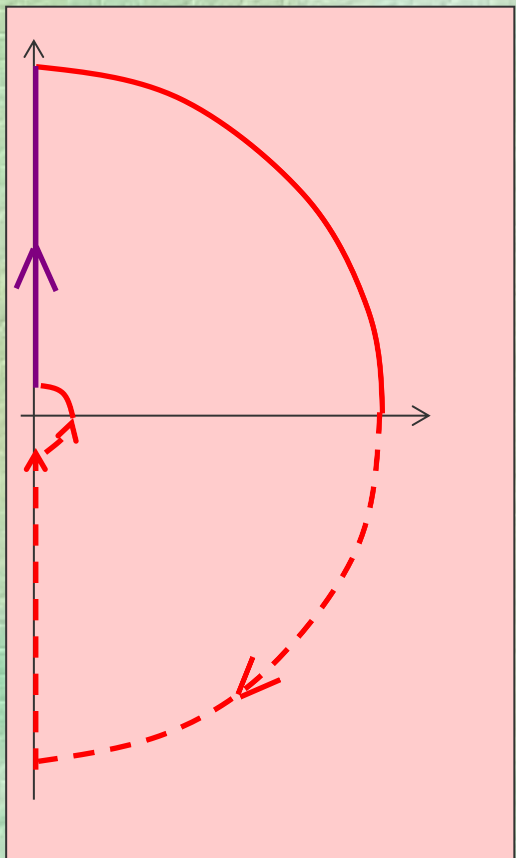


ω	?
$f(j\omega)$	?

Which point is very important?

Example 4: Discuss about the RHP roots of following system.

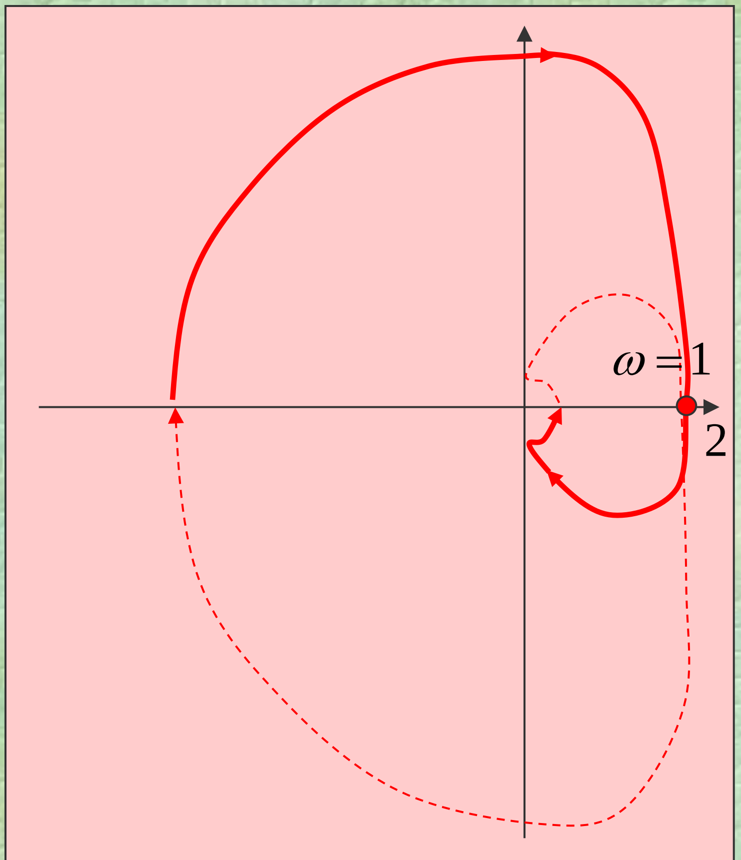
$$1+k\frac{2(s-1)}{s(s+1)}=0$$



$$f(s)=\frac{2(s-1)}{s(s+1)}$$

$$f(j\omega)=\frac{2(j\omega-1)}{j\omega(j\omega+1)}$$

ω	? 1
$f(j\omega)$	? 2

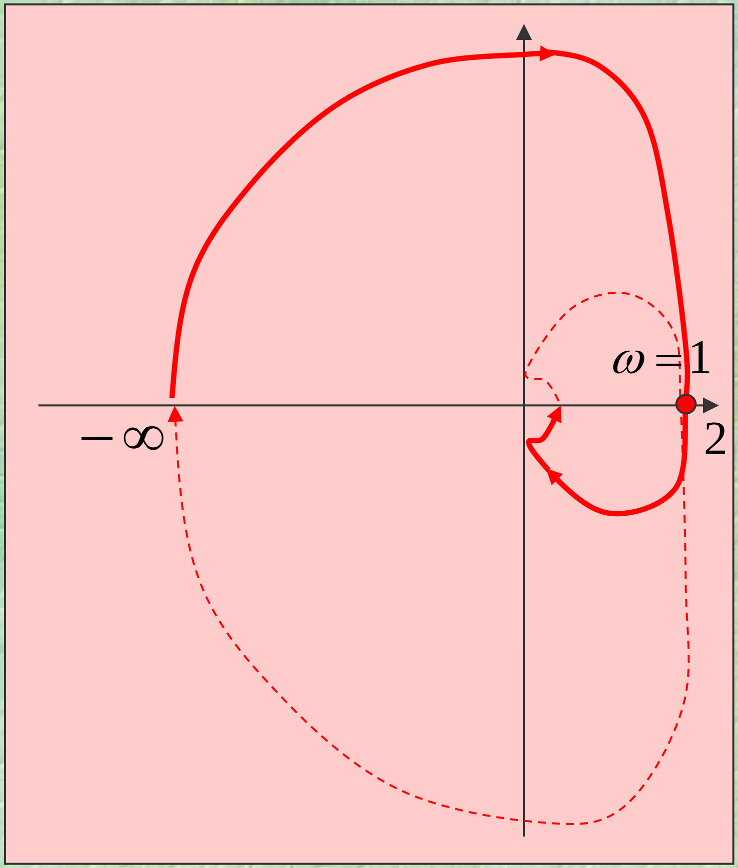


$$\angle f(j\omega)=\angle num-\angle den= \quad 180-\tan^{-1}\omega-(90+\tan^{-1}\omega) \;=90-2\cdot\tan^{-1}\omega$$

$\angle f(j\omega)=0 \quad \rightarrow \quad \omega=1$ Or equivalently let Im(f) = 0 $\quad f(j1)=\dfrac{2(1j-1)}{j(j+1)}=2\angle 0^\circ$

Example 4: Discuss about the RHP roots of following system.

$$1+k\frac{2(s-1)}{s(s+1)}=0$$



• • • •

$k>0$ Unstable(one RHP root)

• • • •

$k<-0.5$ Unstable (two RHP root)

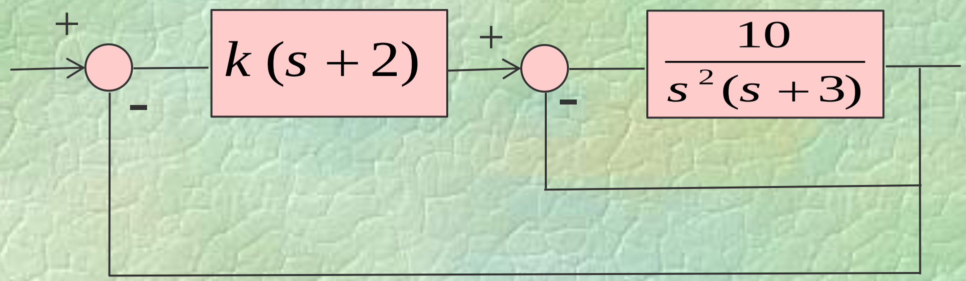
• • • •

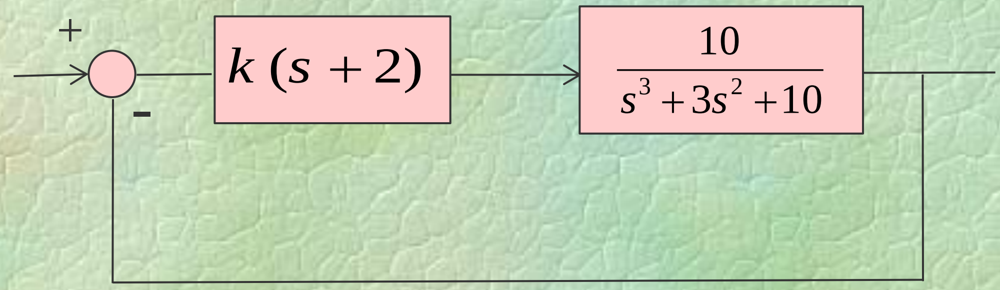
$-0.5<k<0$ Stable

More Study:

$$Z_0 - P_0 = N_0 \quad ??$$

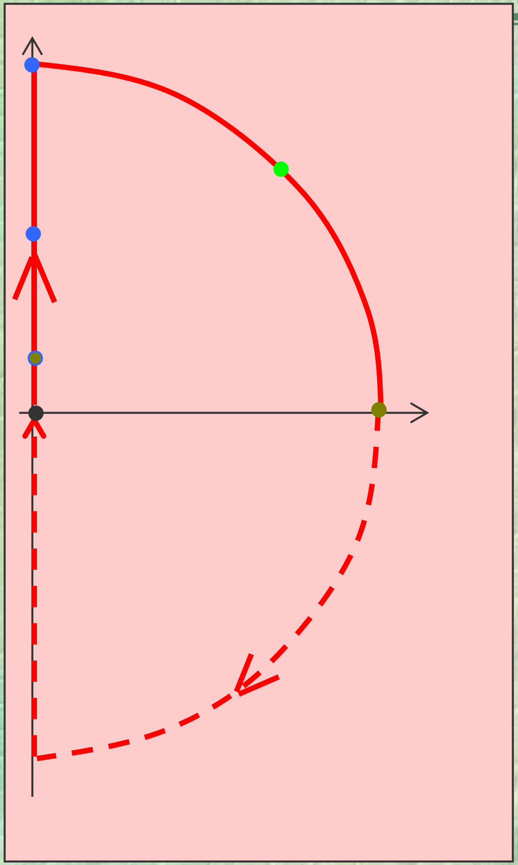
Example 5: Discuss about the stability of the following system for different values of k.





Its standard form is: $$1+k\,\frac{10(s+2)}{s^3+3s^2+10}=0$$

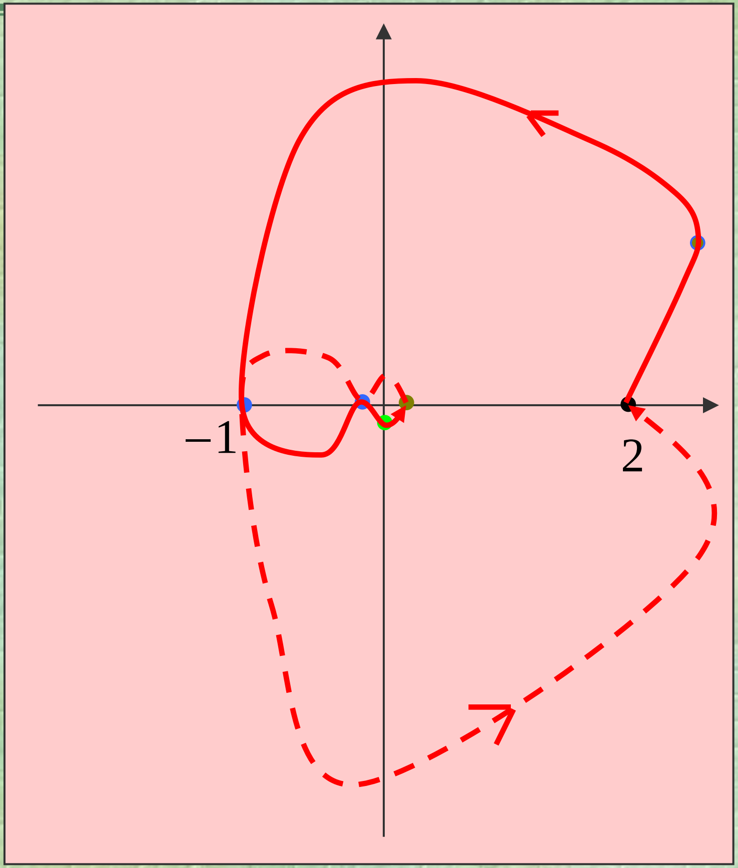
Discuss about the RHP roots of: $1+k\dfrac{10(s+2)}{s^3+3s^2+10}=0$



$$f(s)=\frac{10(s+2)}{s^3+3s^2+10}$$

$$f(s)=\frac{10(0+2)}{0+0+10}=2$$

$$f(j\omega)=\frac{10(j\omega+2)}{(10-3\omega^2)-j\omega^3}$$



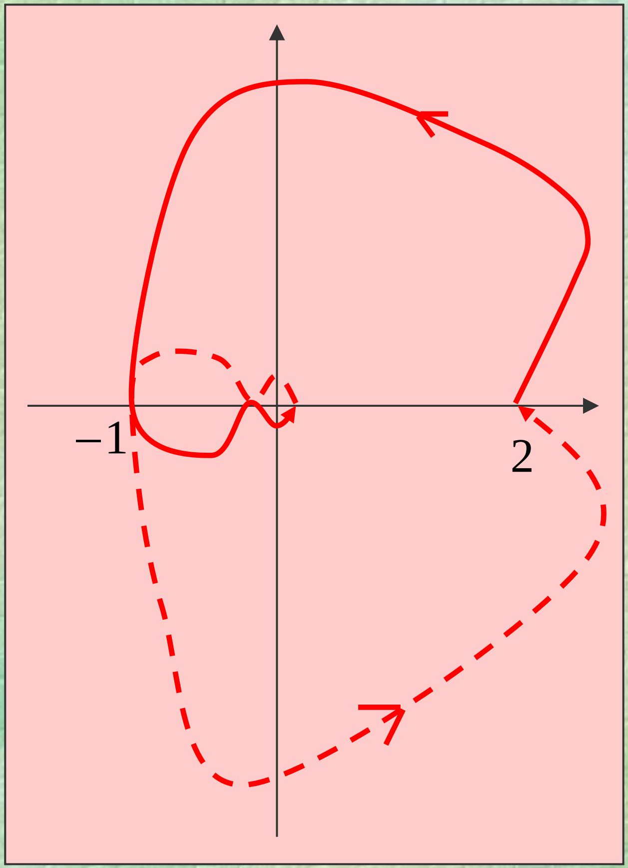
$$f(j\omega)=\frac{10(j\omega+2)}{(10-3\omega^2)-j\omega^3}\,\frac{(10-3\omega^2)+j\omega^3}{(10-3\omega^2)+j\omega^3}$$

$$\mathrm{Im}(f)=\frac{10\omega(10-3\omega^2)+20\omega^3}{(10-3\omega^2)^2+\omega^6}=0 \qquad \omega=0,\pm\sqrt{10}$$

$$f(j\sqrt{10})=\frac{10(j\sqrt{10}+2)}{(10-30)-j10\sqrt{10}}=1\angle 180^\circ$$

How? $f(j1)=\dfrac{10(j1+2)}{(10-3)-j1}=\dfrac{20+10j}{7-j1}=2.6+1.8j$

Discuss about the RHP roots of: $$1+k\frac{10(s+2)}{s^3+3s^2+10}=0$$



$$P_{-1}=P_0=?$$

$$Z_0-P_0=N_0 \qquad 0-P_0=-2 \qquad P_{-1}=P_0=2$$

…….

$0<k<1$ Unstable 2 RHP roots

…….

$k>1$ Stable

…….

$k<-0.5$ Unstable (1 RHP roots)

…….

$-0.5<k<0$ Unstable (2 RHP roots)

Nyquist Stability Criteria

Topics to be covered include:

- Nyquist stability criteria.
- Minimum phase systems.
- Simplified Nyquist stability criterion.

Minimum Phase systems

f(s) is said to be minimum phase if it has no poles and zeros (origin is an exception) and on the RHP and on the jω axis there is no delay.

$$f(s)=\frac{\prod_{i=1}^{n_z}(s+z_i)}{s^{T_y}\prod_{j=1}^{n_p}(s+p_j)}$$

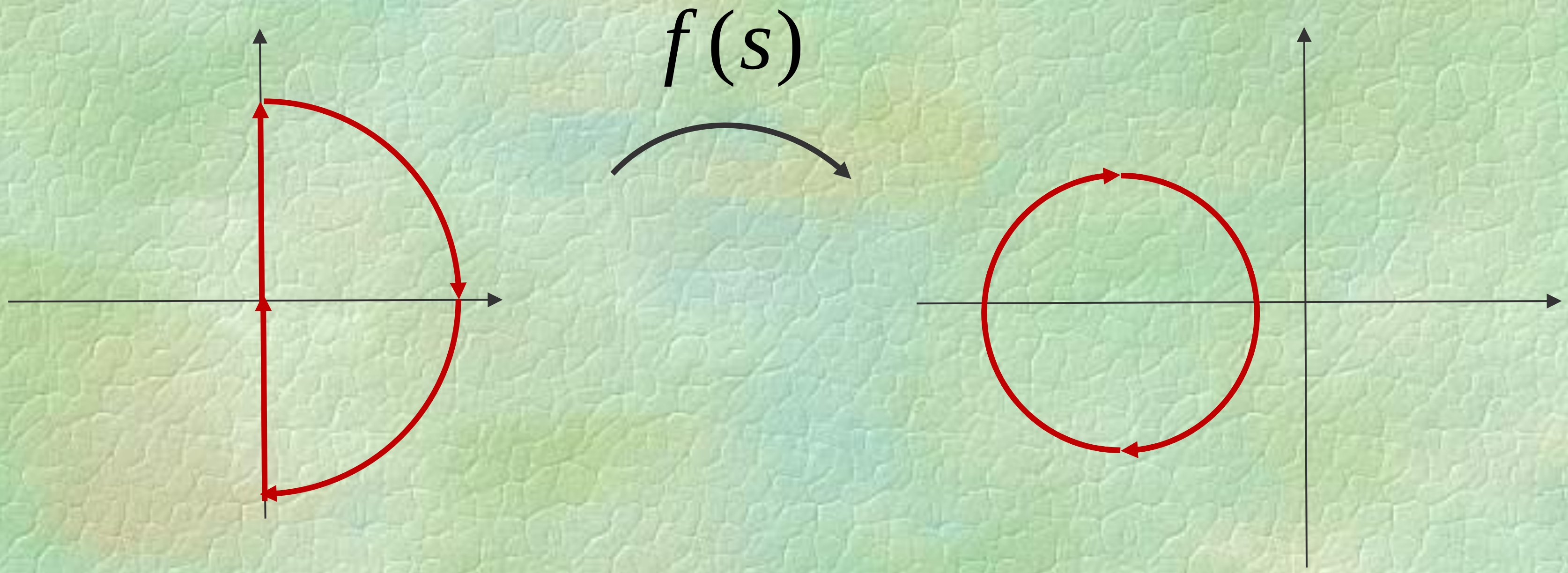
If it was minimum phase

$$z_i\ ,\ p_j>0$$

T_y is type of system

Important note: If f(s) is minimum phase then $Z_0=P_{-1}=P_0=0$

Nyquist fundamental for minimum phase systems



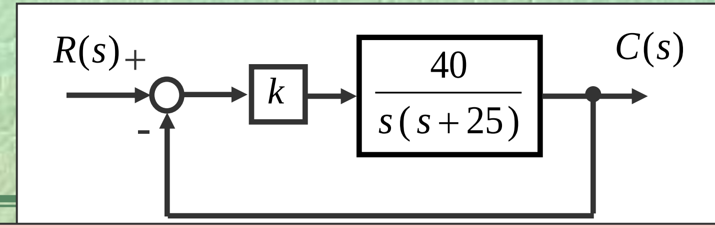
Nyquist path

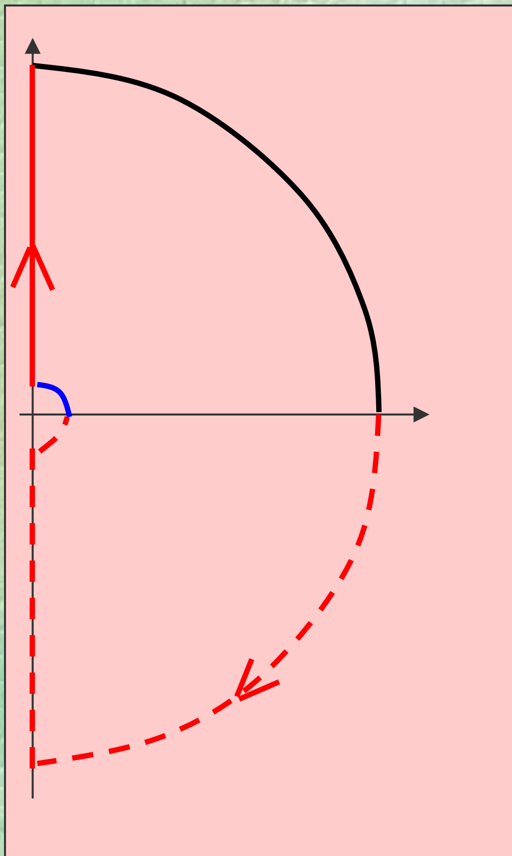
Nyquist plot

$$Z_{-1} - P_{-1} = N_{-\frac{1}{k}}$$

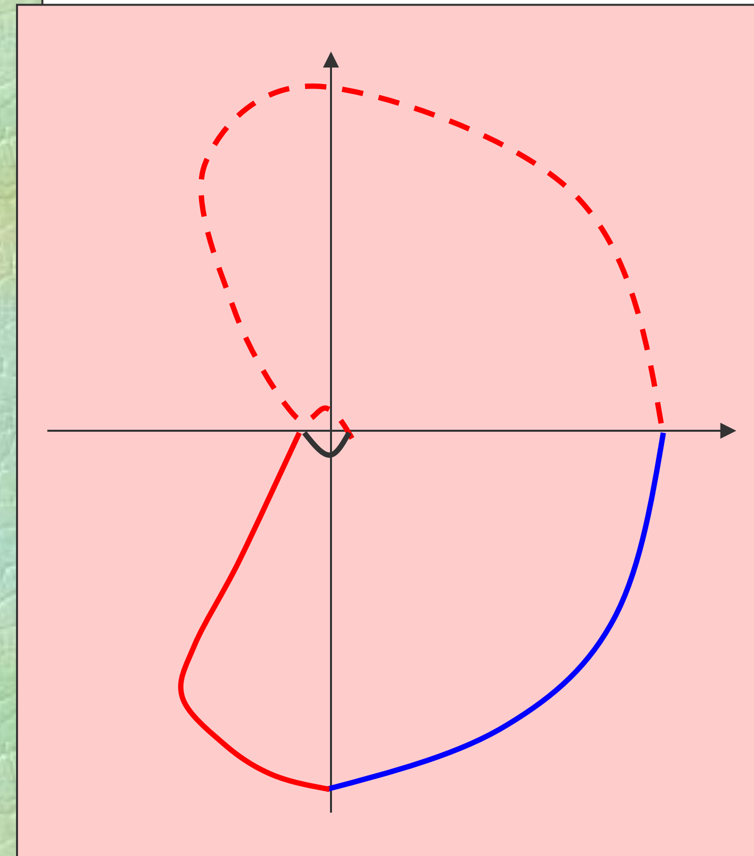
$$Z_{-1} = N_{-\frac{1}{k}}$$

Check the stability of following system by Nyquist method.





$$f(s)=\frac{40}{s(s+25)}$$



System is minimum phase

$$-\frac{1}{k}<0 \quad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(\ 90T_y+\varphi\ )}{360^\circ}=\frac{2(90^\circ-90^\circ)}{360^\circ}=0$$

Stable for $k>0$

$$-\frac{1}{k}>0 \quad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(\ 90T_y+\varphi\ )}{360^\circ}=\frac{2(90^\circ+90^\circ)}{360^\circ}=1$$

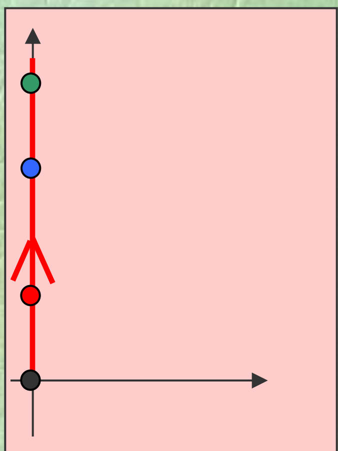
Unstable for $k\leq 0$

One RHP zero

Example 6: Discuss about the RHP roots of the following system for different values of k.

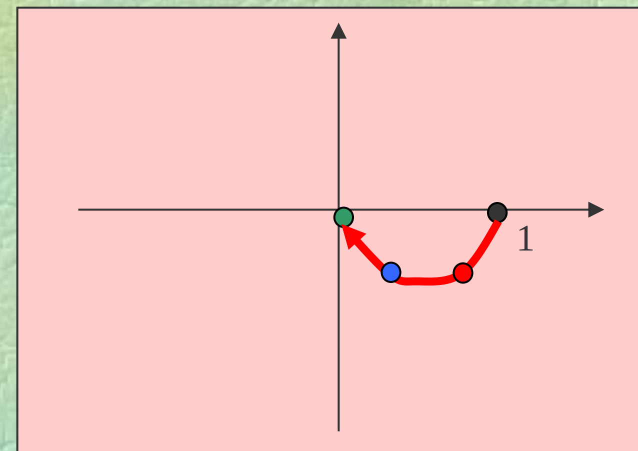
$$1+k\frac{1}{1+\tau s}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{1+\tau s}$$



Polar plot

$$-\frac{1}{k}<0 \qquad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(90T_y+\varphi)}{360^\circ} \qquad Z_{-1}=0 \qquad k>0$$

No RHP root.

$$0<-\frac{1}{k}<1 \qquad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(90T_y+\varphi)}{360^\circ} \qquad Z_{-1}=1 \qquad k<-1$$

One RHP root.

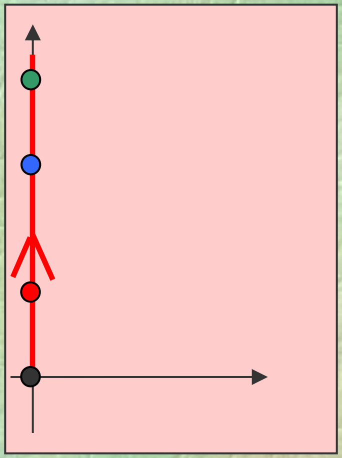
$$-\frac{1}{k}>1 \qquad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(90T_y+\varphi)}{360^\circ} \qquad Z_{-1}=0 \qquad -1<k<0$$

No RHP root.

Example 7: Discuss about the RHP roots of the following system for different values of k.

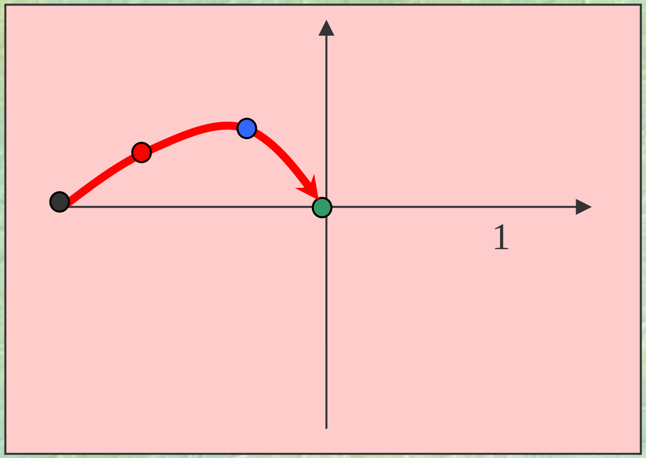
$$1+k\frac{1}{s^2(1+\tau s)}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{s^2(1+\tau s)}$$



Polar plot

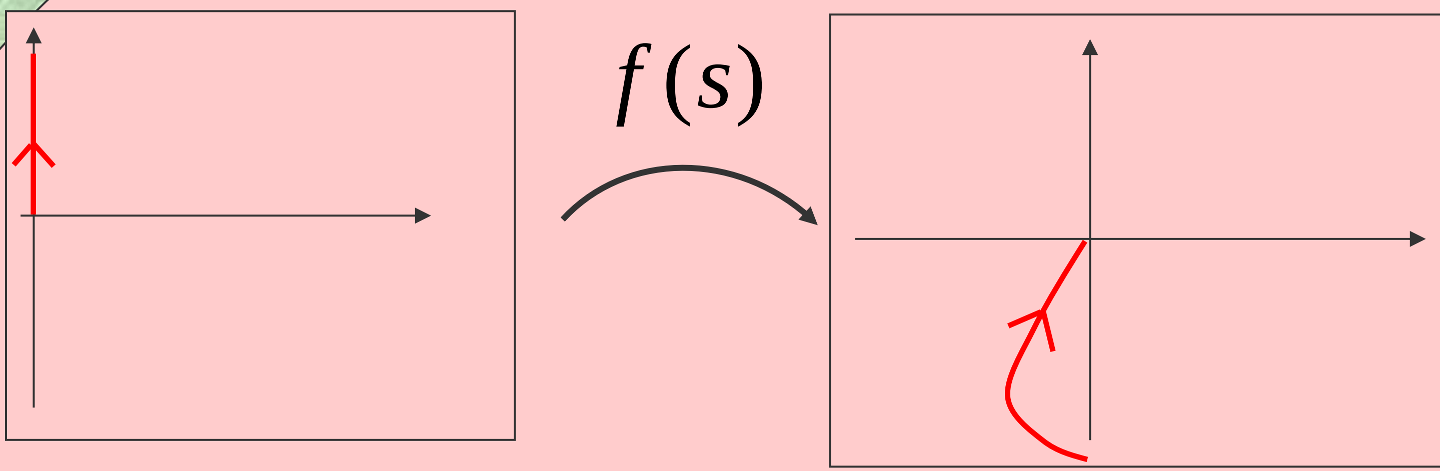
$$-\frac{1}{k}<0 \quad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(90T_y+\varphi)}{360^\circ}=\frac{2(180+180)}{360^\circ}=2 \qquad k>0$$

Two RHP roots.

$$-\frac{1}{k}>0 \quad Z_{-1}=N_{-\frac{1}{k}}=\frac{2(90T_y+\varphi)}{360^\circ}=\frac{2(180+0)}{360^\circ}=1 \qquad k<0$$

One RHP root.

Simplified Nyquist method



Simplified Nyquist path plot

Polar plot

System is minimum phase

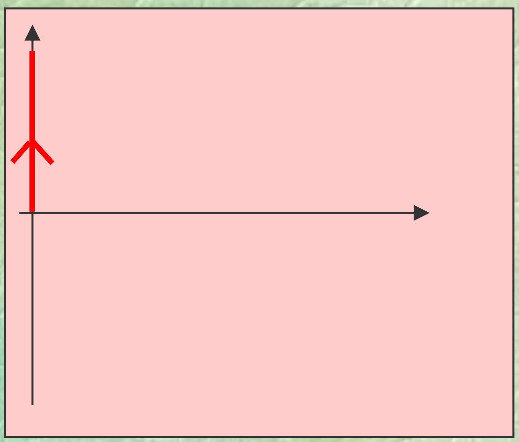
$$Z_{-1} = N_{-\frac{1}{k}} = \frac{2(\;90T_y + \varphi\;)}{360^\circ}$$

φ is the turn angle of polar plot around -1/k

Important remark:

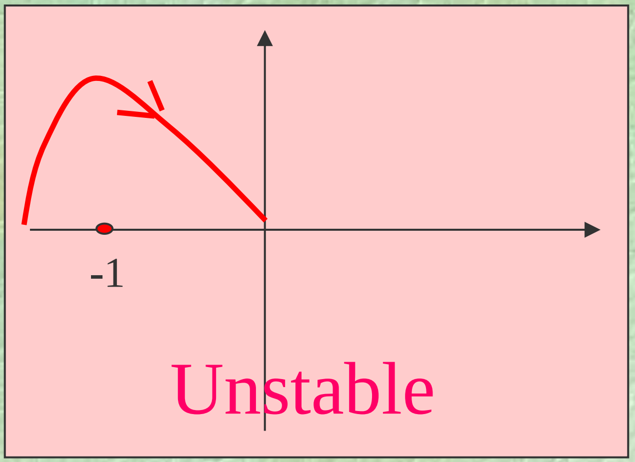
If φ is greater than zero the system is unstable but if they were less than zero one must check it!

Simplified Nyquist method



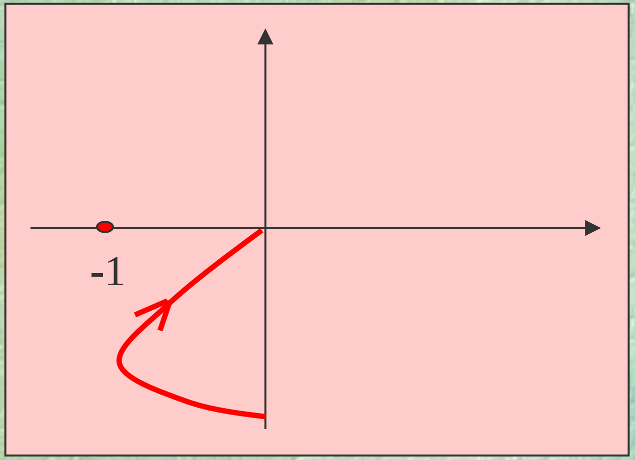
Simplified Nyquist path plot

$kf(s)$



Polar plot

$kf(s)$



Polar plot

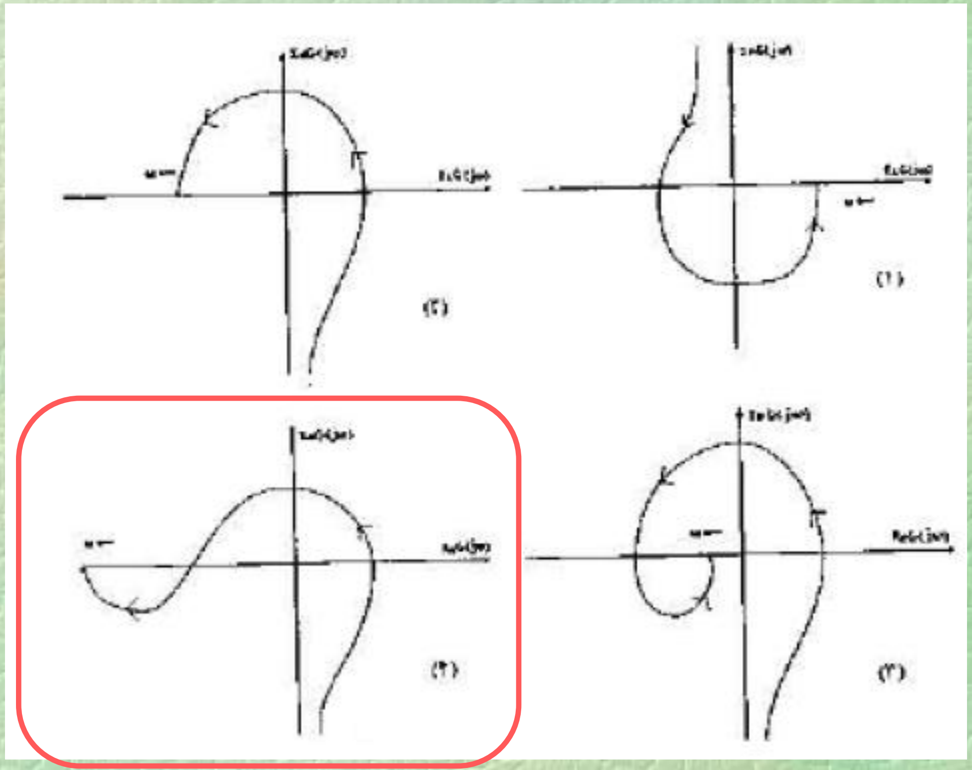
Stability depends on T_y

Simplified Nyquist method

University entrance exam 1393

Example 8: What is the Nyquist plot of following transfer function?

$$G(s)=\frac{-(s+1)(s+2)(s+3)(s+4)}{s^3(s+100)}$$



Exercises

1- The open loop transfer function of a unity-feedback (negative sign) is:

$$G_p(s) = \frac{k}{(s+5)^n}$$

Apply the Nyquist criterion to determine the range of k for stability. Let n=1, 2 and 3

2- The characteristic equation of a linear control system is:

$$s^3 + 2s^2 + 20s + 10k = 0$$

Apply the Nyquist criterion to determine the range of k for stability.

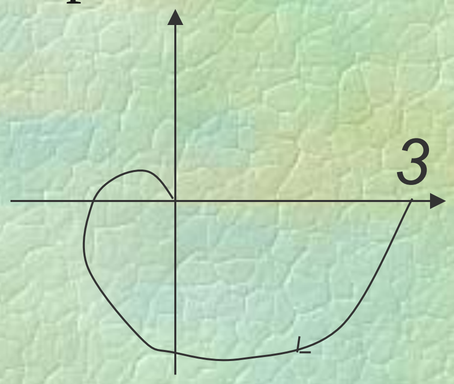
3- The open loop transfer function of a unity-feedback (negative sign) with PD controller is:

$$G_p(s) = \frac{10(K_p + K_d s)}{s^2}$$

Select the value of K_p so that the parabolic error constant be 100. Find the equivalent open-loop transfer function $G_{eq}(s)$ for stability analysis with K_d as a gain factor. Sketch the Nyquist plot and check the stability for different values of K_d.

Exercises

4- The polar plot of an open loop transfer function of a minimum phase system is:



Determine the steady state error of the system to a unit step. $e_{ss} = \frac{1}{4}$

5- The open loop transfer function of a unity-feedback (negative sign) is:

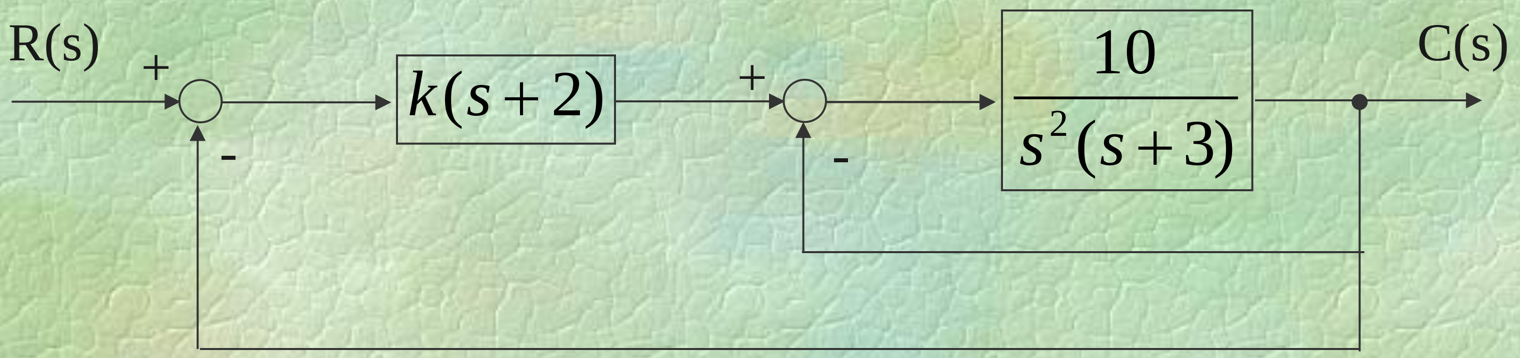
$$G(s) = \frac{ke^{-Ts}}{s+1} \qquad (k > 1)$$

Derive an expression that make the system stable.

$$[T\sqrt{k^2-1} + \tan^{-1}\sqrt{k^2-1}] < \Pi$$

Exercises

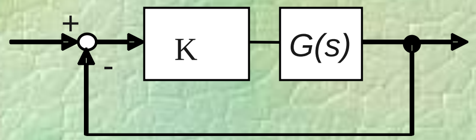
6- Assume the following control system



Find the value of *k* which make the system stable.

Answer: $k>1$

7- Consider the following control system

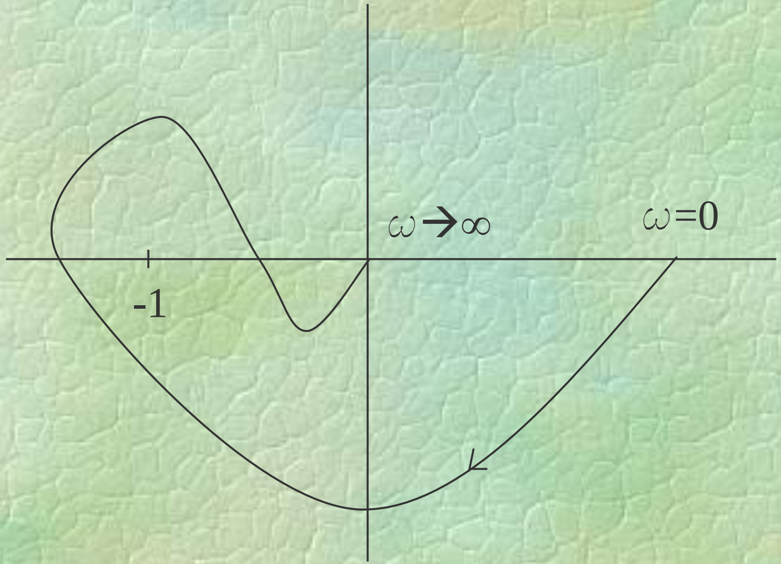


$$G(s)=\frac{s+3}{s(s-1)}$$

Find the value of *k* which make the system stable.

Exercises

8- Polar plot of an open loop transfer function with two RHP poles is shown in the following:

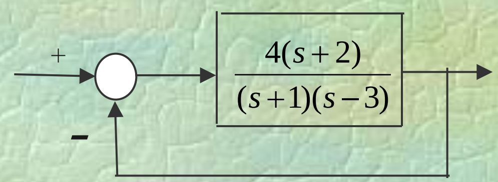


Discuss about the stability of system.

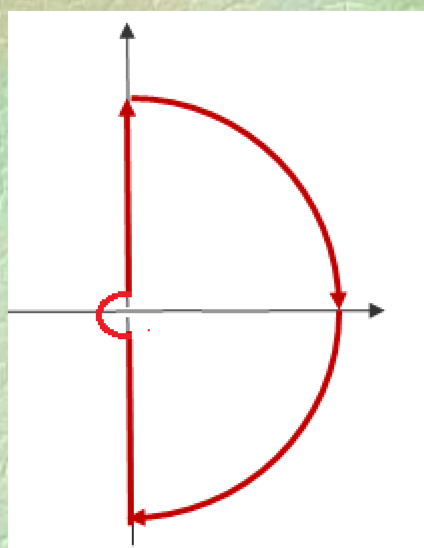
Answer: It is unstable and it has 4 unstable poles.

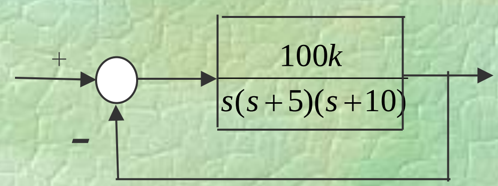
Exercises

9- Discuss the stability of following system by Nyquist criteria.



10- Discuss the stability of following system by Nyquist criteria by use of following Nyquist path (Final 1391).

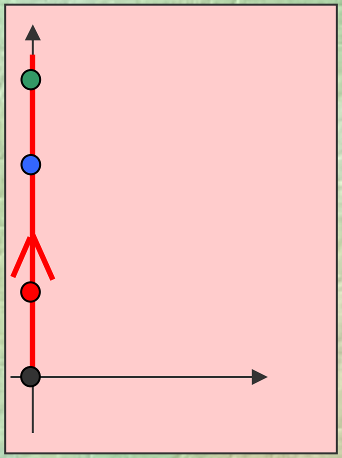




Example 9: Discuss about the RHP roots of the following system for different values of k.

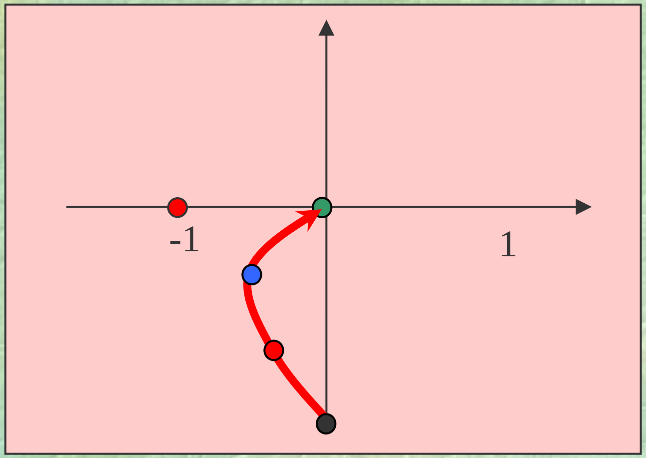
$$1+k\frac{1}{s(1+\tau s)}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{s(1+\tau s)}$$



Polar plot

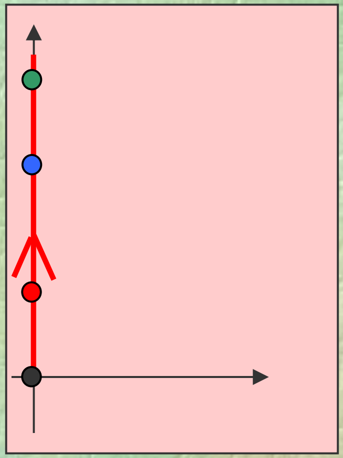
$k>0 \qquad Z_{-1}=N_{-1}=\frac{2(90T_y+\varphi_{-1})}{360^\circ}=\frac{2(90-90)}{360^\circ}=0$ No RHP root.

$k<0 \qquad Z_{-1}=N_{1}=\frac{2(90T_y+\varphi_{1})}{360^\circ}=\frac{2(90+90)}{360^\circ}=1$ One RHP root.

Example 10: Discuss about the RHP roots of following system for different value of k.

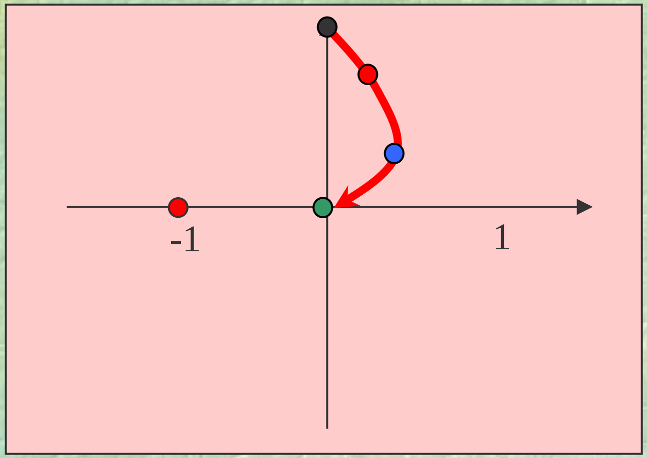
$$1+k\frac{1}{s^3(1+\tau s)}=0 \qquad \tau>0$$

Clearly System is minimum phase so we use simplified Nyquist method



Simplified Nyquist path plot

$$f(s)=\frac{1}{s^3(1+\tau s)}$$



Polar plot

$$k>0 \qquad Z_{-1}=N_{-1}=\frac{2(90T_y+\varphi_{-1})}{360^\circ}=\frac{2(270+90)}{360^\circ}=2$$ Two RHP roots.

$$k<0 \qquad Z_{-1}=N_{1}=\frac{2(90T_y+\varphi_{1})}{360^\circ}=\frac{2(270-90)}{360^\circ}=1$$ One RHP root.