Multivariable Control Systems

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Lecture 4

References are appeared in the last slide.

Poles and Zeros in Multivariable Systems

Topics to be covered include:

- Multivariable Poles and Zeros and System Type
- Direction of Poles and Zeros
- Smith-McMillan Form
- Matrix Fraction Description (MFD) and Smith-McMillan Form
- Transmission Zero Assignment

Different system representation

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

State space model

$$Y(s) = G(s)U(s)$$

Transfer matrix

$$\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix} \begin{bmatrix} \xi(s) \\ -U(s) \end{bmatrix} = \begin{bmatrix} 0 \\ -Y(s) \end{bmatrix}$$

$$\mathrm{P}(s) = \begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix}$$

Rosenbrock’s system matrix

Multivariable Poles and zeros?

Multivariable Poles (Through State Space Description)

State space model

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Definition 4-1: The poles p_i of a system with state-space description $(A, B, C \text{ and } D)$ are eigenvalues $\lambda_i(A), i = 1, 2, \ldots, n$ of the matrix *A*.

The pole polynomial or characteristic polynomial is defined as

$$\phi(s) = |sI - A|$$

Thus the system's poles are the roots of the characteristic polynomial

$$\phi(s) = |sI - A| = 0$$

Note that if A does not correspond to a minimal realization then the poles by this definition will include the poles (eigenvalues) corresponding to uncontrollable and/or unobservable states.

Multivariable Poles (Through Rosenbrock's System Matrix)

Rosenbrock's system matrix

$$\mathrm{P}(s)=\begin{bmatrix} P(s) & Q(s) \\ -R(s) & W(s) \end{bmatrix}$$

Thus the system's poles are the roots of the following polynomial

$$\phi(s)=\left|P(s)\right|=0$$

$$\dot{x}=Ax+Bu$$
$$y=Cx+Du$$

$$\mathrm{P}(s)=\begin{bmatrix} sI-A & B \\ -C & D \end{bmatrix}$$

$$\phi(s)=\left|sI-A\right|=0$$

$$\phi(s)=\left|P(s)\right|=\left|sI-A\right|=0$$

So they are compatible.

Multivariable Poles (Through Transfer Function Description)

Transfer matrix $Y(s) = G(s)U(s)$

Theorem 4-1: Finding pole polynomials through transfer function

The pole polynomial ϕ(s) corresponding to a minimal realization of a system with transfer function *G(s)* is the least common denominator of all non-identically-zero minors of all orders of *G(s).*

A minor of a matrix is the determinant of the square matrix obtained by deleting certain rows and/or columns of the matrix. (note that the numerator and the denominator of each element must be prime).

corresponding to a minimal realization of a system??

Multivariable Poles (Through Transfer Function Description)

Example 4-1

$$G(s)=\frac{1}{(s+1)(s+2)(s-1)}\begin{bmatrix}(s-1)(s+2) & 0 & (s-1)^2\\ -(s+1)(s+2) & (s-1)(s+1) & (s-1)(s+1)\end{bmatrix}$$

The non-identically-zero minors of order 1 are

$$\frac{1}{s+1},\frac{s-1}{(s+1)(s+2)},\frac{-1}{s-1},\frac{1}{s+2},\frac{1}{s+2}$$

The non-identically-zero minor of order 2 are

$$\frac{2}{(s+1)(s+2)},\frac{1}{(s+1)(s+2)},\frac{-(s-1)}{(s+1)(s+2)^2}$$

By considering all minors we find their least common denominator

$$\varphi(s)=(s+1)(s+2)^2(s-1)$$

Multivariable Poles

Exercise 4-1: Consider the state space realization

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} -2.8 & 0.4 & 0 & 0 \\ 0.4 & -2.2 & 0 & 0 \\ -0.8 & -1.6 & -3.0 & 0 \\ 7.6 & -0.8 & 0 & 1.0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

a- Find the poles of the system directly through state space form.

b- Find the transfer function of system (note that the numerator and the denominator of each element must be prime).

c- Find the poles of the system through its transfer function.

d- Compare poles from part “a” and “c” and explain the results.

Multivariable Zeros

Importance of multivariable zeros

- Dynamic response.
- Stability of inverse system.
- Blocking the inputs.
- Not affected by feedback.
- Closed loop poles are on the open loop zeros at high gain.
- Stability analysis by inverse Nyquist

Multivariable Zeros

Different definition of multivariable zeros

- Element zeros.
- Decoupling zeros (input and output).
- Transmission zeros or blocking zeros.
- System zeros.
- Invariant zeros.

Multivariable Zeros

Element zeros

They are not very important in MIMO design.

Decoupling zeros (input and output)

They are clearly subset of poles (which are the roots of |sI-A|=0).

Transmission zeros or blocking zeros

They are the zeros that block the output in special condition (Initial values and direction).

System zeros

$$\{\text{System zeros}\} = \{\text{Transmission zeros}\} + \{\text{i.d. zeros}\} + \{\text{o.d. zeros}\} - \{\text{i.o.d. zeros}\}$$

Invariant zeros

Invariant zeros=System zeros (In the case of square plant)

Multivariable Zeros (Through State Space Description)

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Laplace transform

$$P(s)\begin{bmatrix} x \\ -u \end{bmatrix} = \begin{bmatrix} 0 \\ -y \end{bmatrix}, \qquad P(s) = \begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix}$$

Rosenbrock system matrix

The invariant zeros are then the values of $s=z$ for which the Rosenbrock's system matrix $P(s)$ loses rank.

Let

$$M = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix}, \qquad I_g = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$$

Then the zeros are found as non trivial solutions of

$$(zI_g - M)\begin{bmatrix} x_z \\ u_z \end{bmatrix} = 0 \qquad eig(M, Ig) \text{ if M is square}$$

This is solved as a generalized eigenvalue problem.

Multivariable Zeros (Through State Space Description)

System zero

Rosenbrock system matrix $P(s)=\begin{bmatrix} sI-A & B \\ -C & D \end{bmatrix}$

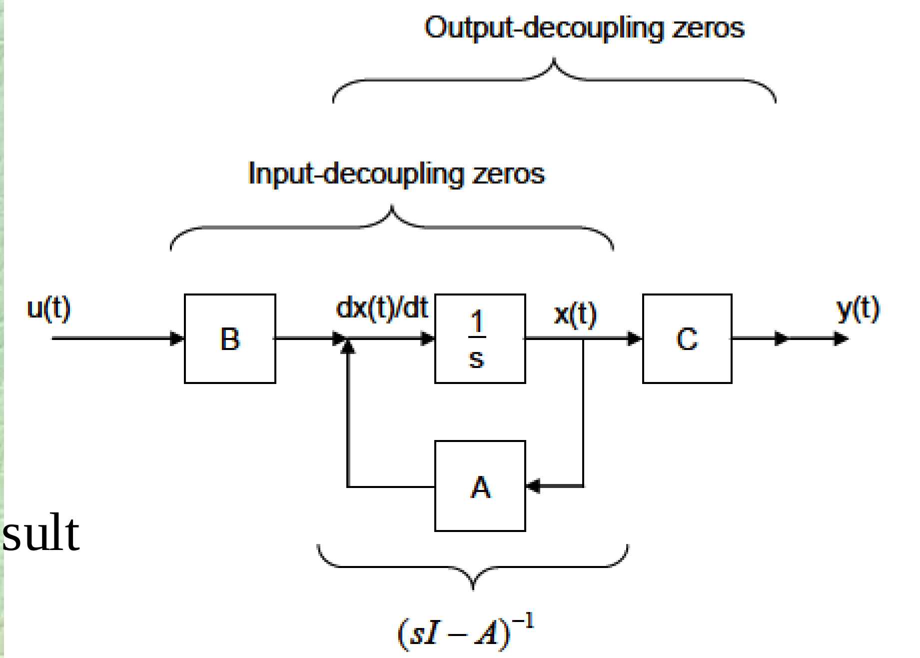
$$\{\text{System zeros}\}=\{\text{Transmission zeros}\}+\{\text{i.d. zeros}\}+\{\text{o.d. zeros}\}-\{\text{i.o.d. zeros}\}$$

Input decoupling zeros (i.d.z.)

The value of s that $\begin{bmatrix} sI-A & B \end{bmatrix}$ losses rank.

Output decoupling zeros (o.d.z.)

The value of s that $\begin{bmatrix} sI-A \\ -C \end{bmatrix}$ losses rank.



Invariant zeros

Invariant zeros are those values of s that result

$$\rho\big(P(s)\big)<\min\big(n+\rho(B),n+\rho(C)\big)$$

Multivariable Zeros (Through State Space Description)

Element zeros

$$G(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{2}{s+3}\\ \frac{1}{s+1} & \frac{1}{s+1}\end{bmatrix}\Rightarrow \text{ No Element Zero}$$

$$G(s)=\begin{bmatrix}\frac{s-1}{s^2+1} & \frac{2}{s+3}\\ \frac{1}{s+1} & \frac{s+1}{(s+3)}\end{bmatrix}\Rightarrow \text{ Element Zeros at } s=\pm 1$$

Decoupling zeros (input and output)

$$\dot{x}=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -2 & 0 & 0\\ 0 & 0 & -3 & 0\\ 0 & 0 & 0 & -4\end{bmatrix}x+\begin{bmatrix}1\\1\\0\\0\end{bmatrix}u$$
$$y=\begin{bmatrix}1 & 0 & 1 & 0\end{bmatrix}x+5u$$

$$g(s)=\frac{(s+2)(s+3)(s+4)}{(s+1)(s+2)(s+3)(s+4)}+5=\frac{5s+6}{s+1}$$

s= -2 and -4 are output decoupling zero (o.d.z)

s= -3 and -4 are input decoupling zero (i.d.z)

s= -4 is input-output decoupling zero (i.o.d.z)

Transmission zeros or blocking zeros

s= -6/5 is transmission or blocking zero

System zeros and invariant zeros

s= -6/5, s=-2, s=-3 and s=-4 are system zero(invariant zeros)

Multivariable Zeros (Through State Space Description)

Let

$$\dot{x} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 & -1 \end{bmatrix} u$$

$$G(s) = \begin{bmatrix} \dfrac{2s+8}{s+3} & \dfrac{-s-4}{s+3} \end{bmatrix}$$

$$P(s) = \left[\begin{array}{ccc|cc} s+5 & 0 & 0 & 0 & 0 \\ 0 & s+2 & 0 & 0 & 1 \\ 0 & 0 & s+3 & 2 & -1 \\ \hline -1 & 0 & -1 & 2 & -1 \end{array}\right]$$

Element zeros

s= -4, -4

Decoupling zeros (input and output)

s= -2 is output decoupling zero (o.d.z)

s= -5 is input decoupling zero (i.d.z)

Transmission zeros or blocking zeros

s= -4 is transmission or blocking zero

Invariant zeros

s= -4, s=-5 are invariant zeros.

System zeros

s= -4, s=-5, s=-2 are system zeros.

Multivariable Zeros (Through State Space Description)

Meaning of invariant zero?

$$\dot{x} = \begin{bmatrix} -5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 2 & -1 \end{bmatrix} u$$

s= -4, s=-5 are invariant zeros.

s= -4, s=-5, s=-2 are system zeros.

Let u-r-Kx

$$K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \end{bmatrix}$$

$$\dot{x} = \left(\begin{bmatrix} -5 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} K \right) x + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix} r$$

$$y = \left(\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 \end{bmatrix} K \right) x + \begin{bmatrix} 2 & -1 \end{bmatrix} r$$

Derive invariant zeros of new system and compare it with old one!?

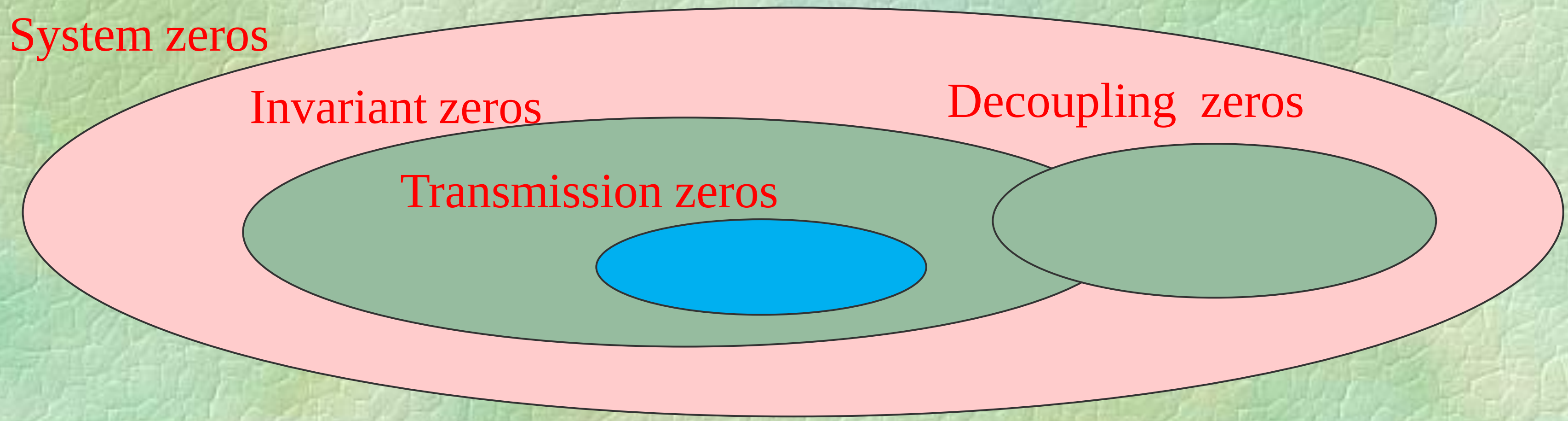
Derive system zeros of new system and compare it with old one!? -2 ???

Multivariable Zeros (Through State Space Description)

Generally we have:

$$\{\text{System zeros}\} = \{\text{Transmission zeros}\} + \{\text{i.d. zeros}\} + \{\text{o.d. zeros}\} - \{\text{i.o.d. zeros}\}$$

$$\{\text{Transmission zeros}\} \subset \{\text{Invariant zeros}\} \subset \{\text{System zeros}\}$$



For square systems with m=p inputs and outputs and n states, limits on the number of transmission zeros are:

$$D \neq 0 : At\ most\ n - m + rank(D)\ zeros$$

$$D = 0 : At\ most\ n - 2m + rank(CB)\ zeros$$

$$D = 0\ and\ rank(CB) = m : Exactly\ n - m\ zeros$$

Multivariable Zeros (Through State Space Description)

Example 4-2: Consider the state space realization

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -6 & -5 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \quad D = 0$$

$$M = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -6 & -5 & -1 \\ 4 & 1 & 0 & 0 \end{bmatrix} \quad I_g = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$eig(M, I_g)$$

$$z = -4$$

$$D \neq 0 : At\ most\ n - m + rank(D)\ zeros$$

$$D = 0 : At\ most\ n - 2m + rank(CB)\ zeros$$

$$D = 0\ and\ rank(CB) = m : Exactly\ n - m\ zeros$$

$$Number\ of\ zeros \leq 3 - 2 \cdot 1 + 0 = 1$$

Multivariable Zeros (Through State Space Description)

Exercise 4-2: Consider the state space realization

$$\dot{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u$$

a- Is this system controllable?

b- Is it observable?

c- Find i.d.z and o.d.z and i.o.d.z.

d- Is there an input decoupling zero which is not an invariant zero? Why?

e- Find a maximum value for the number of transmission zeros of the system.

Multivariable Zeros (Through Transfer Function Description)

Which kind of zeros can be derived from G(s)?

- Element zeros
- Transmission zeros or blocking zeros

Definition 4-2

z_i is a zero (transmission zero) of $G(s)$ if the rank of $G(z_i)$ is less than the normal rank of $G(s)$. The zero polynomial is defined as

$$z(s) = \Pi_{i=1}^{n_z}(s - z_i)$$

Where n_z is the number of finite zeros (transmission zero) of $G(s)$.

Multivariable Zeros (Through Transfer Function Description)

Theorem 4-2

The zero polynomial *z(s)* corresponding to a minimal realization of the system is the greatest common divisor of all the numerators of all order-*r* minors of *G(s)* where *r* is the normal rank of provided that these minors have been adjusted in such a way that they have the pole polynomial $\phi(s)$ as their denominators.

corresponding to a minimal realization of a system??

Multivariable Zeros (Through Transfer Function Description)

Example 4-3

$$G(s)=\frac{1}{(s+1)(s+2)(s-1)}\begin{bmatrix}(s-1)(s+2) & 0 & (s-1)^2\\ -(s+1)(s+2) & (s-1)(s+1) & (s-1)(s+1)\end{bmatrix}$$

according to example 4-1 the pole polynomial is:

$$\varphi(s)=(s+1)(s+2)^2(s-1)$$

The minors of order 2 with $\phi(s)$ as their denominators are

$$\frac{2(s-1)(s+2)}{(s+1)(s+2)^2(s-1)},\ \frac{(s-1)(s+2)}{(s+1)(s+2)^2(s-1)},\ \frac{-(s-1)^2}{(s+1)(s+2)^2(s-1)}$$

The greatest common divisor of all the numerators of all order-2 minors is

$$z(s)=s-1$$

Remark: See rank of G(s) at z=1.

Multivariable Zeros

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Exercise 4-3: Consider the state space realization

$$A = \begin{bmatrix} -2.8 & 0.4 & 0 & 0 \\ 0.4 & -2.2 & 0 & 0 \\ -0.8 & -1.6 & -3.0 & 0 \\ 7.6 & -0.8 & 0 & 1.0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

a- Find the transmission zeros of the system directly through state space form.

b- Find the transfer function of system (note that the numerator and the denominator of each element must be prime).

c- Find the transmission zeros of the system through its transfer function.

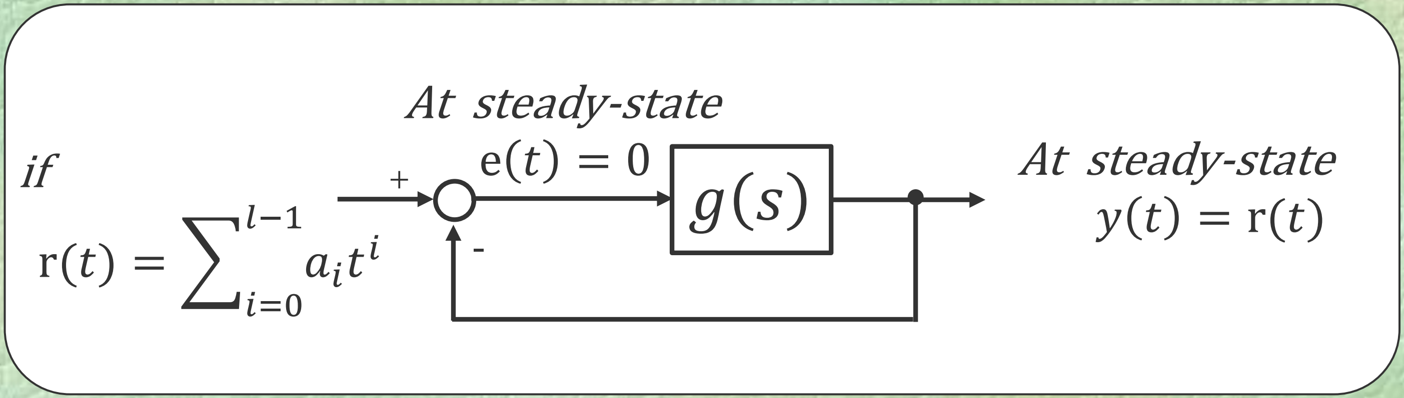
System type in SISO systems

Consider a transfer function g(s) and shows it in following form.

$$g(s) = \frac{1}{s^l} g_1(s)$$

Where l is the largest integer number such that $g_1(s)$ has no zero at 0.

In this condition l is the type of $g(s)$ and its meaning is shown in following figure.



System type in MIMO systems

Consider a transfer matrix G(s) and shows it in following form.

$$G(s)=\begin{bmatrix}\frac{1}{s^{l_1}} & 0 & \cdots & 0\\ 0 & \frac{1}{s^{l_2}} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \frac{1}{s^{l_m}}\end{bmatrix}G_1(s)$$

Where $\{l_1, l_2, \dots, l_m\}$ is the largest integer number such that $G_1(s)$ has no zero at 0.

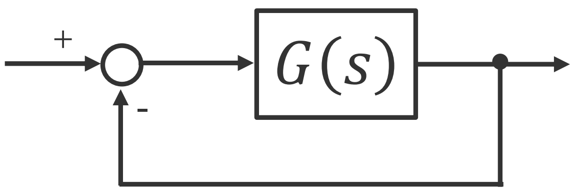
In this condition $\{l_1, l_2, \dots, l_m\}$ is the type of $G(s)$ and its meaning is shown in the next slide.

System type in MIMO systems

Consider $\{l_1, l_2, \dots, l_m\}$ is the type of $G(s)$ then:

if

$$\mathrm{r}(t) = \begin{bmatrix} 0 \\ \vdots \\ \sum_{i=0}^{l_k-1} a_i t^i \\ \vdots \\ 0 \end{bmatrix}$$



At steady-state $y(t) = \mathrm{r}(t)$

System type in MIMO systems

Example 4-4: Derive type of following system and explain it by simulation.

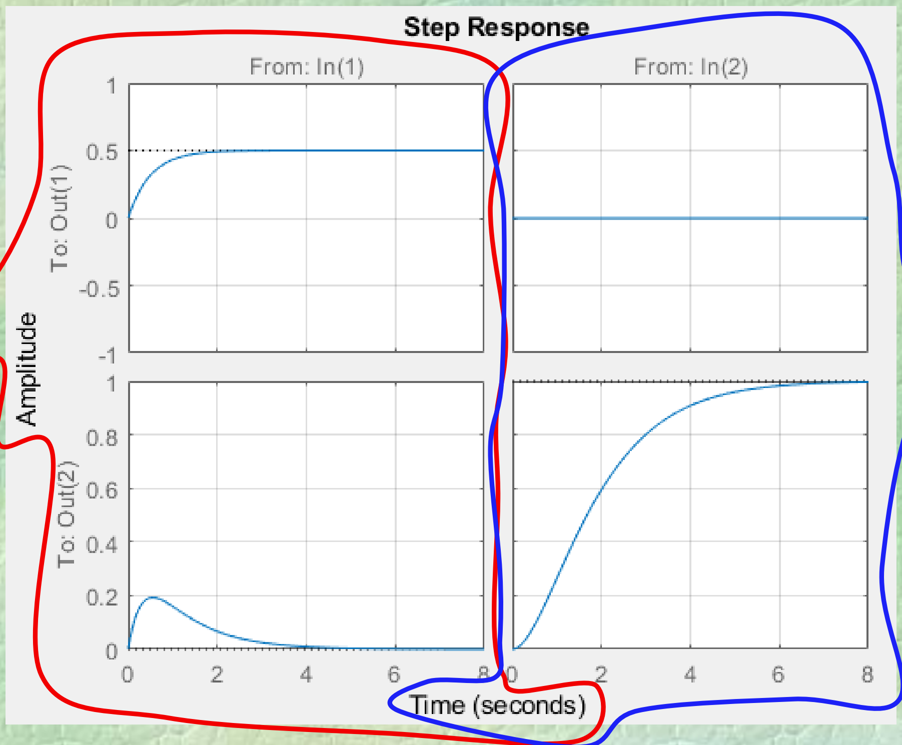
$$G(s)=\begin{bmatrix}\dfrac{1}{s+1} & 0\\ \dfrac{1}{s+3} & \dfrac{1}{s(s+2)}\end{bmatrix}$$

Solution: We rewrite G(s) as:

$$G(s)=\begin{bmatrix}1 & 0\\ 0 & 1/s\end{bmatrix}\begin{bmatrix}\dfrac{1}{s+1} & 0\\ \dfrac{s}{s+3} & \dfrac{1}{s+2}\end{bmatrix}$$

So $\{0,1\}$ is the type of $G(s)$ and now:

Step to input one:



Step to input two:

Poles and Zeros in Multivariable Systems

Topics to be covered include:

- Multivariable Poles and Zeros and System Type
- Direction of Poles and Zeros
- Smith-McMillan Form
- Matrix Fraction Description (MFD) and Smith-McMillan Form
- Transmission zero assignment

Directions of Poles and Zeros

Zero directions:

Let *G(s)* have a zero at *s = z*, Then *G(s)* losses rank at *s = z* and

there will exist nonzero vectors u_z and y_z such that

$$G(z)u_z = 0 \qquad y_z^H G(z) = 0$$

u_z is input zero direction and y_z is output zero direction

We usually normalize the direction vectors to have unit norm

$$\|u_z\|_2 = 1 \qquad \|y_z\|_2 = 1$$

Directions of Poles and Zeros

Pole directions:

Let $G(s)$ have a pole at $s = p$. Then $G(p)$ is infinite and we may

somewhat crudely write

$$G(p)u_p = \infty \qquad y_p^H G(p) = \infty$$

u_p is input pole direction and y_p is output pole direction

$$At_p = pt_p \qquad q_p^H A = pq_p^H$$

$$y_p = Ct_p \qquad u_p = B^H q_p$$

Directions of Poles and Zeros

Example4-5:

$$G(s)=\frac{1}{s+2}\begin{bmatrix} s-1 & 4 \\ 4.5 & 2(s-1) \end{bmatrix}$$

It has a zero at z = 4 and a pole at p = -2 .

$$G(s_0)=G(3)=\frac{1}{5}\begin{bmatrix} 2 & 4 \\ 4.5 & 4 \end{bmatrix}=\begin{bmatrix} 0.588 & -0.809 \\ 0.809 & 0.588 \end{bmatrix}\begin{bmatrix} 1.475 & 0 \\ 0 & 0.271 \end{bmatrix}\begin{bmatrix} 0.653 & 0.757 \\ 0.757 & -0.653 \end{bmatrix}^H$$

$$G(z)=G(4)=\frac{1}{6}\begin{bmatrix} 3 & 4 \\ 4.5 & 6 \end{bmatrix}=\begin{bmatrix} 0.55 & -0.83 \\ 0.83 & 0.55 \end{bmatrix}\begin{bmatrix} 1.50 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}^H$$

y_z — **Output zero direction**

u_z — **Input zero direction**

Now let the input as:

$$u(t)=\begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}e^{4t} \Rightarrow Y(s)=G(s)U(s)=\frac{1}{s+2}\begin{bmatrix} s-1 & 4 \\ 4.5 & 2(s-1) \end{bmatrix}\begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}\frac{1}{s-4}=\frac{1}{s+2}\begin{bmatrix} -0.8 \\ 1.2 \end{bmatrix}$$

$$y(t)=\begin{bmatrix} -0.8e^{-2t} \\ 1.2e^{-2t} \end{bmatrix}$$

which does not contain any component of the input signal e^{4t}

Directions of Poles and Zeros

Example4-6:

$$G(s)=\frac{1}{s+2}\begin{bmatrix} s-1 & 4 \\ 4.5 & 2(s-1) \end{bmatrix}$$

It has a zero at *z = 4* and a pole at *p = -2* .

$$G(p)=G(-2)=\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$$

??!!

$$G(p+\varepsilon)=G(-2+\varepsilon)=\frac{1}{\varepsilon}\begin{bmatrix} -3+\varepsilon & 4 \\ 4.5 & 2(-3+\varepsilon) \end{bmatrix}$$

Let for example $\varepsilon = 0.001$

$$G(-2+0.001)=\begin{bmatrix} -0.55 & 0.83 \\ 0.83 & 0.55 \end{bmatrix}\begin{bmatrix} 9010 & 0 \\ 0 & 0.00 \end{bmatrix}\begin{bmatrix} 0.6 & 0.8 \\ -0.8 & 0.6 \end{bmatrix}^H$$

y_p — **Output pole direction**

u_p — **Input pole direction**

One Property of Zero Direction

System zero

$$P(s) = \begin{bmatrix} sI - A & B \\ -C & D \end{bmatrix}$$

Rosenbrock system matrix

The value of s that $P(s)$ losses rank.

Thus we have a zero at s=z:

$$\begin{bmatrix} zI - A & B \\ -C & D \end{bmatrix} \begin{bmatrix} x_z \\ -u_z \end{bmatrix} = 0 \qquad \exists \begin{bmatrix} x_z \\ -u_z \end{bmatrix} \neq 0$$

Now show that the output of following system is zero for all t

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du \qquad \Rightarrow \qquad y(t) = 0, \quad for\ all\ \ t \geq 0$$

$$x(0) = x_z, \quad u(t) = u_z e^{zt}$$

Minimality

A state space system (A, B, C and D) is minimal if it is a system with the least number of states giving its transfer function.

If there is another system (A_1, B_1, C_1 and D_1) with fewer states having the same transfer function, the given system is not minimal.

For SISO systems, a system is minimal if and only if the transfer function numerator polynomial and denominator polynomial have no common roots.

For MV systems one must use the definition of MV zeros.

Theorem4-3 . A system (A, B, C and D) is minimal if and only if it has no input decoupling zeros and no output decoupling zeros.

Poles and Zeros in Multivariable Systems

Topics to be covered include:

- Multivariable Poles and Zeros and System Type
- Direction of Poles and Zeros
- Smith-McMillan Form
- Matrix Fraction Description (MFD) and Smith-McMillan Form
- Transmission zero assignment

Smith Form of a Polynomial Matrix

Suppose that $\Pi(s)$ is a polynomial matrix.

Smith form of $\Pi(s)$ is denoted by $\Pi_s(s)$, and it is a pseudo diagonal in the following form

$$\Pi_s(s) = \begin{bmatrix} \Pi_{ds}(s) & 0 \\ 0 & 0 \end{bmatrix}$$

Where $\Pi_{ds}(s) = diag\{\bar{\varepsilon}_1(s), \bar{\varepsilon}_2(s), \ldots\ldots, \bar{\varepsilon}_r(s)\}$

$\bar{\varepsilon}_i(s)$ is a factor of $\bar{\varepsilon}_{i+1}(s)$ and $\bar{\varepsilon}_i(s) = \dfrac{\chi_i}{\chi_{i-1}}$

Smith Form of a Polynomial Matrix

Where $\Pi_{ds}(s) = diag\left\{\bar{\varepsilon}_1(s), \bar{\varepsilon}_2(s), ..., \bar{\varepsilon}_r(s)\right\}$

$\bar{\varepsilon}_i(s)$ is a factor of $\bar{\varepsilon}_{i+1}(s)$ and $\bar{\varepsilon}_i(s) = \dfrac{\chi_i}{\chi_{i-1}}$

$\chi_0 = 1$

$\chi_1 =$ **gcd {all monic minors of degree 1}**

$\chi_2 =$ **gcd {all monic minors of degree 2}**

……………………………………………………..

……………………………………………………..

$\chi_r =$ **gcd {all monic minors of degree *r*}**

Smith Form of a Polynomial Matrix

The three elementary operations for a polynomial matrix are used to find Smith form.

- Multiplying a row or column by a non-zero constant;
- Interchanging two rows or two columns; and
- Adding a non-zero polynomial multiple of a row or column to another row or column.

$$\Pi_s(s) = L_{n2}(s).......L_2(s)L_1(s)\Pi(s)R_1(s)R_2(s).....R_{n1}(s)$$

$$\Pi_s(s) = L(s)\Pi(s)R(s)$$

Smith Form of a Polynomial Matrix

Example 4-7

$$\Pi(s) = \begin{bmatrix} 4 & -(s+2) \\ 2(s+2) & -\dfrac{1}{2} \end{bmatrix}$$

$$\chi_0 = 1 \qquad \chi_1 = \gcd\{1, s+2, s+2, 1\} = 1$$

$$\chi_2 = \gcd\{s^2 + 4s + 3\} = (s+1)(s+3)$$

$$\bar{\varepsilon}_1(s) = \frac{\chi_1}{\chi_0} = 1 \qquad \bar{\varepsilon}_2(s) = \frac{\chi_2}{\chi_1} = (s+1)(s+3)$$

$$\Pi_s(s) = \begin{bmatrix} 1 & 0 \\ 0 & (s+1)(s+3) \end{bmatrix}$$

Exercise 4-4: Derive R(s) and L(s) that convert $\Pi(s)$to $\Pi_s(s)$

Smith Mc-Millan Form of a Transfer Function Matrix

Theorem 4-4 (Smith-McMillan form)

Let $G(s)=[g_{ij}(s)]$ be an $m\times p$ matrix transfer function, where $g_{ij}(s)$ are rational scalar transfer functions, *G(s)* can be represented by:

$$G(s)=\frac{1}{d(s)}\Pi(s)$$

Where *Π(s)* is an $m\times p$ polynomial matrix of rank *r* and *d(s)* is the least common multiple of the denominators of all elements of *G(s)* .

Then, $\tilde{G}(s)$ is Smith McMillan form of *G(s)* and can be derived directly by

$$\tilde{G}(s)=\frac{1}{d(s)}\Pi_s(s)=\frac{1}{d(s)}\begin{bmatrix}\Pi_{ds}(s) & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}M(s) & 0\\ 0 & 0\end{bmatrix}$$

Smith Mc-Millan Form of a Transfer Function Matrix

Theorem 4-4 (Smith-McMillan form)

$$G(s)=\frac{1}{d(s)}\Pi(s) \qquad \tilde{G}(s)=\frac{1}{d(s)}\Pi_s(s)=\frac{1}{d(s)}\begin{bmatrix}\Pi_{ds}(s) & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}M(s) & 0\\ 0 & 0\end{bmatrix}$$

$$M(s)=diag\left\{\frac{\varepsilon_1(s)}{\delta_1(s)},\frac{\varepsilon_2(s)}{\delta_2(s)},........,\frac{\varepsilon_r(s)}{\delta_r(s)}\right\}$$

$$\tilde{G}(s)=L(s)G(s)R(s) \qquad G(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)$$

The matrices $\tilde{L}(s)$, $L(s)$, $\tilde{R}(s)$ and $R(s)$ are unimodular and

$$\tilde{L}(s)=L(s)^{-1} \quad , \quad \tilde{R}(s)=R(s)^{-1}$$

Smith Mc-Millan Form of a Transfer Function Matrix

Example 4-8

$$G(s)=\begin{bmatrix}\dfrac{4}{(s+1)(s+2)} & \dfrac{-1}{(s+1)} \\ \dfrac{2}{(s+1)} & \dfrac{-1}{2(s+1)(s+2)}\end{bmatrix}$$

$$G(s)=\frac{1}{d(s)}\Pi(s)\ , \qquad \Pi(s)=\begin{bmatrix}4 & -(s+2) \\ 2(s+2) & -0.5\end{bmatrix}, \qquad d(s)=(s+1)(s+2)$$

$$\Pi_s(s)=\begin{bmatrix}1 & 0 \\ 0 & (s+1)(s+3)\end{bmatrix}$$

$$\tilde{G}(s)=\frac{1}{d(s)}\Pi_s(s)=\begin{bmatrix}\dfrac{1}{(s+1)(s+2)} & 0 \\ 0 & \dfrac{s+3}{s+2}\end{bmatrix}$$

Smith Mc-Millan Form of a Transfer Function Matrix

Example 4-9

$$G(s)=\frac{1}{d(s)}\Pi(s)=\frac{1}{(s+1)(s+2)}\begin{bmatrix}1 & -1\\ s^2+s-4 & 2s^2-s-8\\ s^2-4 & 2s^2-8\end{bmatrix}$$

$$\Pi(s)=\begin{bmatrix}1 & -1\\ s^2+s-4 & 2s^2-s-8\\ s^2-4 & 2s^2-8\end{bmatrix}$$

$$\Pi_1(s)=\begin{bmatrix}1 & -1\\ 0 & 3(s^2-4)\\ 0 & 3(s^2-4)\end{bmatrix}$$

$$\Pi_2(s)=\begin{bmatrix}1 & -1\\ 0 & 3(s^2-4)\\ 0 & 0\end{bmatrix}$$

$$\Pi_3(s)=\begin{bmatrix}1 & 0\\ 0 & 3(s^2-4)\\ 0 & 0\end{bmatrix}$$

$$\Pi_4(s)=\Pi_s(s)=\begin{bmatrix}1 & 0\\ 0 & (s^2-4)\\ 0 & 0\end{bmatrix}$$

$$\Pi_s(s)=L_2(s)L_1(s)\Pi(s)R_3(s)R_4(s)$$

$$G(s)=\frac{1}{d(s)}\Pi(s)=\frac{1}{d(s)}\tilde{L}(s)\Pi_s(s)\tilde{R}(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)$$

$$\tilde{G}(s)=\begin{bmatrix}\frac{1}{(s+1)(s+2)} & 0\\ 0 & \frac{s-2}{s+1}\\ 0 & 0\end{bmatrix}$$

Poles and Zeros in Multivariable Systems

Topics to be covered include:

- Multivariable Poles and Zeros and System Type
- Direction of Poles and Zeros
- Smith-McMillan Form
- Matrix Fraction Description (MFD) and Smith-McMillan Form
- Transmission zero assignment

Matrix Fraction Description (MFD)

Matrix Fraction Description for Transfer Matrix

$$\mathrm{G}(s) = \frac{1}{d(s)} N(s)$$

Suppose G is an p×q matrix so

$$\mathrm{G}(s) = \left(d(s) I_p\right)^{-1} N(s) = D_L^{-1}(s) N_L(s)$$

Left Matrix Fraction Description (LMFD)

$$\mathrm{G}(s) = N(s)\left(d(s) I_q\right)^{-1} = N_R(s) D_R^{-1}(s)$$

Right Matrix Fraction Description (RMFD)

But this forms are not irreducible.

Irreducible RMFD and LMFD can be derived directly through SMM form.

Matrix Fraction Description & Smith-McMillan form

Let $G(s)$ is a $m\times m$ matrix and its the Smith McMillan is $\tilde{G}(s)$

Let define: $N(s)\overset{\Delta}{=}diag\left(\varepsilon_1(s),\ldots,\varepsilon_r(s),0,\ldots,0\right)$ $D(s)\overset{\Delta}{=}diag\left(\delta_1(s),\ldots,\delta_r(s),1,\ldots,1\right)$

$$\tilde{G}(s)=N(s)D(s)^{-1} \quad \text{or} \quad \tilde{G}(s)=D(s)^{-1}N(s)$$

We know that

$$G(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)=\tilde{L}(s)N(s)D(s)^{-1}\tilde{R}(s)=\tilde{L}(s)N(s)\left(R(s)D(s)\right)^{-1}=N_R(s)D_R(s)^{-1}$$

$$G(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)=\tilde{L}(s)D(s)^{-1}N(s)\tilde{R}(s)=\left(D(s)L(s)\right)^{-1}N(s)\tilde{R}(s)=D_L(s)^{-1}N_L(s)$$

It is easy to see that when a RMFD is irreducible, then

* $s=z$ is a transmission zero of $G(s)$ if and only if $N_L(s)$ or $N_R(s)$ losses rank at $s=z$

* $s=p$ is a pole of $G(s)$ if and only if $D_L(s)$ or $D_R(s)$ is singular at $s=p$

This means that the pole polynomial of G(s) is $\varphi(s)=\det\left(G_D(s)\right)$

Matrix Fraction Description & Smith-McMillan form

Example 4-10

$$G(s)=\begin{bmatrix}\dfrac{1}{(s+1)(s+2)} & \dfrac{-1}{(s+1)(s+2)}\\ \dfrac{s^2+s-4}{(s+1)(s+2)} & \dfrac{2s^2-s-8}{(s+1)(s+2)}\\ \dfrac{s-2}{(s+1)} & \dfrac{2s-4}{(s+1)}\end{bmatrix}$$

$$G(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)=\begin{bmatrix}1 & 0 & 0\\ s^2+s-4 & 1 & 0\\ s^2-4 & 1 & 1\end{bmatrix}\begin{bmatrix}\dfrac{1}{(s+1)(s+2)} & 0\\ 0 & \dfrac{s-2}{s+1}\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}=$$

$$\begin{bmatrix}1 & 0 & 0\\ s^2+s-4 & 1 & 0\\ s^2-4 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & s-2\\ 0 & 0\end{bmatrix}\begin{bmatrix}(s+2)(s+1) & 0\\ 0 & s+1\end{bmatrix}^{-1}\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}=\begin{bmatrix}1 & 0\\ s^2+s-4 & s-2\\ s^2-4 & s-2\end{bmatrix}\begin{bmatrix}(s+2)(s+1) & \dfrac{s+1}{3}\\ 0 & \dfrac{s+1}{3}\end{bmatrix}^{-1}$$

RMFD: $N_R(s)$ $D_R(s)$

Matrix Fraction Description & Smith-McMillan form

Example 4-10

$$G(s)=\begin{bmatrix}\frac{1}{(s+1)(s+2)} & \frac{-1}{(s+1)(s+2)}\\ \frac{s^2+s-4}{(s+1)(s+2)} & \frac{2s^2-s-8}{(s+1)(s+2)}\\ \frac{s-2}{(s+1)} & \frac{2s-4}{(s+1)}\end{bmatrix}$$

$$G(s)=\tilde{L}(s)\tilde{G}(s)\tilde{R}(s)=\begin{bmatrix}1 & 0 & 0\\ s^2+s-4 & 1 & 0\\ s^2-4 & 1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{(s+1)(s+2)} & 0\\ 0 & \frac{s-2}{s+1}\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}=$$

$$\begin{bmatrix}1 & 0 & 0\\ s^2+s-4 & 1 & 0\\ s^2-4 & 1 & 1\end{bmatrix}\begin{bmatrix}(s+2)(s+1) & 0 & 0\\ 0 & s+1 & 0\\ 0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0\\ 0 & s-2\\ 0 & 0\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 3\end{bmatrix}=\begin{bmatrix}(s+2)(s+1) & 0 & 0\\ (s+1)(4-s^2-s) & s+1 & 0\\ s & -1 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & -1\\ 0 & 3(s-2)\\ 0 & 0\end{bmatrix}$$

LMFD: $D_L(s)$ $N_L(s)$

Poles and Zeros in Multivariable Systems

Topics to be covered include:

- Multivariable Poles and Zeros and System Type
- Direction of Poles and Zeros
- Smith-McMillan Form
- Matrix Fraction Description (MFD) and Smith-McMillan Form
- Transmission zero assignment

Transmission zero assignment

Pole assignment

$$G(s)\begin{cases}\dot{x} = Ax + Bu\\ y = Cx\end{cases}$$

State feedback

$$u = r - Kx$$

New system

$$G_f(s)\begin{cases}\dot{x} = (A\text{-}BK)x + Br\\ y = Cx\end{cases}$$

When can one assign the poles in arbitrary place?

Transmission zero assignment?

Transmission zero assignment

Zeros position depends on the location of sensors and actuators.

Changing zero position can affect the output response.

Theorem4-5: Transmission zeros cannot be assigned by state feedback.

Proof

System is defined by $P(s)=\begin{bmatrix} sI-A & B \\ -C & D \end{bmatrix}$

System with feedback is: $P_f(s)=\begin{bmatrix} sI-A+BK & B \\ -C+DK & D \end{bmatrix}$

$$P_f(s)=\begin{bmatrix} sI-A+BK & B \\ -C+DK & D \end{bmatrix}=\begin{bmatrix} sI-A & B \\ -C & D \end{bmatrix}\begin{bmatrix} I & 0 \\ K & I \end{bmatrix}=P(s)\begin{bmatrix} I & 0 \\ K & I \end{bmatrix}$$

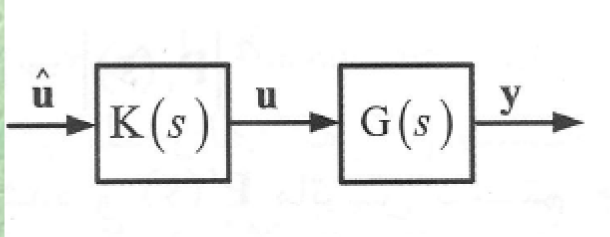
$$\rho\left(P_f(s)\right)=\rho\left(P(s)\right)$$

Theorem4-6: Transmission zeros cannot be assigned by output feedback.

Transmission zero assignment

What about following structures?

$$G(s)\begin{cases}\dot{x} = Ax + Bu \\ y = Cx\end{cases} \qquad K(s)\begin{cases}\dot{w} = Fw + G\hat{u} \\ u = Hw + D\hat{u}\end{cases}$$



Theorem4-7: Transmission zeros cannot be assigned by series compensation.

$$P_s(s) = \begin{bmatrix} sI - F & 0 & G \\ -BH & sI - A & BD \\ 0 & -C & 0 \end{bmatrix}$$

$$= \begin{bmatrix} I & 0 & 0 \\ 0 & sI - A & B \\ 0 & -C & 0 \end{bmatrix} \begin{bmatrix} sI - F & 0 & G \\ 0 & I & 0 \\ -H & 0 & D \end{bmatrix}$$

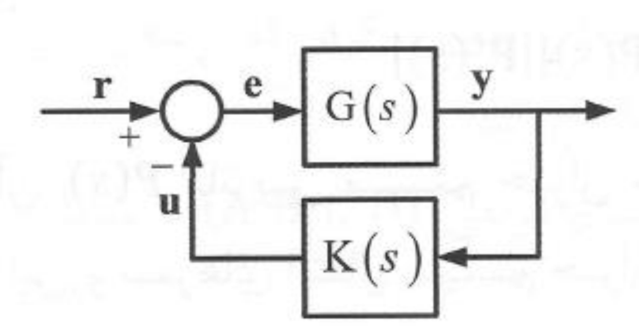
$$|P_s(s)| = |P(s)||P'(s)|$$

$$\{z_{new}\} = \{z_{system}\} \cup \{z_{controller}\}$$

Transmission zero assignment

What about following structures?

$$G(s)\begin{cases}\dot{x}=Ax+B(r-u)\\ y=Cx\end{cases}\qquad K(s)\begin{cases}\dot{w}=Fw+Gy\\ u=Hw+Dy\end{cases}$$



Theorem4-8: Transmission zeros cannot be assigned by dynamic feedback.

$$P_f(s)=\begin{bmatrix} sI-F & -GC & 0\\ BH & sI-A+BDC & B\\ 0 & -C & 0\end{bmatrix}$$

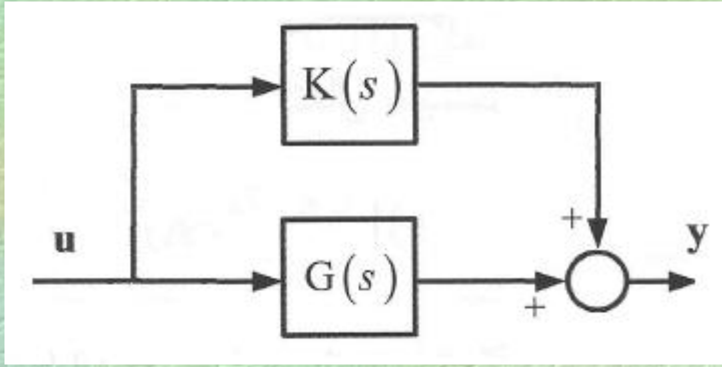
$$=\begin{bmatrix} sI-F & 0 & G\\ BH & I & BD\\ 0 & 0 & I\end{bmatrix}\begin{bmatrix} I & 0 & 0\\ 0 & sI-A & B\\ 0 & -C & 0\end{bmatrix}$$

$$|P_f(s)|=|sI-F|\,|P(s)|$$

$$\{z_{new}\}=\{p_{controller}\}\cup\{z_{system}\}$$

Transmission zero assignment

A method to assign transmission zero.



Basic idea!

Let:

$$G(s)=\frac{s+2}{s+1}$$

Clearly there is one pole at s= -1 and one transmission zero at s= -2.

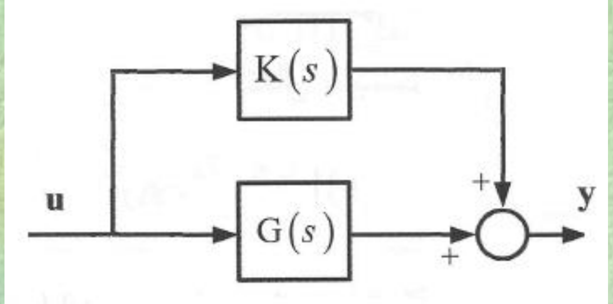
Now let:

$$K(s)=k \qquad G_{new}(s)=\frac{s+2}{s+1}+k=\frac{(k+1)s+2+k}{s+1}$$

Where is the pole and transmission zero of new system?

Transmission zero assignment

A method to assign transmission zero.



Let G is a square matrix with m inputs and m outputs.

In the case of 2m>n a static controller (K(s)=K) can help. Otherwise, a dynamic controller can help.

$$G(s)\begin{cases}\dot{x}=Ax+Bu\\ y_g=Cx+Du\end{cases}$$

$$G(s)+K\begin{cases}\dot{x}=Ax+Bu\\ y=Cx+(K+D)u\end{cases}$$

If K+D is nonsingular(K assigned by designer)

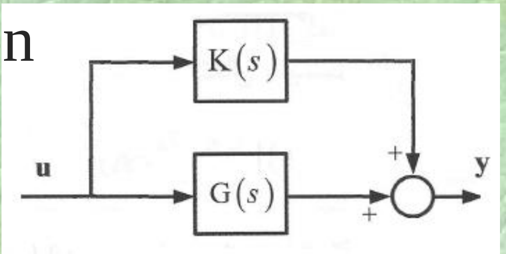
$$(G(s)+K)^{-1}\begin{cases}\dot{z}=\left(A-B(K+D)^{-1}C\right)z+B(K+D)^{-1}y\\ u=-(K+D)^{-1}Cz+(K+D)^{-1}y\end{cases}$$

Now assign poles of inverse system by suitable K.

This is simply an output feedback for ??

Transmission zero assignment

Example 4-11: Suppose we want to assign transmission zeros of following system at roots of $(s+2)(s^2+3s+4)$



$$G(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{2}{s+3}\\ \frac{1}{s+1} & \frac{1}{s+1}\end{bmatrix}$$

s=1 is transmission zero of system. State space of system is:

$$A=\begin{bmatrix}1 & -8 & -8\\ 1 & -5 & -4\\ 0 & 0 & -1\end{bmatrix}\quad B=\begin{bmatrix}0 & 0\\ -1 & 0\\ 1 & 1\end{bmatrix}\quad C=\begin{bmatrix}0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}$$

Now try to assign poles of:

$$A-B(K+D)^{-1}C$$

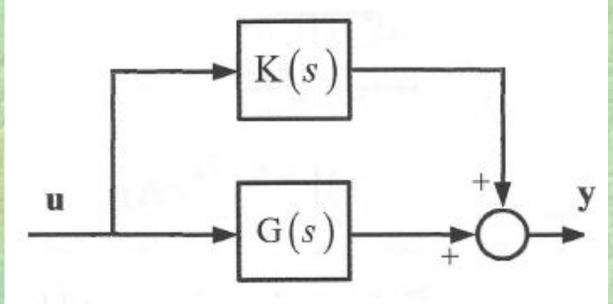
Answer is:

$$K=\begin{bmatrix}0 & -1\\ 1 & 1\end{bmatrix}$$

Exercise 4-5: Assign the zeros at -1, -2 and -3.(Arbitrary)

Transmission zero assignment

A method to assign transmission zero.



Let G is a square matrix with m inputs and m outputs.

In the case of $2m \le n$ a dynamic controller can help.

$$G(s)\begin{cases}\dot{x} = Ax + Bu \\ y_g = Cx + Du\end{cases}$$

$$K(s)\begin{cases}\dot{z} = Fz + Gu \\ y_k = Hz + Eu\end{cases}$$

$$Augmented \Rightarrow \begin{cases}\begin{bmatrix}\dot{x} \\ \dot{z}\end{bmatrix} = \begin{bmatrix}A & 0 \\ 0 & F\end{bmatrix}\begin{bmatrix}x \\ z\end{bmatrix} + \begin{bmatrix}B \\ G\end{bmatrix}u \\ y = \begin{bmatrix}C & H\end{bmatrix}\begin{bmatrix}x \\ z\end{bmatrix} + (E + D)u\end{cases}$$

We want to assign transmission zeros ??

Now assign poles of inverse system by suitable F,G,H and E.

Try that E+D is nonsingular(E assigned by designer).

Transmission zero assignment

A method to assign transmission zero.

$$G(s)+K(s) \Rightarrow \begin{cases} \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ G \end{bmatrix} u \\ y = \begin{bmatrix} C & H \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + (E+D)u \end{cases}$$

If E+D is nonsingular(E assigned by designer)

Now assign poles of inverse system by suitable F,G,H and E.

$$(G(s)+K)^{-1} \begin{cases} \dot{w} = \left(\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - \begin{bmatrix} B \\ G \end{bmatrix} (E+D)^{-1} \begin{bmatrix} C & H \end{bmatrix} \right) w + \begin{bmatrix} B \\ G \end{bmatrix} (E+D)^{-1} y \\ u = -(E+D)^{-1} \begin{bmatrix} C & H \end{bmatrix} w + (E+D)^{-1} y \end{cases}$$

Now try to assign poles of following matrix by suitable F,G,H and E.

$$\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - \begin{bmatrix} B \\ G \end{bmatrix} (E+D)^{-1} \begin{bmatrix} C & H \end{bmatrix}$$

Transmission zero assignment

A method to assign transmission zero.

Now try to assign poles of following by suitable F,G,H and E.

$$\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - \begin{bmatrix} B \\ G \end{bmatrix} (E+D)^{-1} \begin{bmatrix} C & H \end{bmatrix}$$

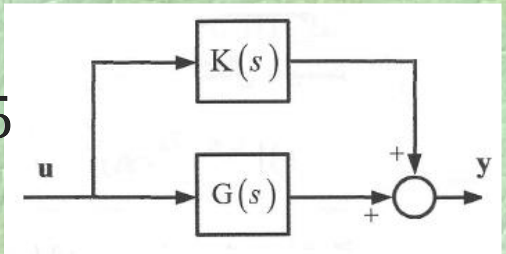
We have:

$$\begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} - \begin{bmatrix} B \\ G \end{bmatrix} (E+D)^{-1} \begin{bmatrix} C & H \end{bmatrix}$$
$$= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} (E+D)^{-1} & (E+D)^{-1} H \\ G(E+D)^{-1} & -F + G(E+D)^{-1} H \end{bmatrix} \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$$

This is simply an output feedback for ??

Transmission zero assignment

Example 4-12: Suppose we want to assign transmission zeros of following system at -1, -4 and -5



$$G(s)=\frac{-s+1}{s^2+5s+6}$$

s=1 is transmission zeros of system. State space of system is:

$$A=\begin{bmatrix}0 & 1\\ -6 & -5\end{bmatrix}\quad B=\begin{bmatrix}0\\ 1\end{bmatrix}\quad C=\begin{bmatrix}1 & -1\end{bmatrix}\quad D=0$$

2=2 so we need dynamic compensation with r = 1, so, try to assign poles of:

$$\hat{A}=\begin{bmatrix}0 & 1 & 0\\ -6 & -5 & 0\\ 0 & 0 & 0\end{bmatrix}\quad \hat{B}=\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}\quad \hat{C}=\begin{bmatrix}1 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}(E+D)^{-1} & (E+D)^{-1}H\\ G(E+D)^{-1} & -F+G(E+D)^{-1}H\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 10 & 4\end{bmatrix}$$

$E=-1$ $\quad$ $H=0$

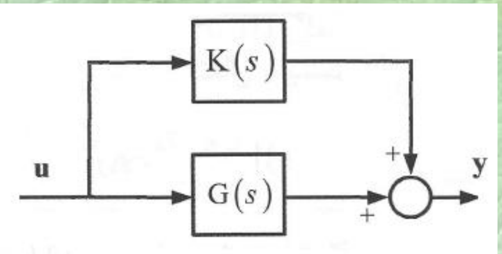
$G=-10$ $\quad$ $F=-4$

$$K(s)\begin{cases}\dot{z}=Fz+Gu=-4z-10u\\ y_k=Hz+Eu=-u\end{cases}$$

Check final system!!??

Transmission zero assignment

Exercise 4-6: Suppose we want to assign transmission zeros of following system at -0.5, -1, -1.5, -2 and -2.5 (Arbitrary)



$$G(s) = \frac{5s^2 - 9s - 5}{s^3 - 8s^2 + 14s - 5}$$

For zero assignment procedure see following papers.

A. Khaki Sedigh "Transmission zero assignment for linear Multivariable plans" 10th IASTED International Symposium, 1991.

R.V Patel and P. Misra "Transmission zero assignment in linear multivariable systems" ACC/WM5 1992.

Exercises

Exercise 4-1 till 4-6: Mentioned in the lecture.

Exercise 4-7: Consider following system. (Arbitrary)

$$\dot{x}=\begin{bmatrix}1 & 3 & -1\\ -2 & 0 & 1\\ 3 & 1 & 2\end{bmatrix}x+\begin{bmatrix}1\\0\\0\end{bmatrix}u$$
$$y=\begin{bmatrix}1 & 0 & 0\end{bmatrix}x$$

a) Find the SMM form of the system.
b) Find the pole and zero polynomial of the system.
c) Find the RMFD and LMFD of the system.

Exercise 4-8: Consider following transfer function.

$$G(s)=\begin{bmatrix}\dfrac{1}{s+2}\\ \dfrac{s-2}{s+4}\\ 2\end{bmatrix}$$

a) Find the SMM form of the system system.
b) Find the pole and zero polynomial of the system.
c) Find the RMFD and LMFD of the system.

Exercises(Continue)

Exercise 4-9: Consider following transfer function.

$$G(s)=\begin{bmatrix}\frac{1}{s+2} & \frac{s+2}{s+4}\\ \frac{s-2}{s+4} & \frac{s+1}{s-3}\\ \frac{3}{s+2} & \frac{4}{2s-1}\end{bmatrix}$$

a) Find the SMM form of the system.
b) Find the pole and zero polynomial of the system.
c) Find the RMFD and LMFD of the system.

Exercise 4-10: Find the degree and the characteristic polynomials of the following proper rational matrices.

$$\text{a.}\ \begin{bmatrix}\frac{1}{(s+1)^2} & \frac{s+3}{s+2} & \frac{1}{s+5}\\ \frac{1}{(s+3)^2} & \frac{s+1}{s+4} & \frac{1}{s}\end{bmatrix} \qquad \text{b.}\ \begin{bmatrix}\frac{1}{(s+1)^2} & \frac{1}{(s+2)(s+1)}\\ \frac{1}{s+2} & \frac{1}{(s+2)(s+1)}\end{bmatrix}$$

Exercise 4-11: Assign the zeros of following system on -2 and -4.(Final)

$$G(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{2}{s+3}\\ \frac{1}{s+1} & 0\end{bmatrix}$$

Ans: K=diag(0.5,4).

References

References

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