LINEAR CONTROL SYSTEMS

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Lecture 8 – Part I

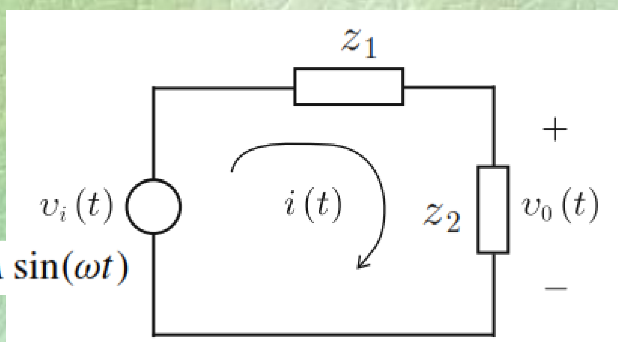
Frequency domain analysis

Topics to be covered include:

- Introduction.
- Frequency domain charts.
  - Bode plot.
  - Nichols chart.
  - Polar plot.
- Stability analysis.
  - Gain margin.
  - Phase margin.
  - Crossover frequencies.
- Effect of adding poles and zeros on loop transfer function.

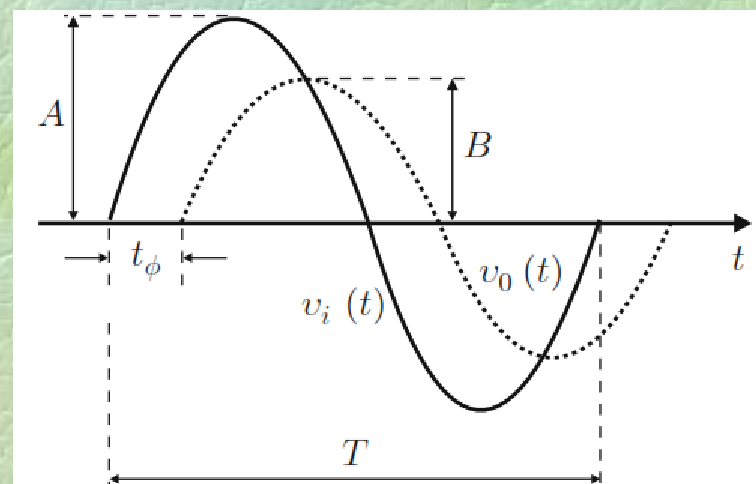
Introduction

The frequency response is the steady-state response of a system to a sinusoidal input where the frequency is varied from zero to infinity.



$v_i(t) = A\sin(\omega t)$

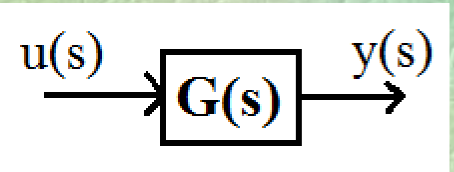
$v_0(t) = B\sin(\omega t + \phi)$



Introduction

Why frequency domain?

- It is just for LTI systems.
- It is very easy and near to reality.
- It is suitable for high order systems.
- Time response is important for us but there is a good relation …..



$a\sin(\omega t + \delta)$

SS response??

$a|G(j\omega)|\sin(\omega t + \delta + \angle G(j\omega))$

Advantages of the Frequency Response Method

- **Simplicity**: It simplifies the analysis of complex systems by focusing on their behavior in the frequency domain, which can be easier to interpret than time-domain responses.

- **Insight into System Behavior**: It provides clear insights into how a system responds to different frequencies, helping to understand resonances, bandwidth, and stability.

- **Design and Tuning**: It aids in the design and tuning of controllers by allowing engineers to shape the system’s frequency response to meet specific performance criteria.

- **Stability and Robustness**: It helps assess stability margins and robustness by analyzing gain and phase margins in the frequency domain.

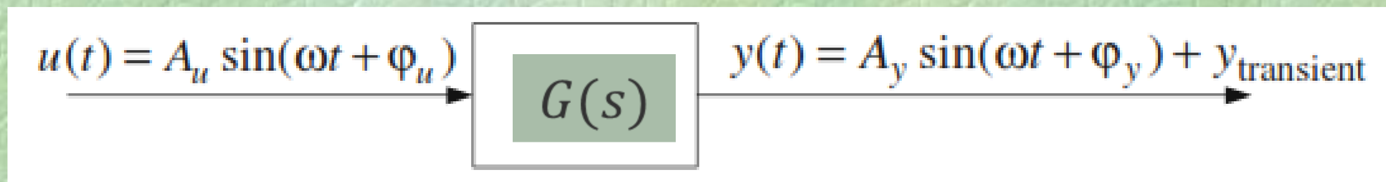
Advantages of the Frequency Response Method

•**Tool Availability**: There are many powerful tools and methods available for analyzing and designing systems in the frequency domain, such as Bode plots, Nyquist plots, and root locus.

•**Noise and Disturbance Analysis**: It can help analyze the effects of noise and disturbances on system performance, as it separates the system's response from the input signal.

•**Frequency Response Testing**: It allows for straightforward testing of system response to sinusoidal inputs, which can be practical for both theoretical and experimental analysis.

Steady-State Response to a Sinusoidal Input



$$y(t)=y_{steady}(t)+y_{transient}(t).$$

Steady-State Output of the System:

$$y_{steady}(t)=A_y\sin(\omega t+\varphi_y).$$

Steady-State Response to a Sinusoidal Input

Example 1: Consider the following system. Determine the steady-state output for a sinusoidal input as follows:"

$$G(s) = \frac{k}{s+1} \qquad x(t) = X.\sin(wt)$$

Solution:

$$G(s) = \frac{k}{s+1} \Rightarrow G(j\omega) = \frac{k}{j\omega+1}$$

Steady-State Output of the System

$$y(t) = y\sin(\omega t + \theta)$$

?? ??

Reminder:
$$|a + jb| = \sqrt{a^2 + b^2}$$

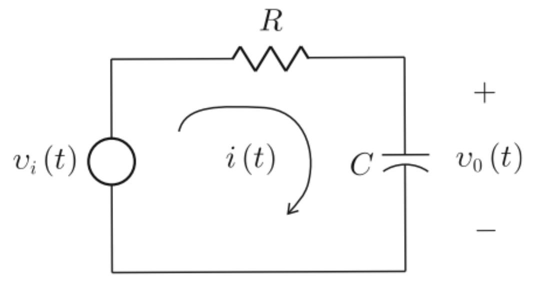
Reminder:
$$\angle(a + jb) = \tan^{-1}\frac{b}{a}$$

$$|G(j\omega)| = \frac{k}{\sqrt{1+\omega^2}} \qquad \angle G(j\omega) = -\tan^{-1}\omega$$

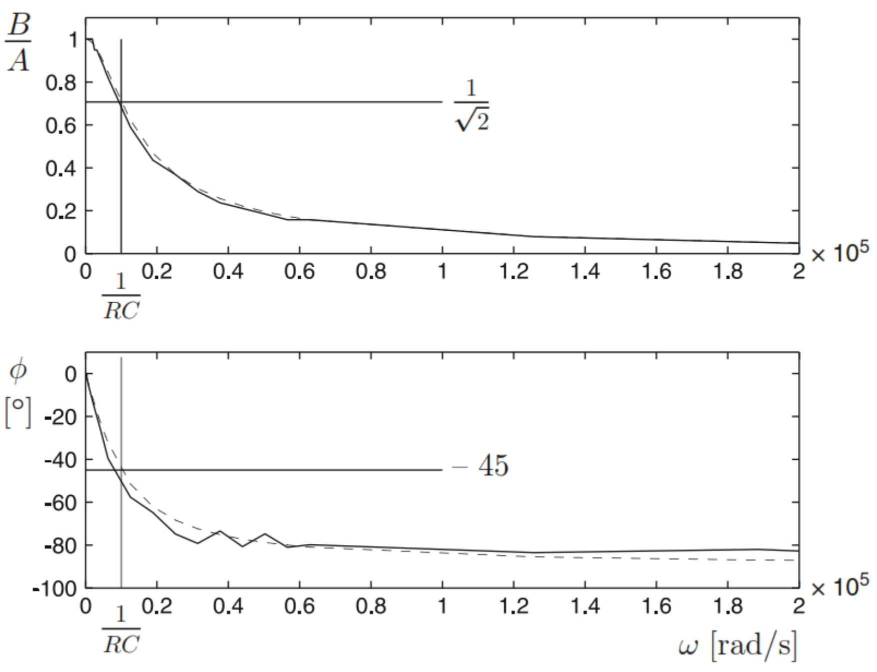
$$y(t) = y\sin(\omega t + \theta) = \frac{kx}{\sqrt{1+\omega^2}} \cdot \sin(\omega t - \tan^{-1}(\omega))$$

Steady-State Response to a Sinusoidal Input

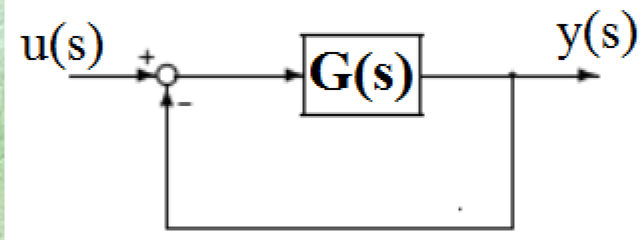
Example 2: Laboratory Method for Plotting the Frequency Response.



f [Hz]	$\omega = 2\pi f [\times 10^5 \text{rad/s}]$	A [V]	B [V]	ϕ [°]
50	0.0031	4	4	0
60	0.0038	4	4	−2.3
70	0.0044	4	4	−2.5
80	0.0050	4	4	−2.8
90	0.0057	4	4	−3.8
100	0.0063	4	4	−4.3
200	0.0126	4	4	−8.5
300	0.0188	4	4	−12.9
⋮				
8000	0.5027	3.8	0.7	−74.7
9000	0.5655	3.8	0.6	−81
10000	0.6283	3.8	0.6	−79.8
20000	1.2566	3.8	0.3	−83.5
30000	1.8850	3.8	0.2	−82
40000	2.5133	3.9	0.1	−86
50000	3.1416	3.9	0.1	−87



Introduction

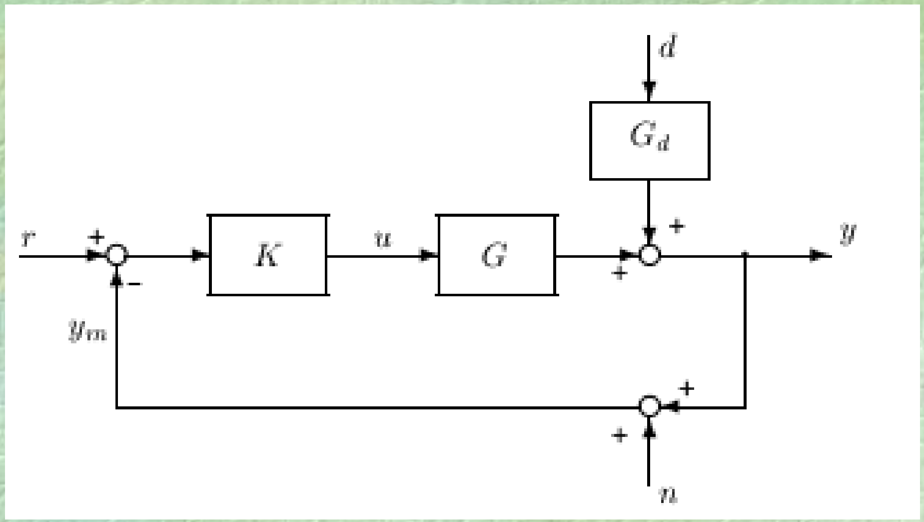


$$G(s) = \frac{e^{-s}}{s(s+1)}$$

$$u(t) = 5\sin(2t + \frac{\pi}{4})$$

$$y(t) = ??$$

One degree-of-freedom configuration



Different transfer functions:

S(s), T(s) and L(s)

Control targets:

Command tracking, ………..

Disturbance rejection and ..……….

Noise attenuation. ………..

Frequency domain charts:

Bode plot, ……….

Nichols chart and ……….

polar plot. ……….

One degree-of-freedom configuration

Frequency domain specifications:

Peak of resonance(M_P)

Resonance frequency(ω_P)

Closed-loop bandwidth

Open-loop bandwidth

Gain crossover frequencies.

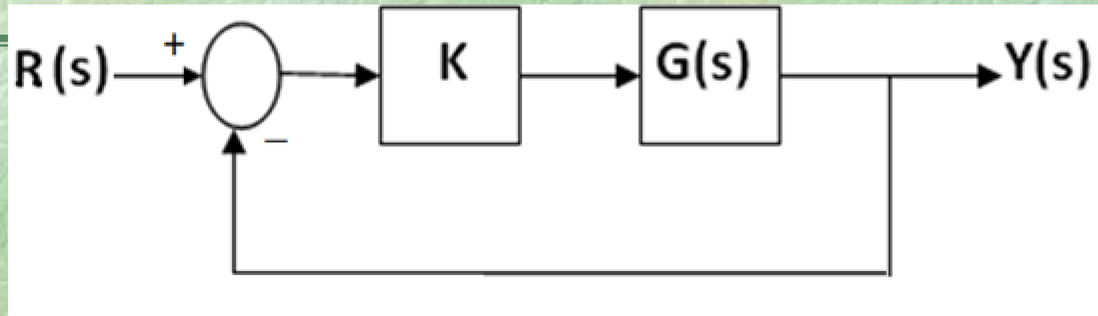
Phase crossover frequencies.

Gain margin (GM)

Phase margin (PM)

Sensitivity Peak (M_s)

Stability Analysis Using the Bode Diagram



Characteristic equation $1+KG(s)=0$

$$K(G(j\omega)) = -1$$

$$20log|K(G(j\omega)| = 0$$

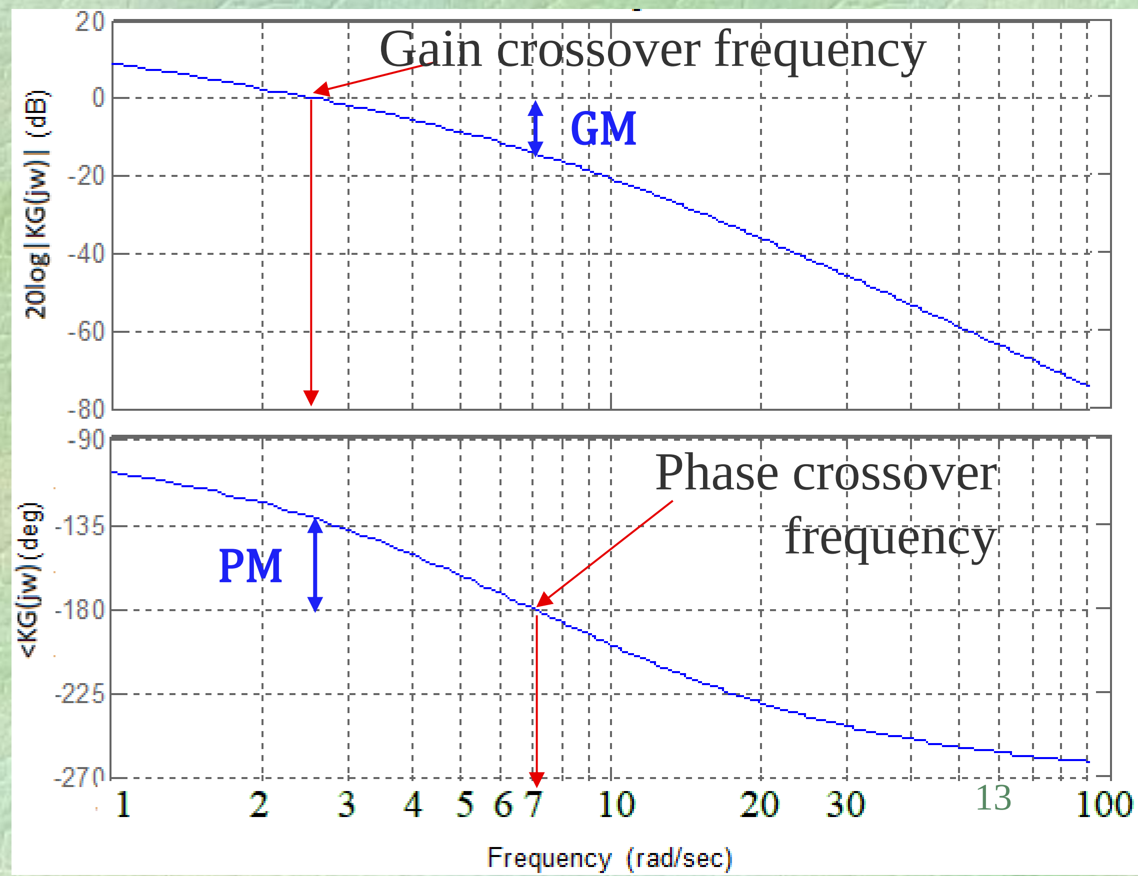
$$< |K(G(j\omega)| = -180$$

Gain crossover frequency=2.5 rad/sec

Phase Margin=50°

Phase crossover frequency=7 rad/sec

Gain Margin=15 dB



Stability Analysis Using the Bode Diagram

Gain crossover frequency ω_c: The frequency at which the magnitude plot intersects the 0 dB line (the magnitude of the transfer function equals 1).

Phase crossover frequency ω_{180} : The frequency at which the phase plot intersects the -180 degree line.

Gain Margin (*GM*): The gain margin is the difference between the system gain at the phase crossover frequency and 0 db line.

$$GM = -20\log|G(j\omega_b)|$$

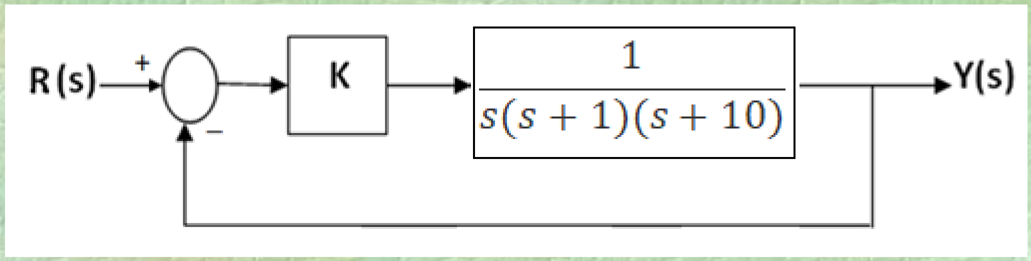
Phase Margin (*GM*): The phase margin is the difference between the system phase at the gain crossover frequency and -180 degree.

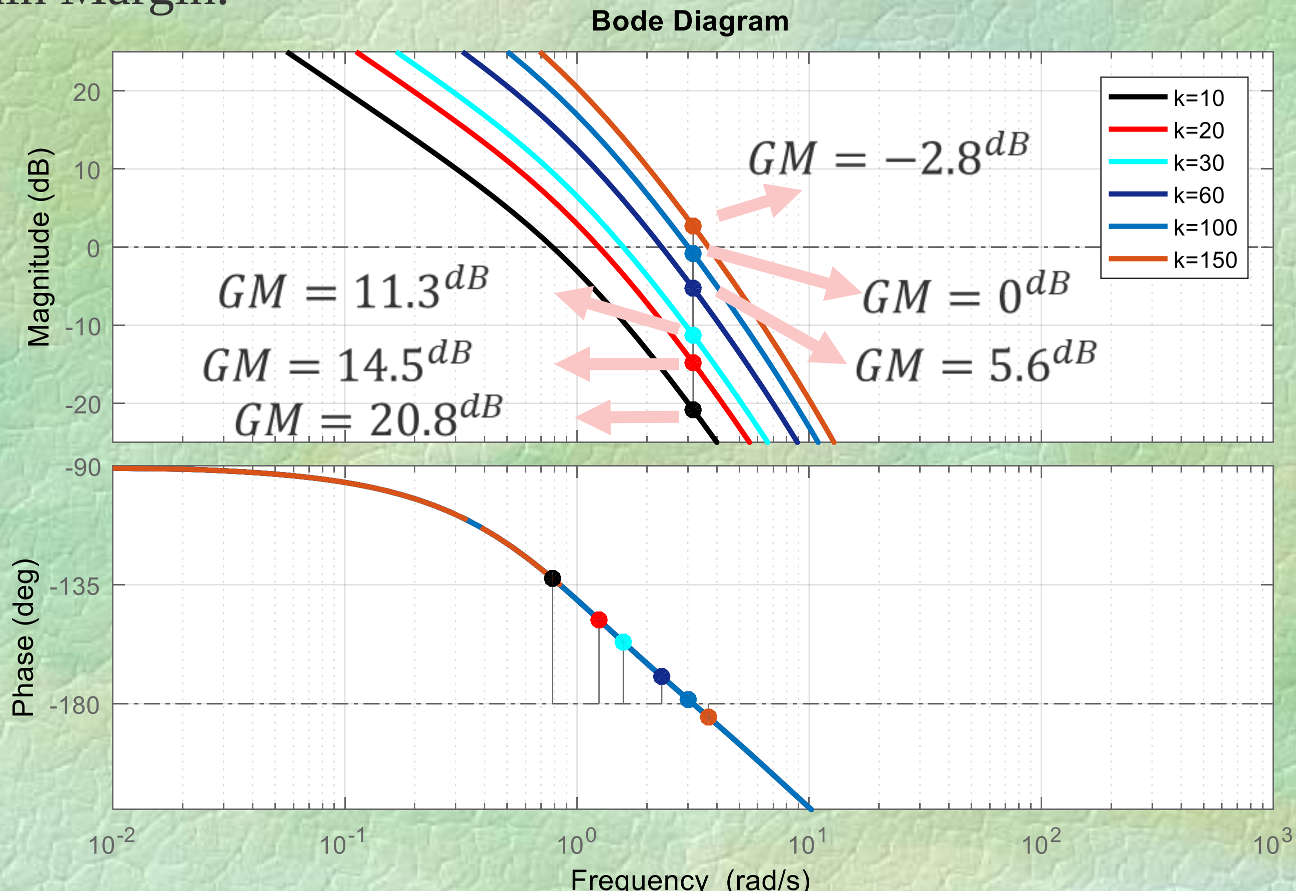
$$PM =< \mathrm{G}(\mathrm{j}\omega_c) + 180$$

In stable systems, both Phase Margin (PM) and Gain Margin (GM) are positive.

Stability Analysis Using the Bode Diagram

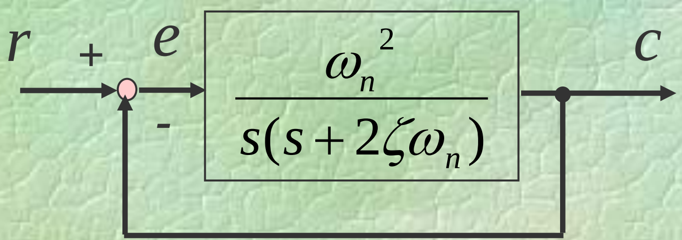
The concept of Gain Margin is: The maximum amount of k in dB by which the system can remain stable in a closed-loop configuration is called the Gain Margin.

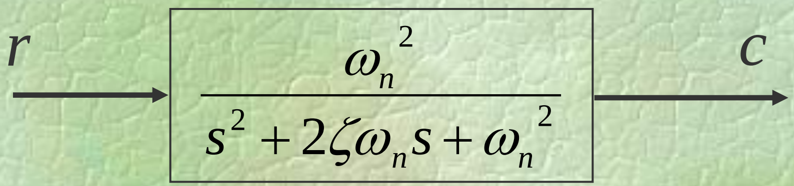


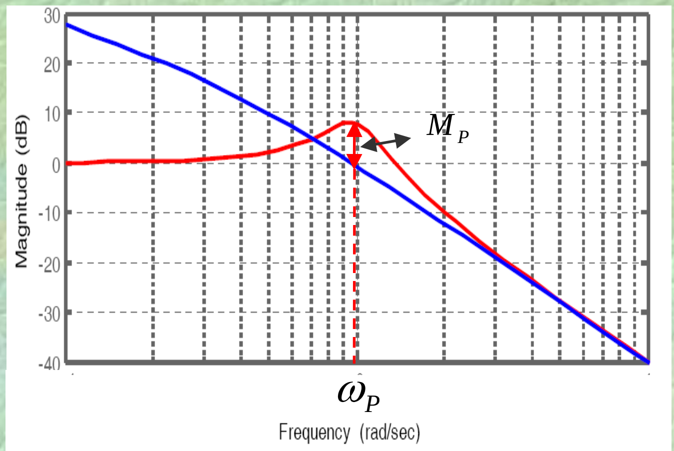


Dr. Ali Karimpour Aug 2024

Introducing a prototype second order system.







Resonance frequency (ω_P)

$$\omega_p = \begin{cases} \omega_n\sqrt{1-2\xi^2} & \xi < 0.707 \\ 0 & \xi > 0.707 \end{cases}$$

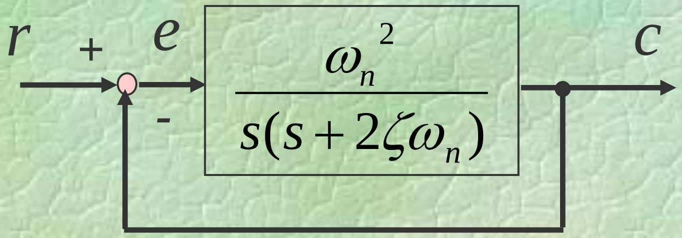
Peak of resonance (M_P)

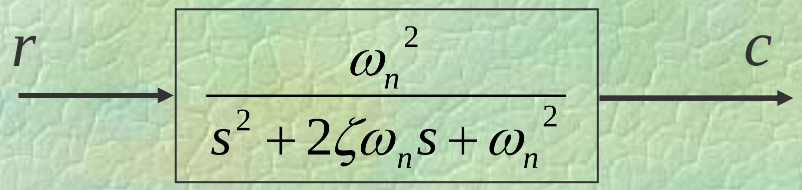
$$M_p = \begin{cases} \dfrac{1}{2\xi\sqrt{1-\xi^2}} \;\; or \;\; 20\log\left(\dfrac{1}{2\xi\sqrt{1-\xi^2}}\right) & \xi < 0.707 \\ 1 \;\; or \;\; 0\,db & \xi > 0.707 \end{cases}$$

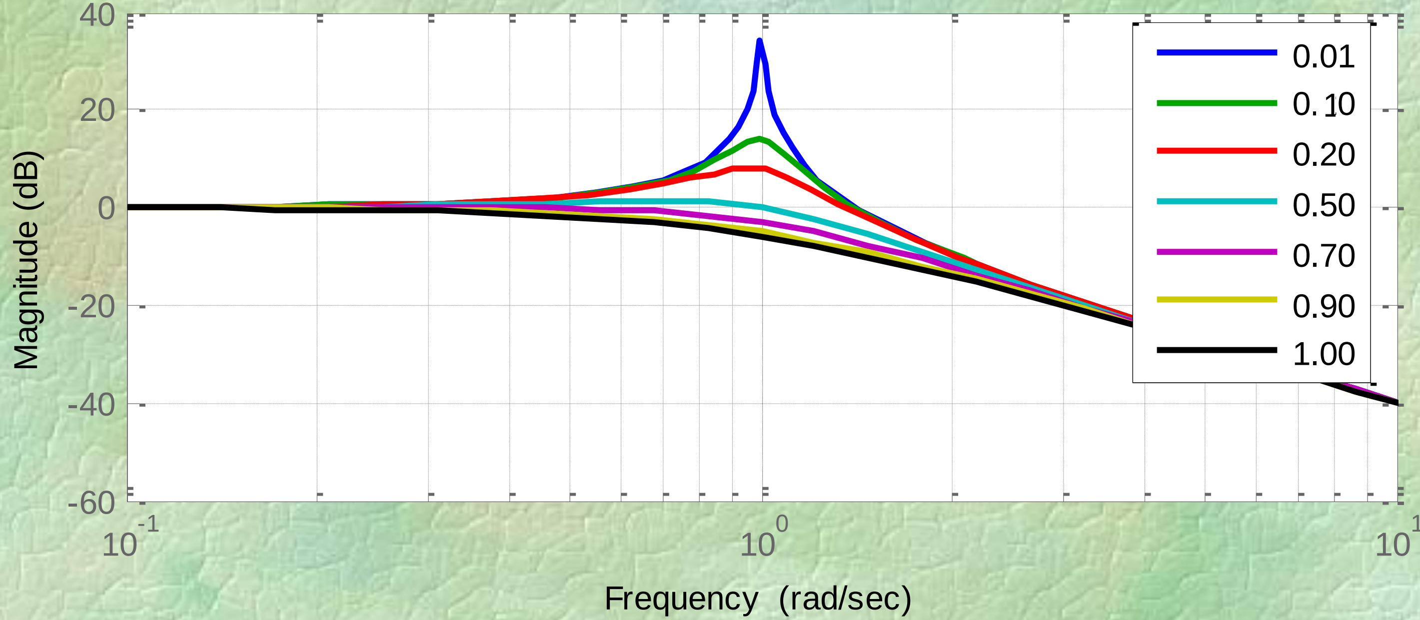
Closed-loop bandwidth (ω_B)

$$\omega_{\mathbf{B}} = \omega_n\left((1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}\right)^{1/2}$$

Introducing a prototype second order system.







$$Magnitude\ \ plot\ \ for\ different\ \ \xi\ \ and\ \ \omega_n = 1$$

Frequency domain analysis

- Introduction.
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  - Phase margin.
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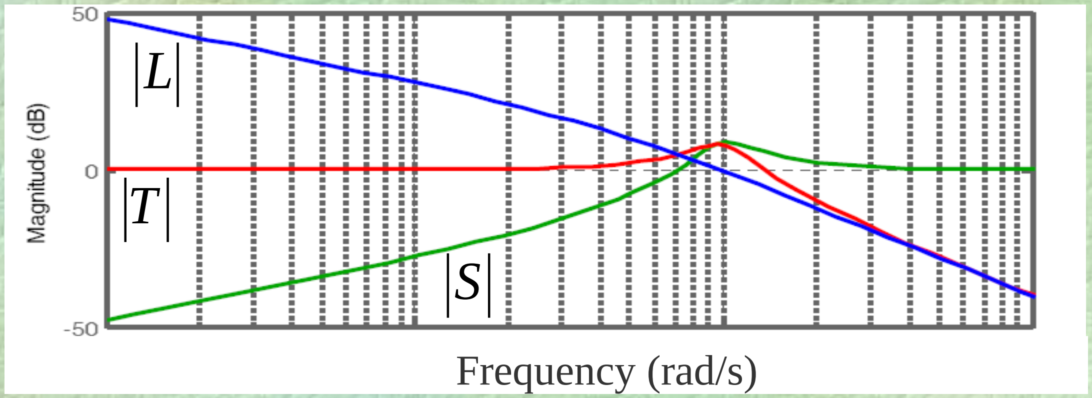
One degree-of-freedom configuration

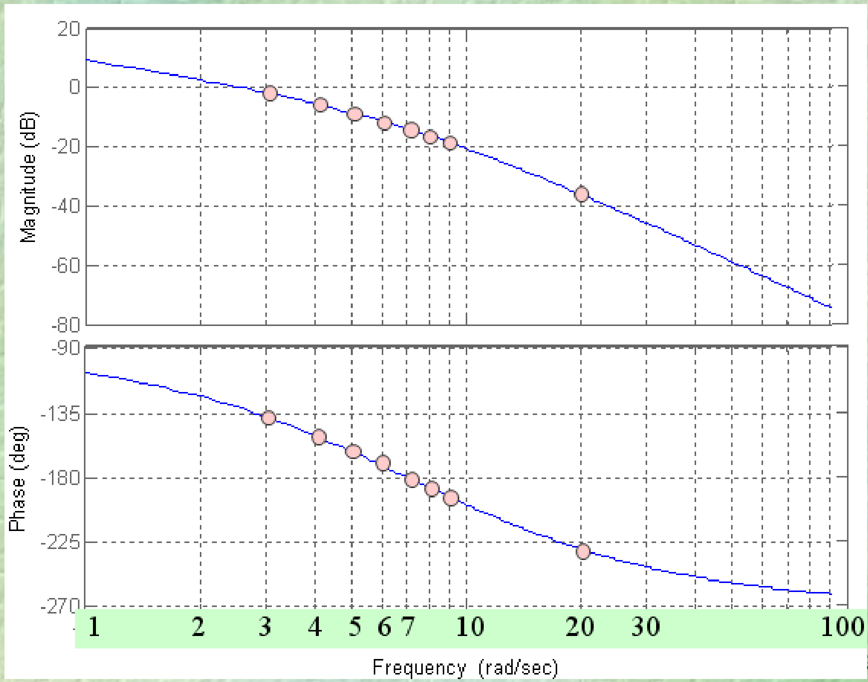


Remarks:

1- This is absolute value versus frequency, what about phase?

2- What is Bode plot?

3- How to derive Bode plot?



Frequency domain analysis

Example 3: Design Bode plot for given system.

$$G(s)=\frac{2500}{s(s+25)}=\frac{100}{s(s/25+1)}$$

Remarks:

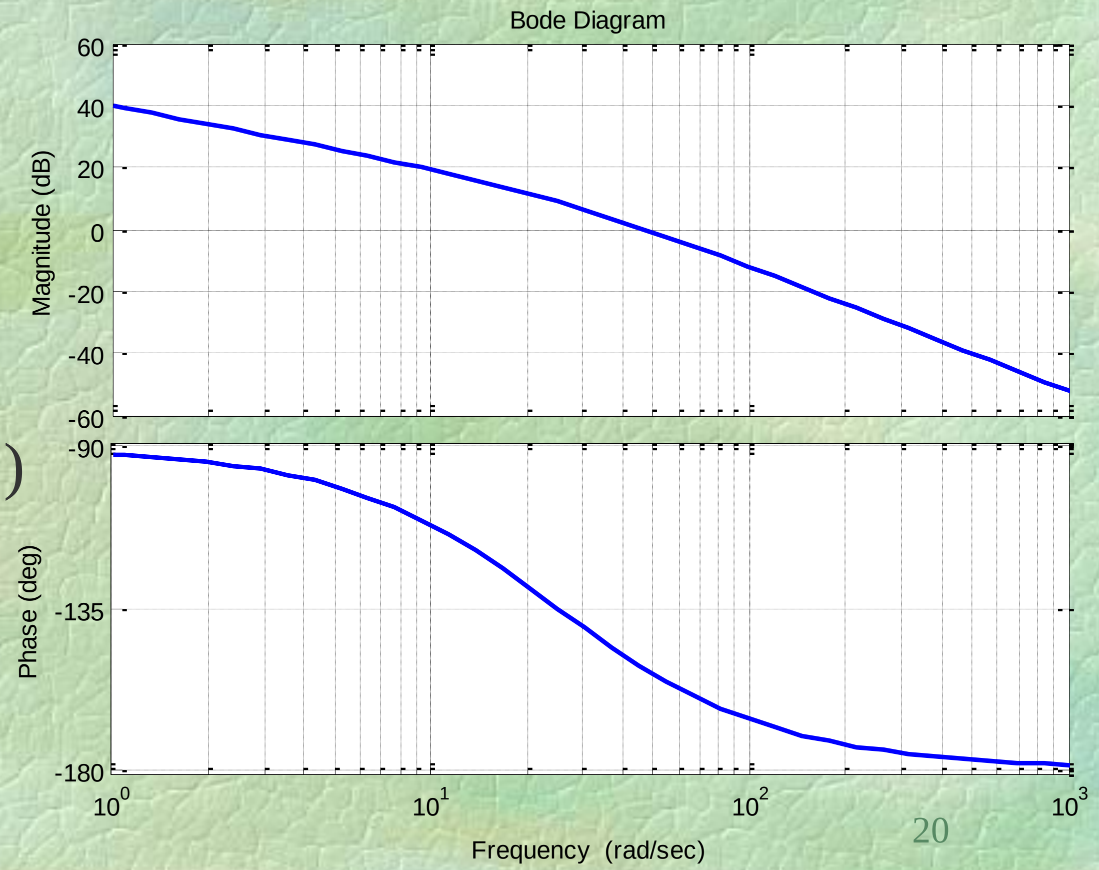
- Frequency range? 1-1000

MATLAB

bode(2500,[1 25 0])

bode(2500,[1 25 0],{10,100})

grid on



University entrance exam 2014

Example 4: Which transfer function has least phase variation?

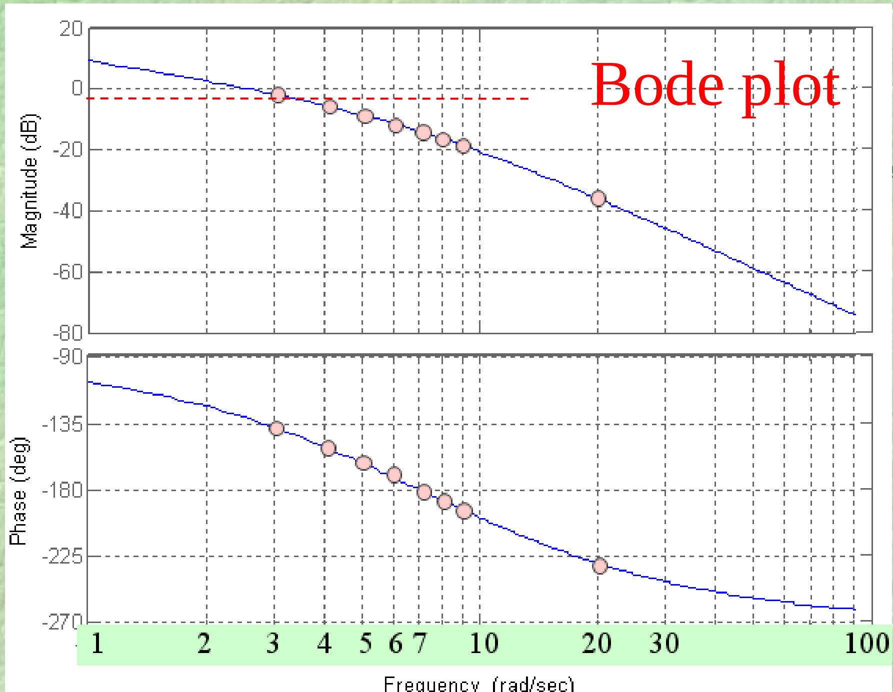
$$g(s)=\frac{s+1}{(s-1)^2} \quad (2$$

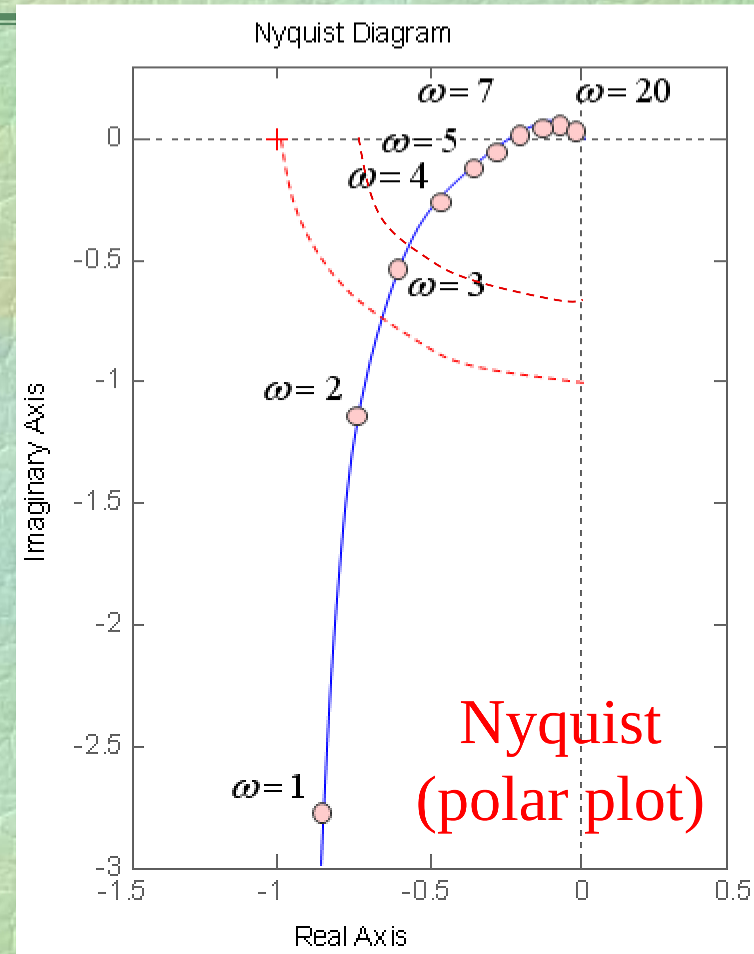
$$g(s)=\frac{1}{s^2-1} \quad (1$$

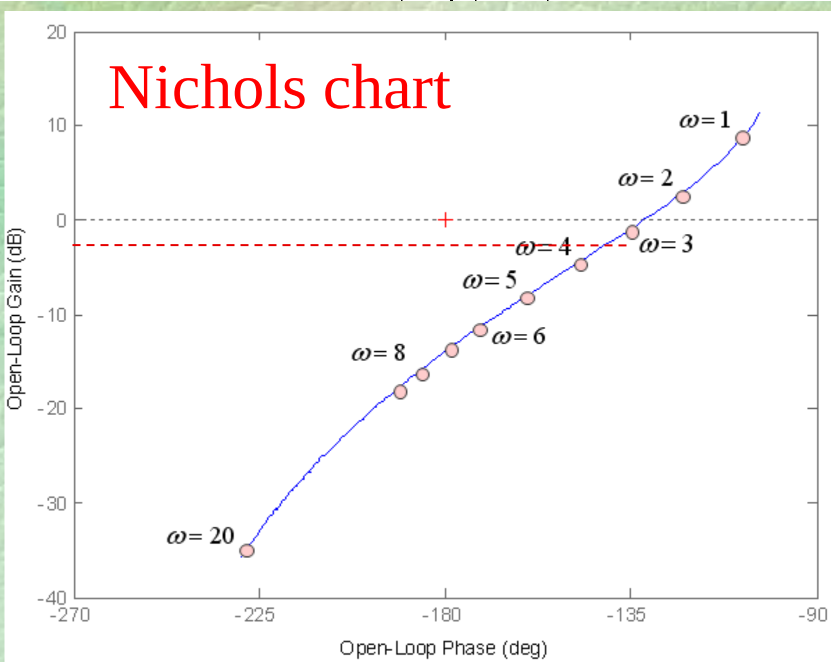
$$g(s)=\frac{1}{(s+1)^2} \quad (4$$

$$g(s)=\frac{1}{s^2+1} \quad (3$$

Frequency domain charts



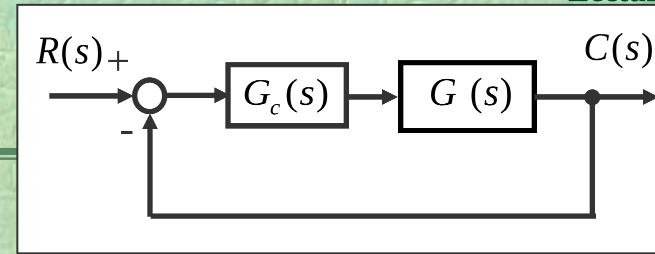




Open - loop bandwidth (ω_o)

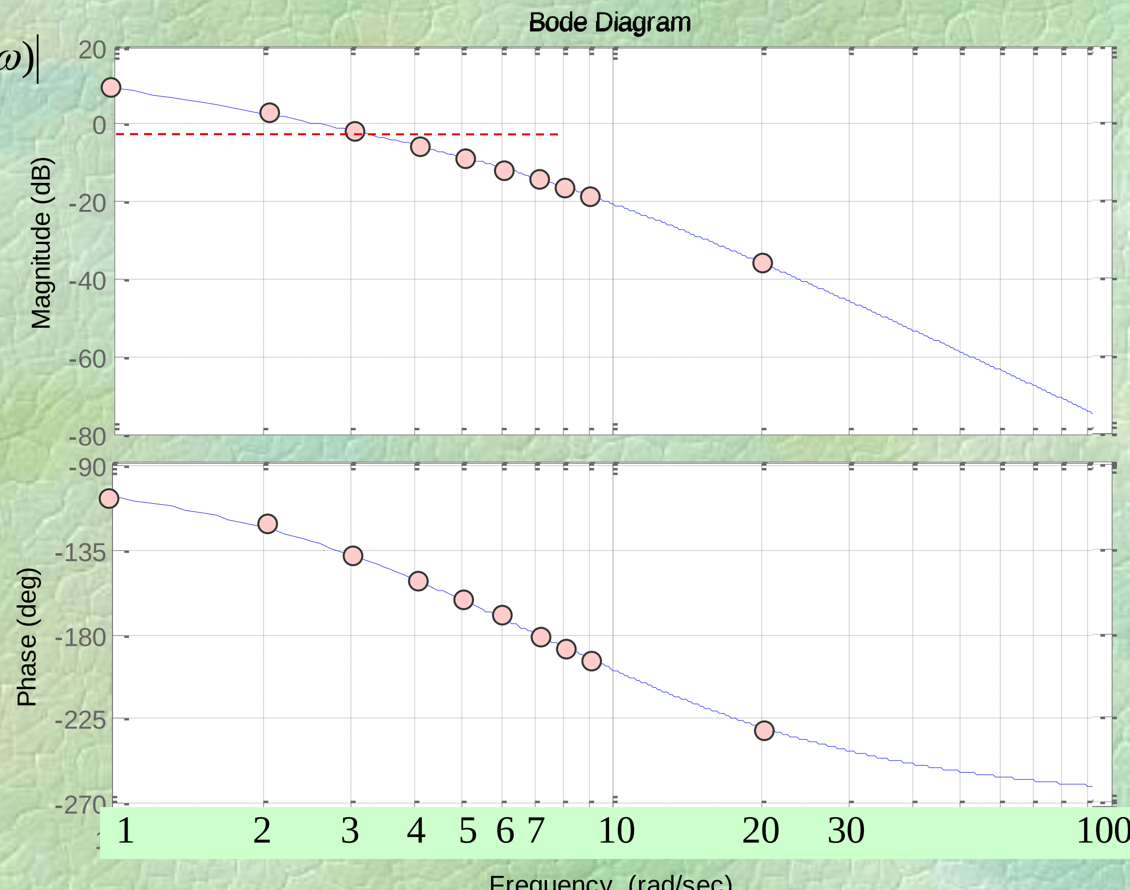
Gain crossover frequency (ω_c)

Bode plot



Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$

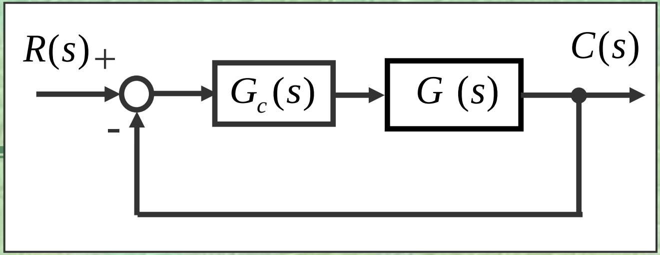
ω	$G_c(j\omega)G(j\omega)$	$20\log\lvert G_c(j\omega)G(j\omega)\rvert$
1	$2.93\angle -107^\circ$	$9.33\ db$
2	$1.37\angle -123^\circ$	$2.71\ db$
3	$0.82\angle -138^\circ$	$-1.71\ db$
4	$0.54\angle -151^\circ$	$-5.29\ db$
5	$0.38\angle -162^\circ$	$-8.42\ db$
6	$0.27\angle -171^\circ$	$-11.23\ db$
7	$0.20\angle -179^\circ$	$-13.80\ db$
8	$0.16\angle -187^\circ$	$-16.18\ db$
9	$0.12\angle -193^\circ$	$-18.39\ db$
20	$0.02\angle -229^\circ$	$-35.77\ db$



Open - loop bandwidth (ω_o)

Gain crossover frequency (ω_c)

Nichols chart (gain phase plot)



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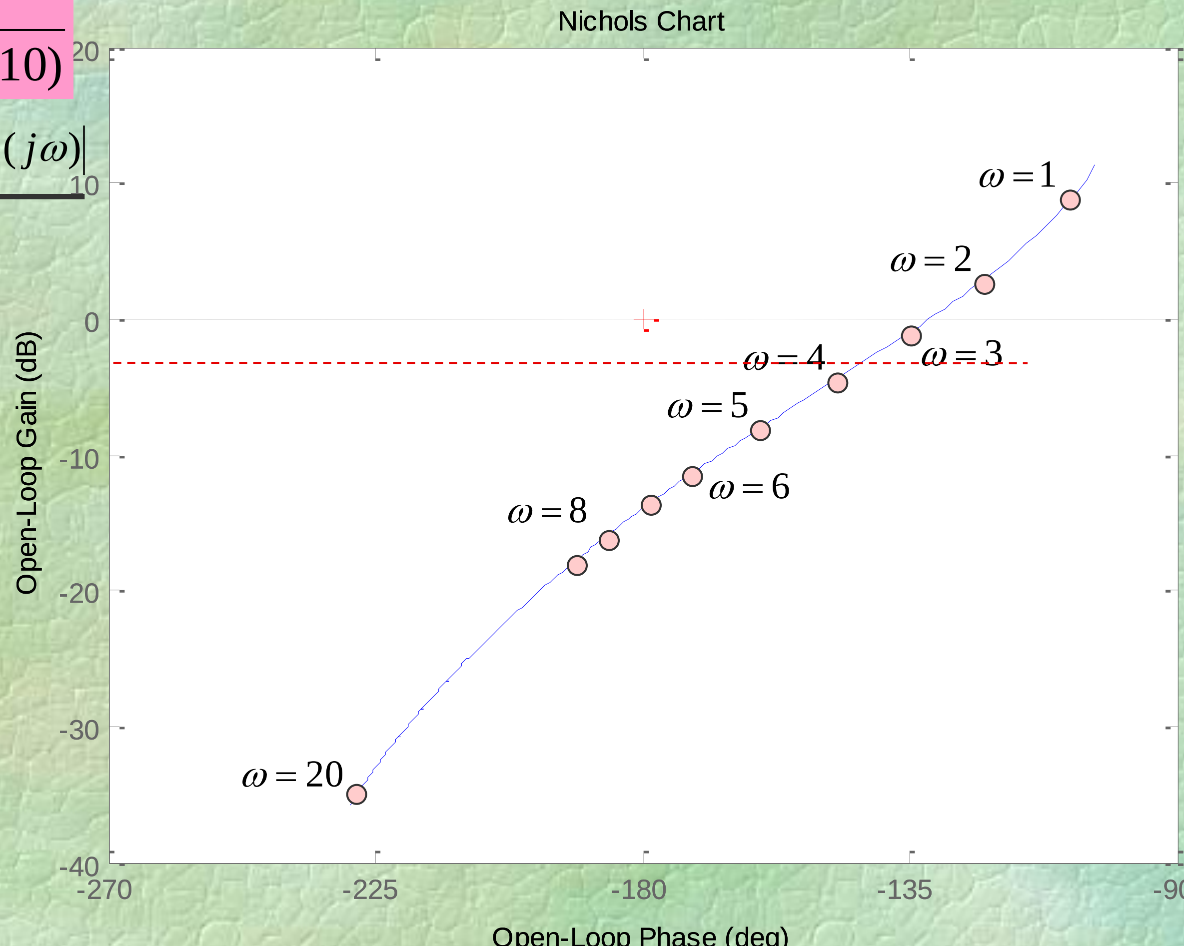
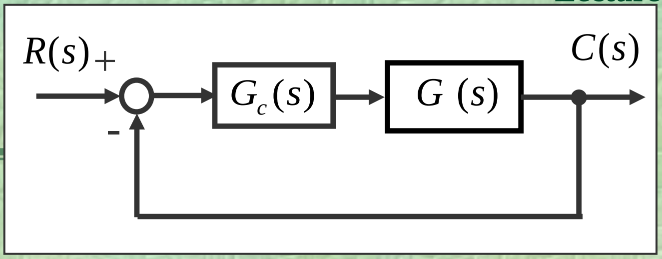


Open - loop bandwidth (ω_o)

Gain crossover frequency (ω_c)

Nyquist chart (polar plot)



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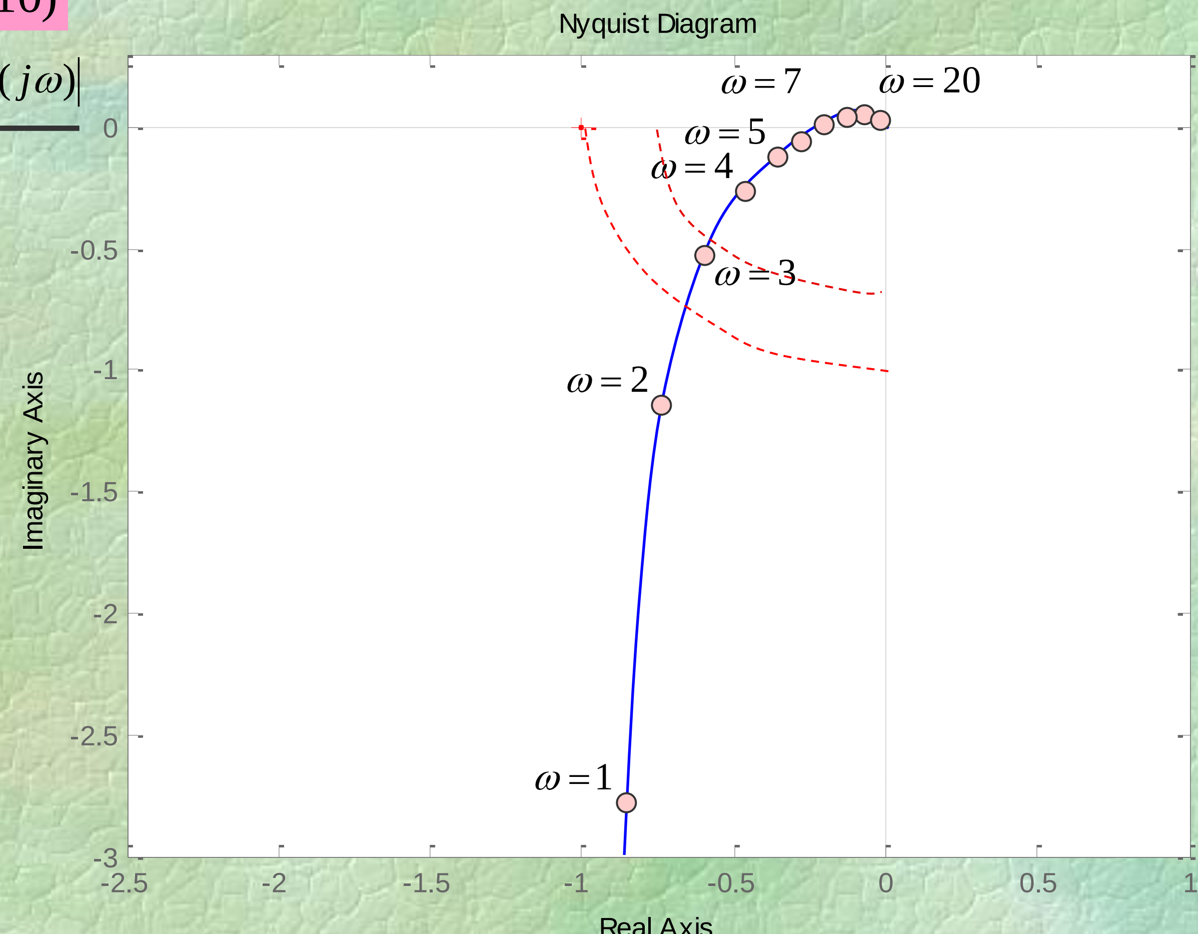
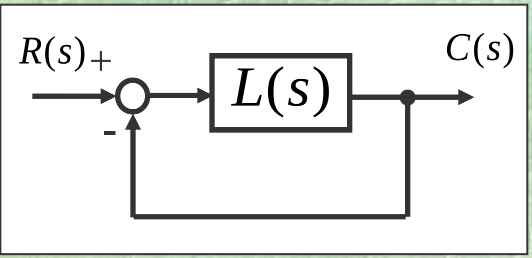
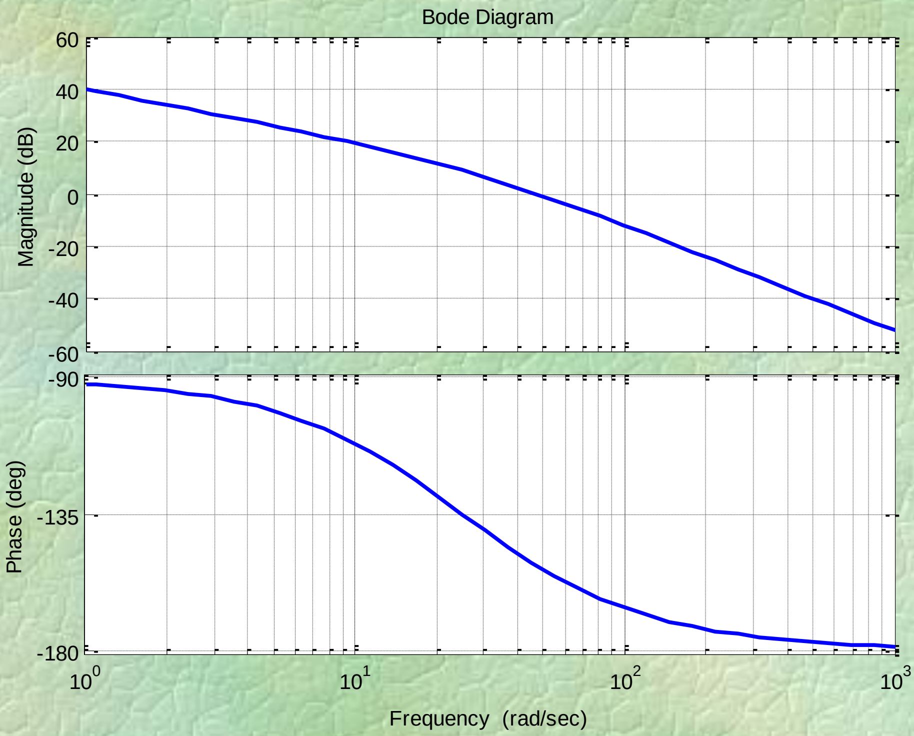


Open - loop bandwidth (ω_o)

Gain crossover frequency (ω_c)

Frequency domain specification





1- Peak of resonance (M_P)

2- Resonance frequency (ω_P)

3- Open - loop bandwidth (ω_o)

4- Closed-loop bandwidth (ω_B)

5- Gain Crossover frequency (ω_c)

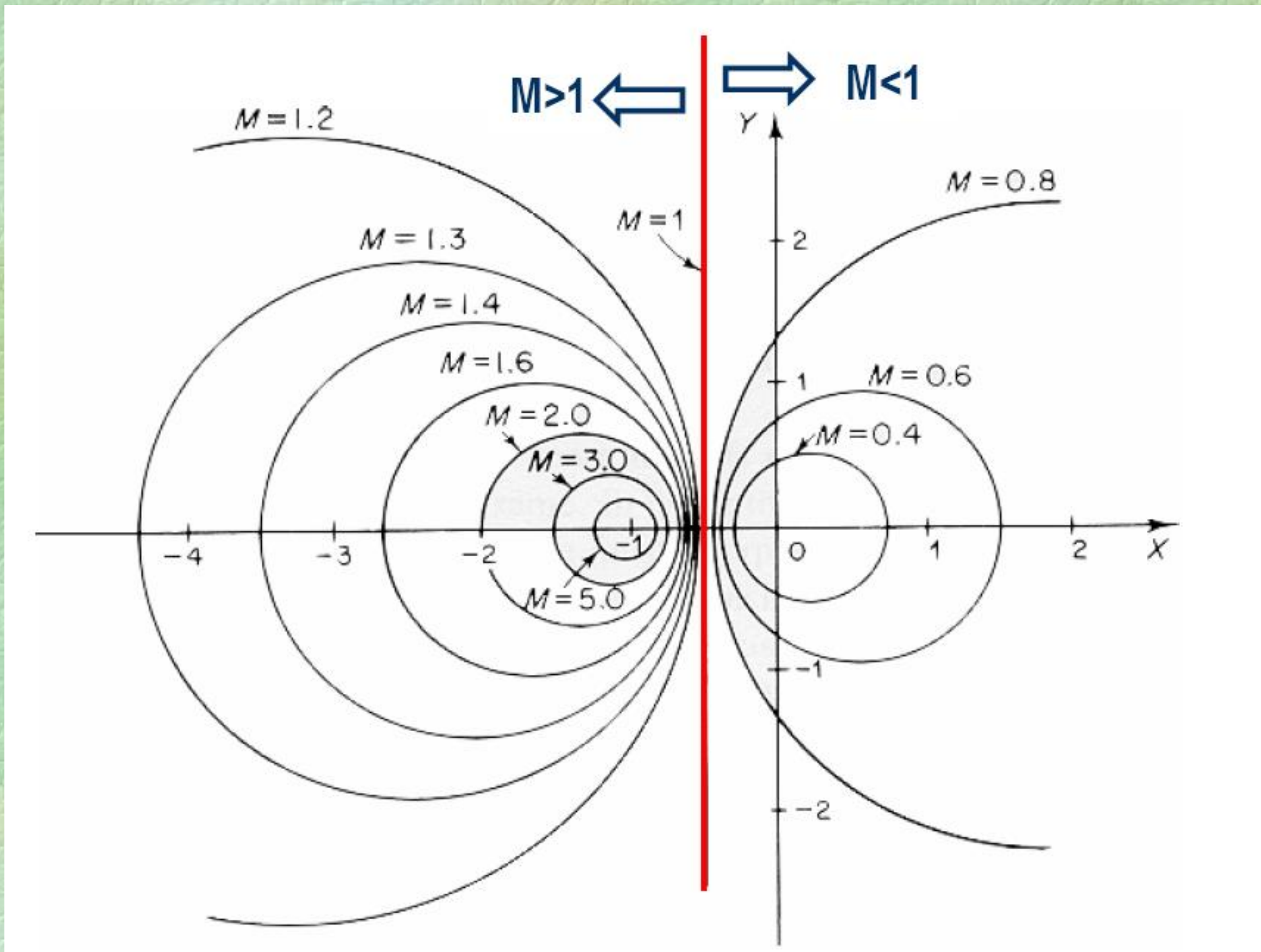
6- Phase Margin (PM)

7- Phase crossover frequency (ω_{180})

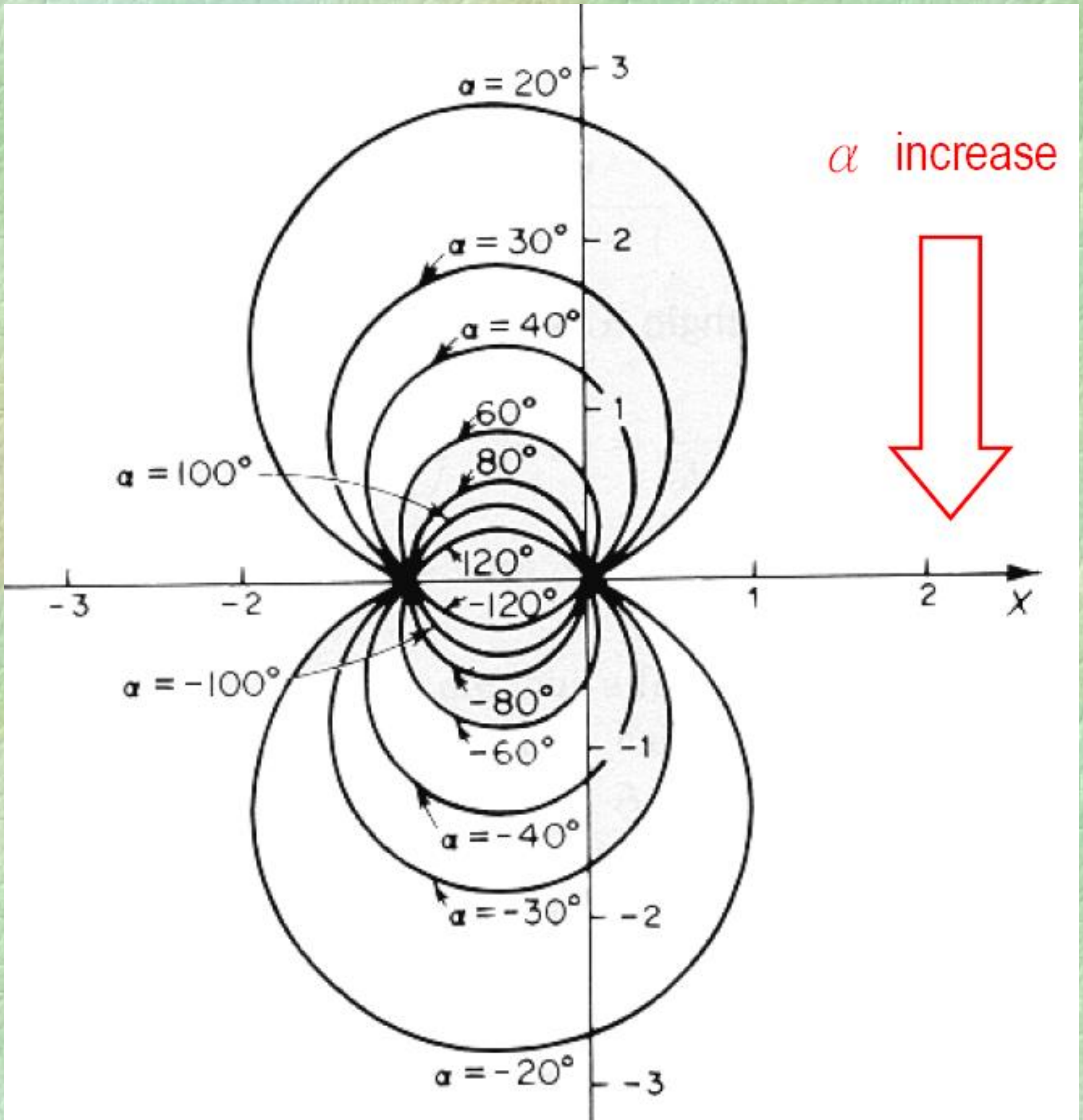
8- Gain Margin (GM)

9- Sensitivity Peak (M_s)

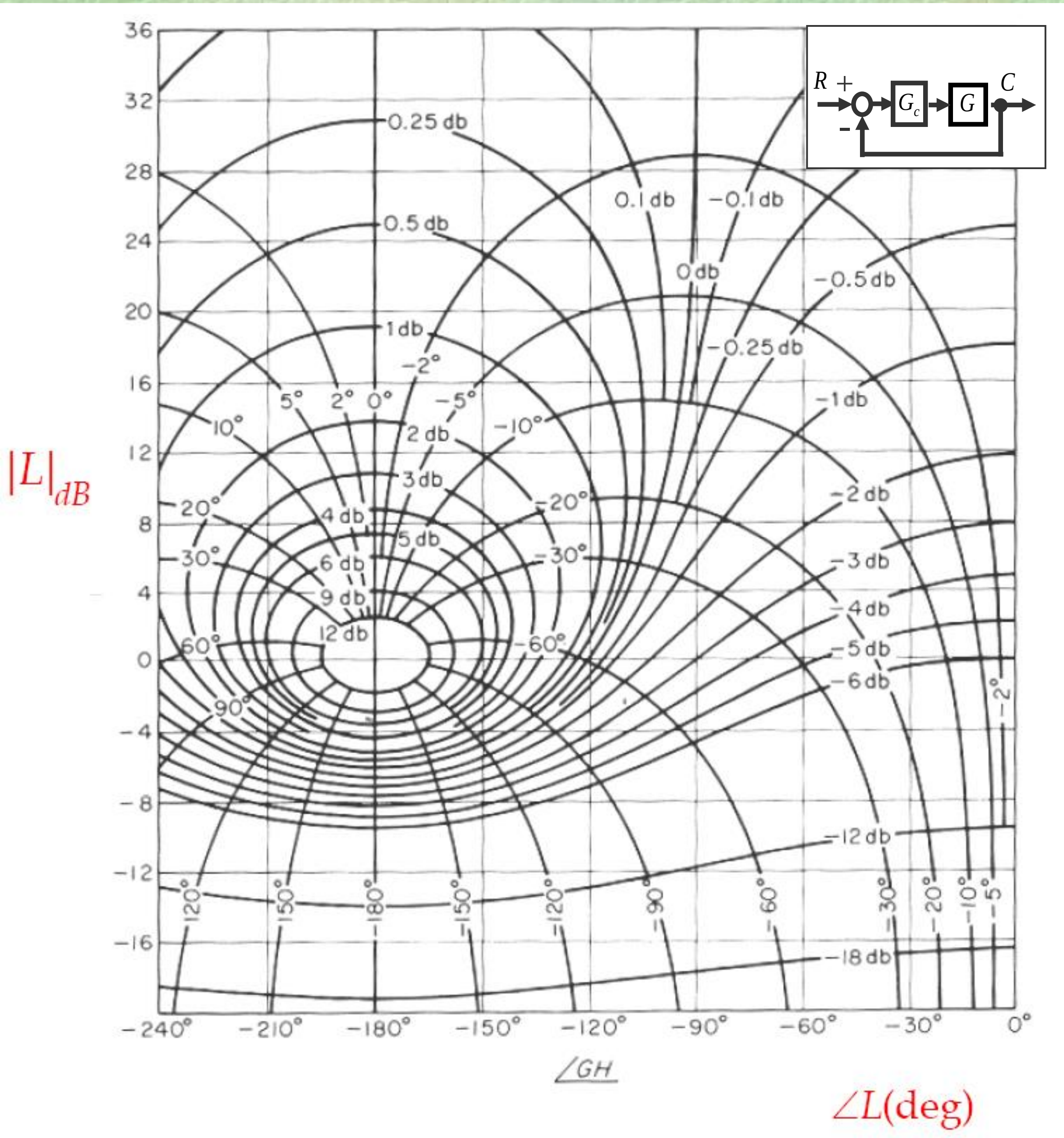
M circles (constant magnitude of T)



N circles (constant phase of T)

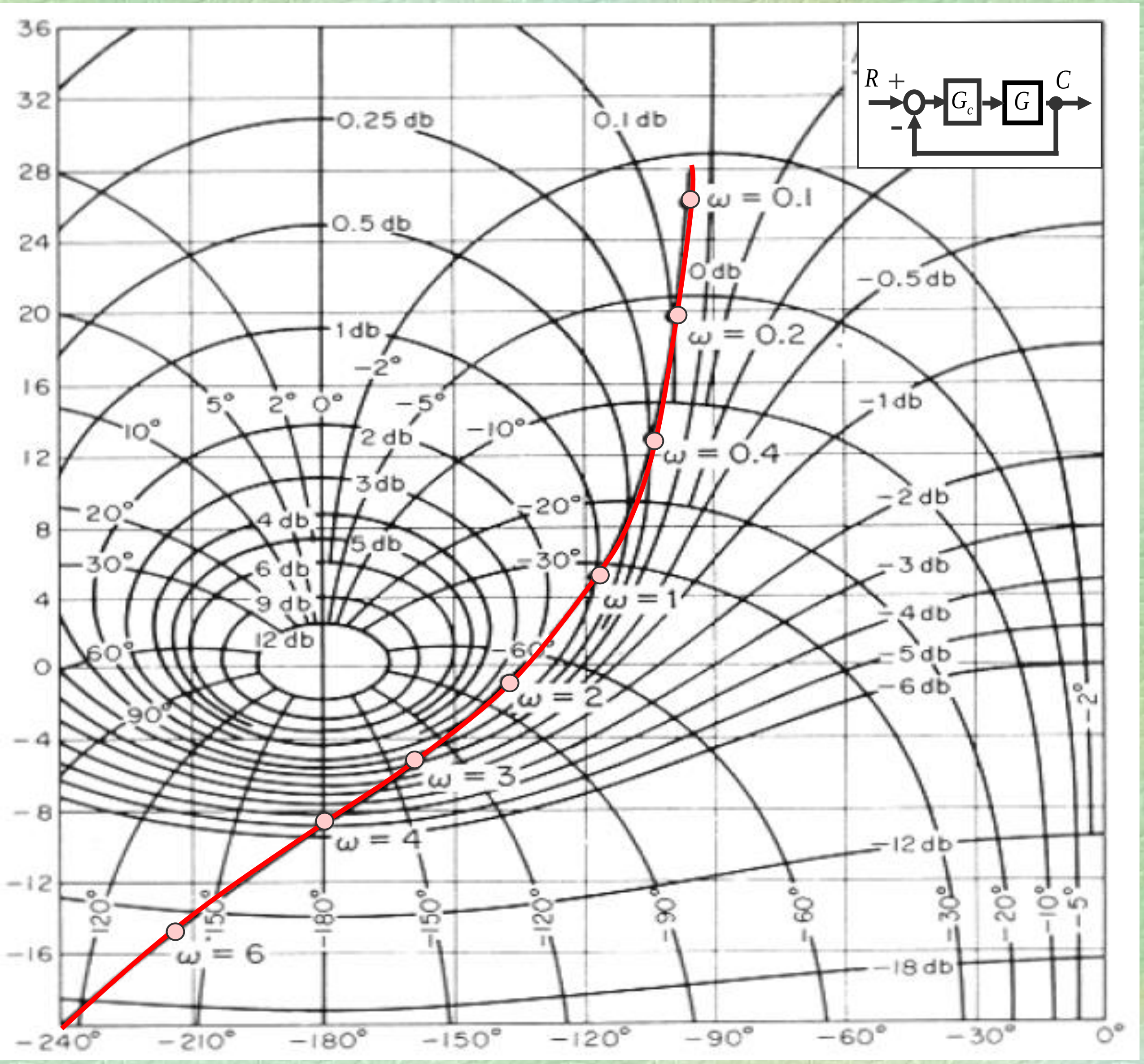


Constant gain and phase loci in Nichols chart



M circles and N circles on Nichols chart

Nichols chart specification



How to derive gain crossover frequencies?

How to derive open loop bandwidth?

Deriving T ?

How to derive M_p?

How to derive ω_p?

How to derive closed loop bandwidth?

Type of system?

How to derive error constants?

Frequency domain analysis

- Introduction.
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Frequency domain specification

1- Peak of resonance (M_P)

2- Resonance frequency (ω_P)

3- Open - loop bandwidth (ω_o)

4- Closed-loop bandwidth (ω_B)

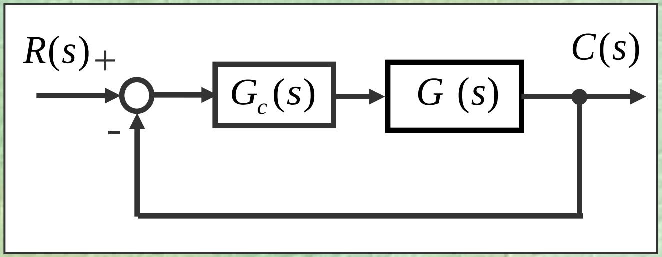
5- Gain Crossover frequency (ω_c)

6- Phase Margin (PM)

7- Phase crossover frequency (ω_{180})

8- Gain Margin (GM)

9- Sensitivity Peak (M_s)

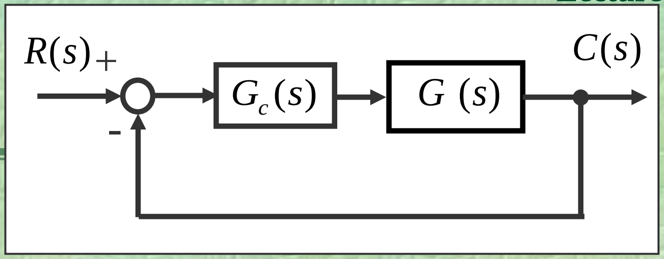


Gain Crossover frequency phase Margin (PM)??

$$PM = \varphi_m = 180 + \angle G(j\omega_c)G_c(j\omega_c)$$

Physical meaning ?

Nyquist chart (polar plot)



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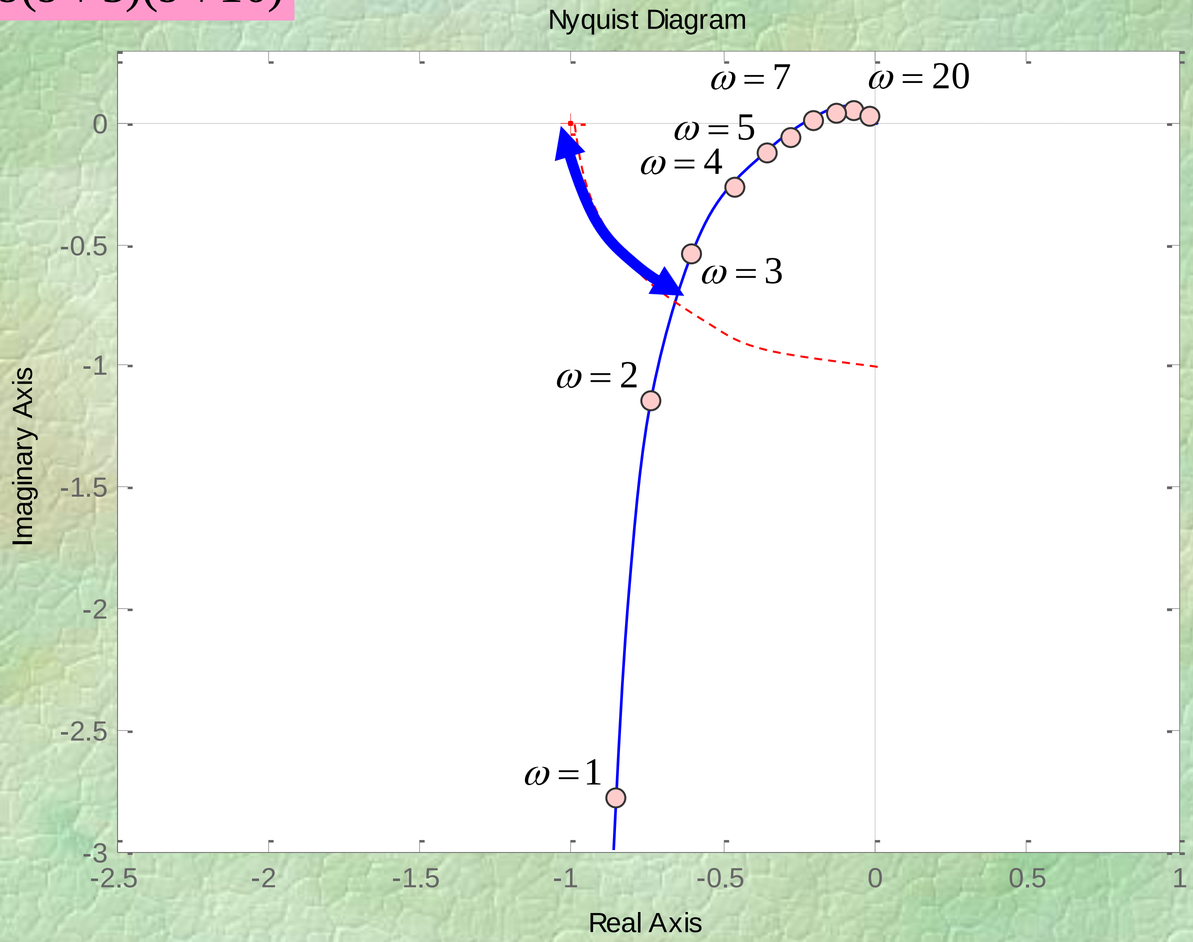
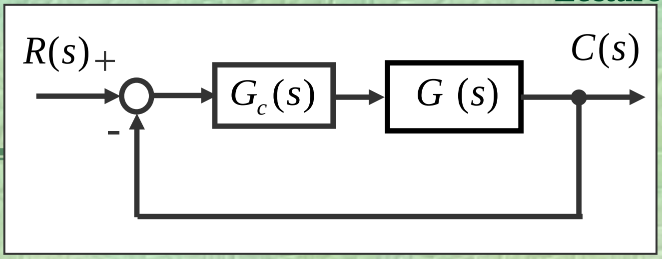
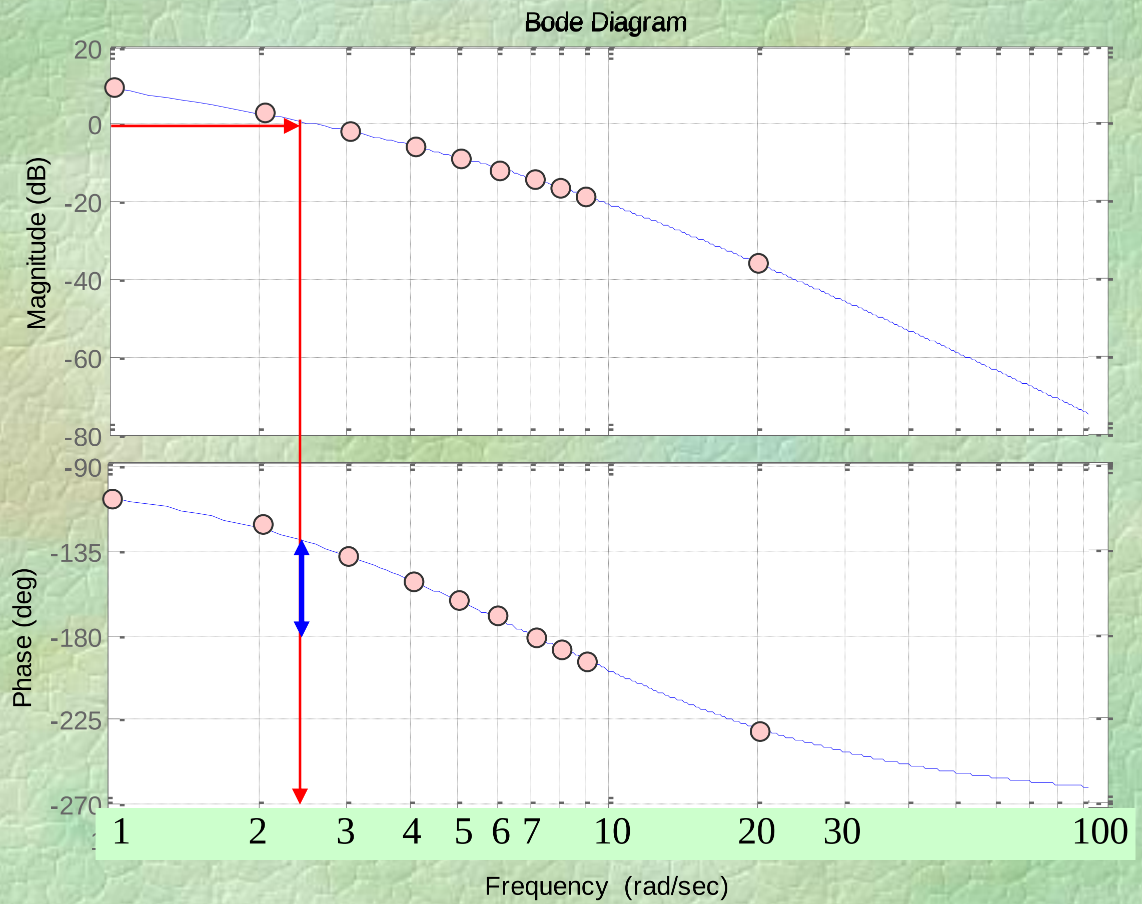


$\omega_c = ?$ $PM = ?$

Bode plot



Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$



$\omega_c = ?$ $PM = ?$

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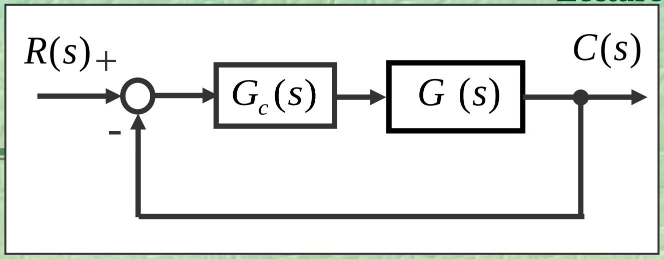
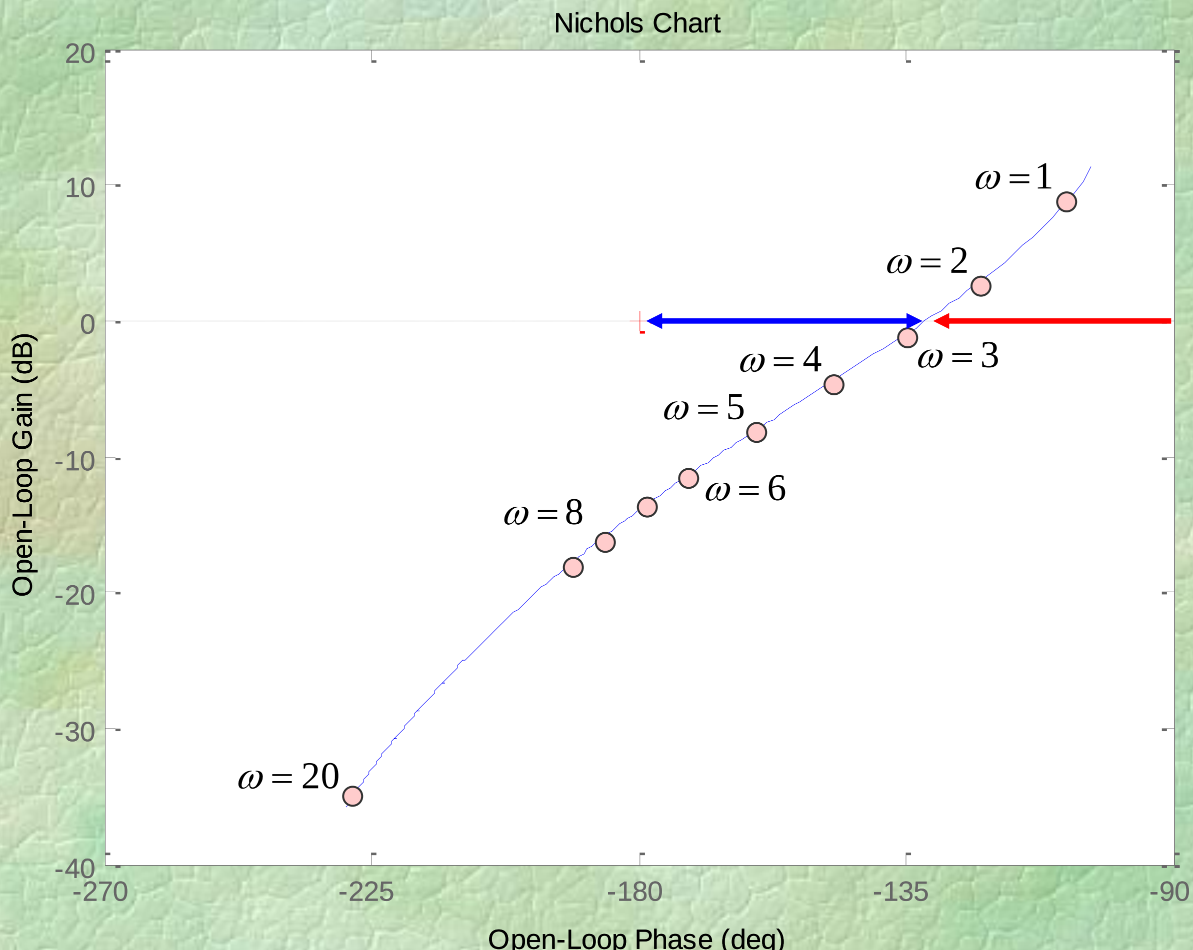
Nichols chart (gain phase plot)



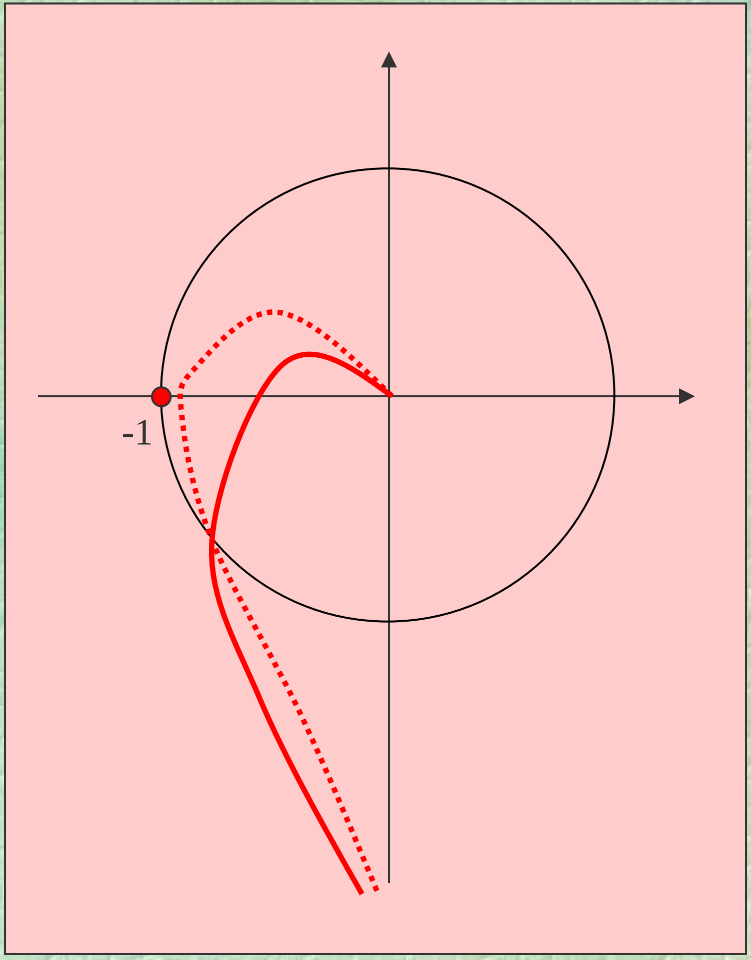
Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$



$\omega_c = ?$ $PM = ?$

Stability margins

Phase and Gain Margin



Same Phase Margin ??!!

Another criteria ??

Frequency domain specification

1- Peak of resonance (M_P)

2- Resonance frequency (ω_P)

3- Open - loop bandwidth (ω_o)

4- Closed-loop bandwidth (ω_B)

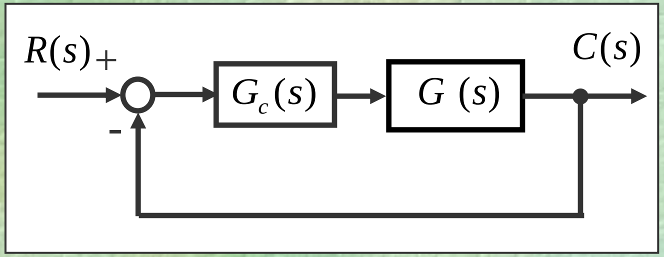
5- Gain Crossover frequency (ω_c)

6- Phase Margin (PM)

7- Phase crossover frequency (ω_{180})

8- Gain Margin (GM)

9- Sensitivity Peak (M_s)



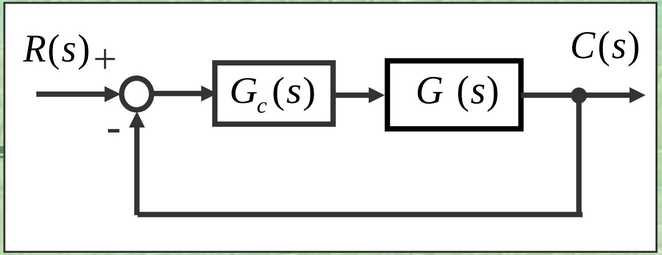
Phase crossover frequency and GM ??

$$GM = 1/\left|G(j\omega_{180})G_c(j\omega_{180})\right|$$

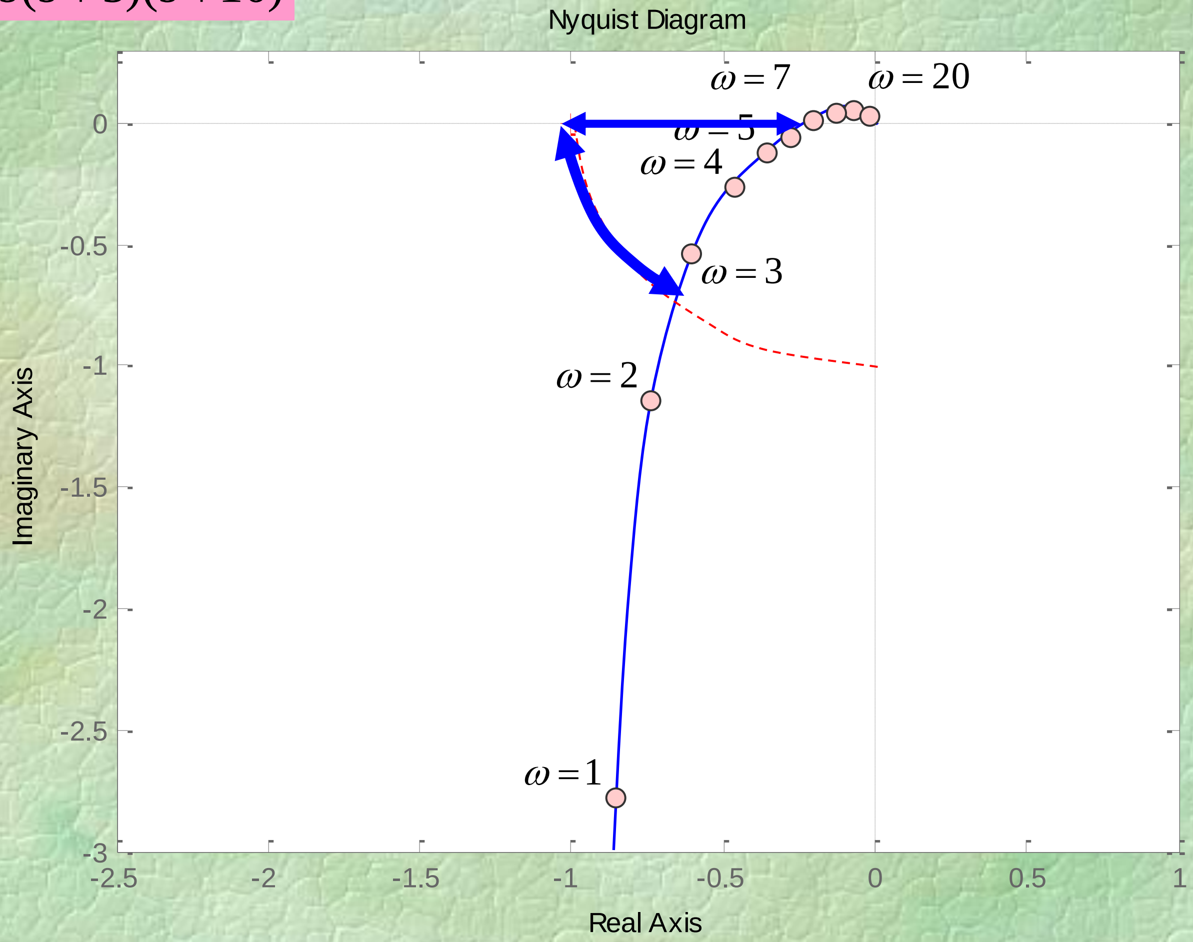
$$GM = -20\log\left(\left|G(j\omega_{180})G_c(j\omega_{180})\right|\right)$$

Physical meaning ?

Nyquist chart (polar plot)

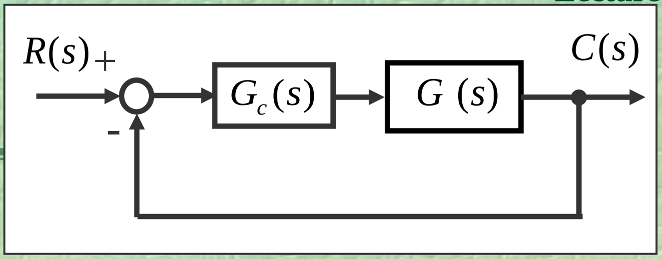


Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$

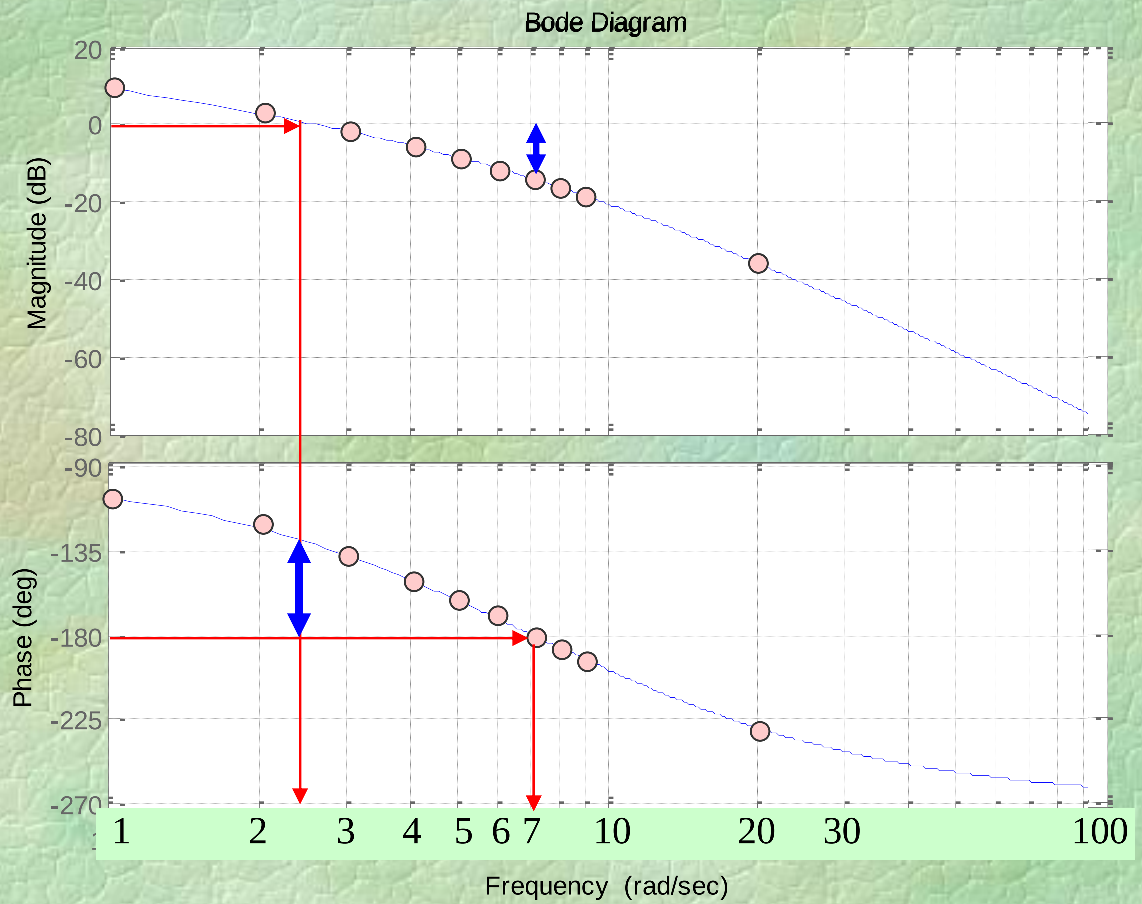


$\omega_c = ?$ $PM = ?$ $\omega_{180} = ?$ $GM = ?$

Bode plot

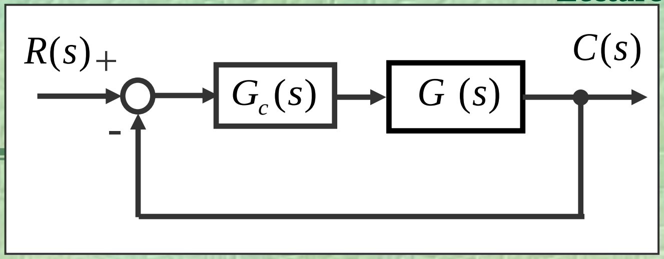


Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$

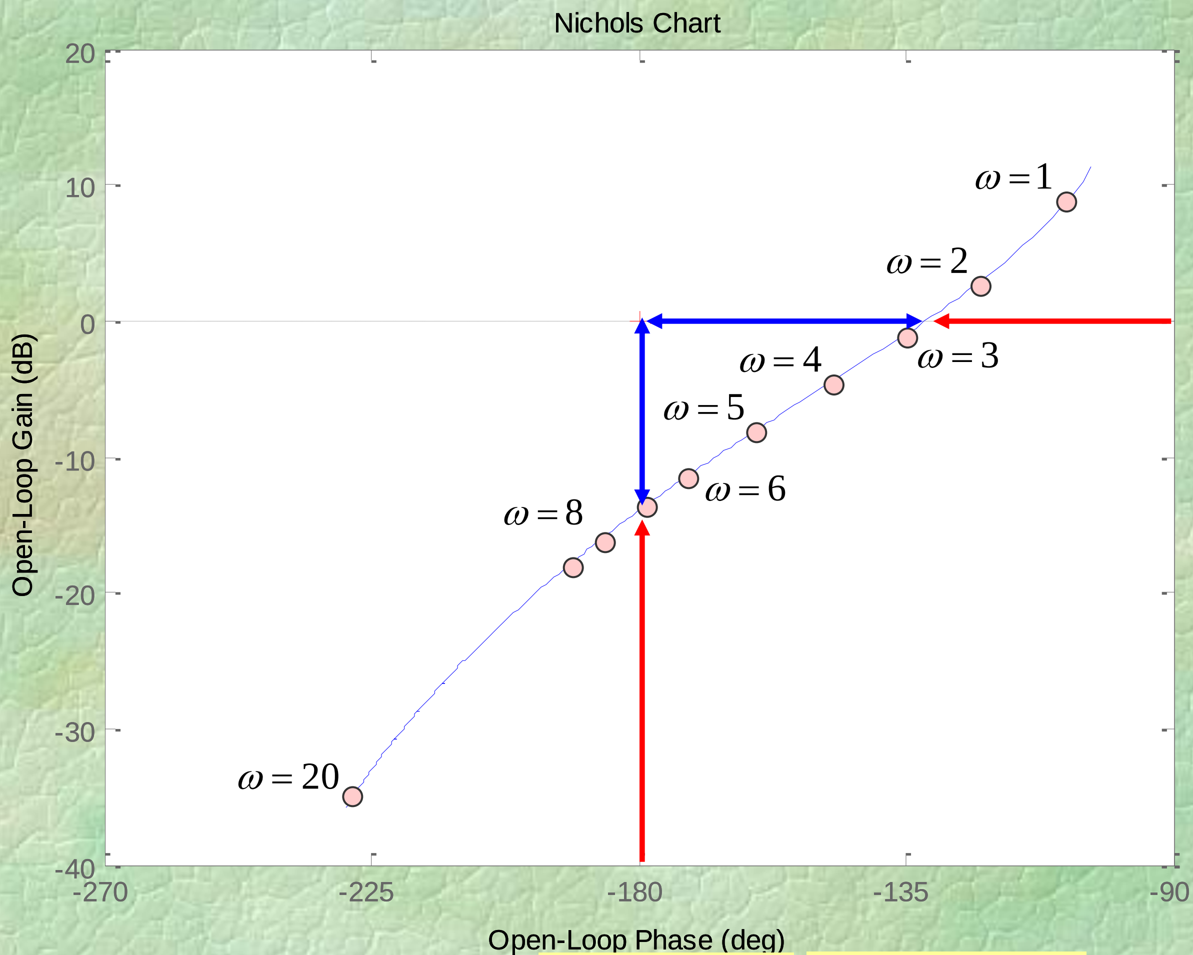


$\omega_c = ?$ $PM = ?$ $\omega_{180} = ?$ $GM = ?$

Nichols chart (gain phase plot)



Let $G_c(s)G(s) = \dfrac{150}{s(s+5)(s+10)}$



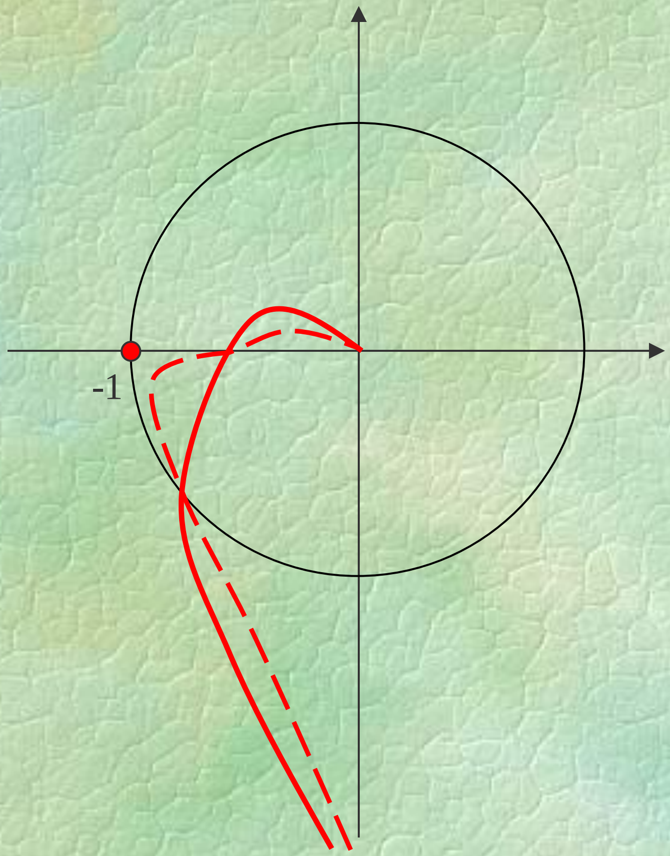
$\omega_c = ?$ $PM = ?$ $\omega_{180} = ?$ $GM = ?$

Stability margins

Phase Margin and Gain Margin

Same Phase Margin ??!!

Same Gain Margin ??!!



Thus we need another measure of relative stability.

Frequency domain specification

1- Peak of resonance (M_P)

2- Resonance frequency (ω_P)

3- Open - loop bandwidth (ω_o)

4- Closed-loop bandwidth (ω_B)

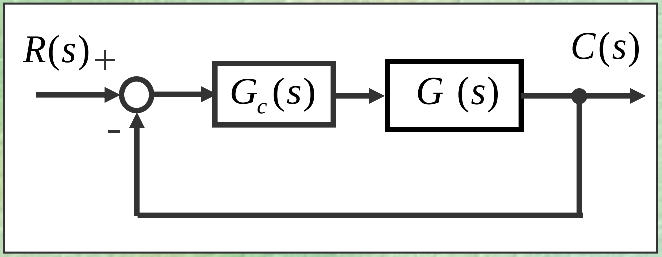
5- Gain Crossover frequency (ω_c)

6- Phase Margin (PM)

7- Phase crossover frequency (ω_{180})

8- Gain Margin (GM)

9- Sensitivity Peak (M_s)

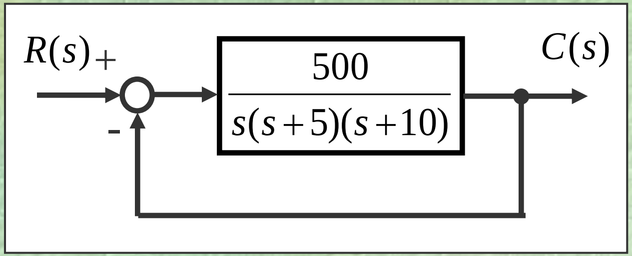


Sensitivity Peak ??

$$M_s = \max_{\omega} |S(j\omega)| = \frac{1}{|1+G(j\omega_s)G_c(j\omega_s)|}$$

Stability margins

Example 5: Derive PM and GM and crossover frequencies of following system.



$$G(s)=\frac{500}{s(s+5)(s+10)}$$

$\omega_c = 6.5\ rad/\sec$ $PM = 10^\circ$

$\omega_{180} = 7.07\ rad/\sec$ $GM = 1.5$ $GM = 3.5\ db$

PM and gain crossover frequency from Nyquist (polar plot)

PM and gain crossover frequency from Bode plot

PM and gain crossover frequency from Nichols (gain phase plot)

University entrance exam 2014

Example 6: Polar plot of the given system is shown in two different situation. Which expression is correct?

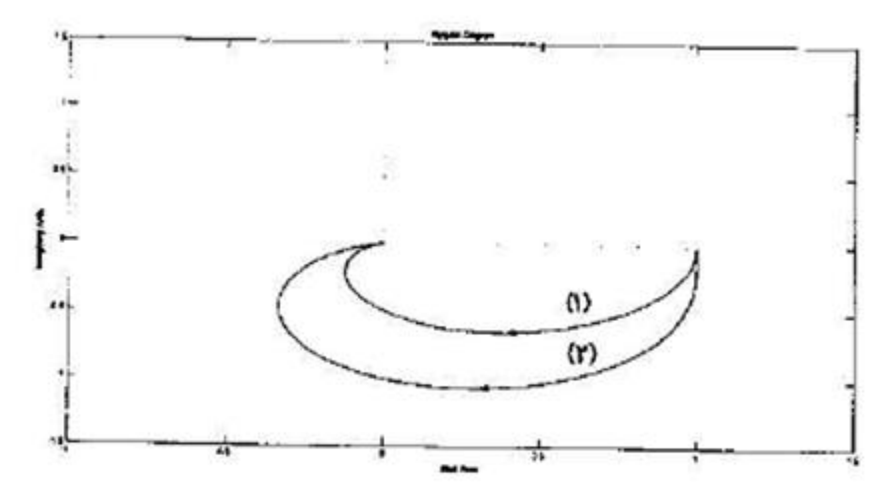
$$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

1- System 1 has more overshot than 2.

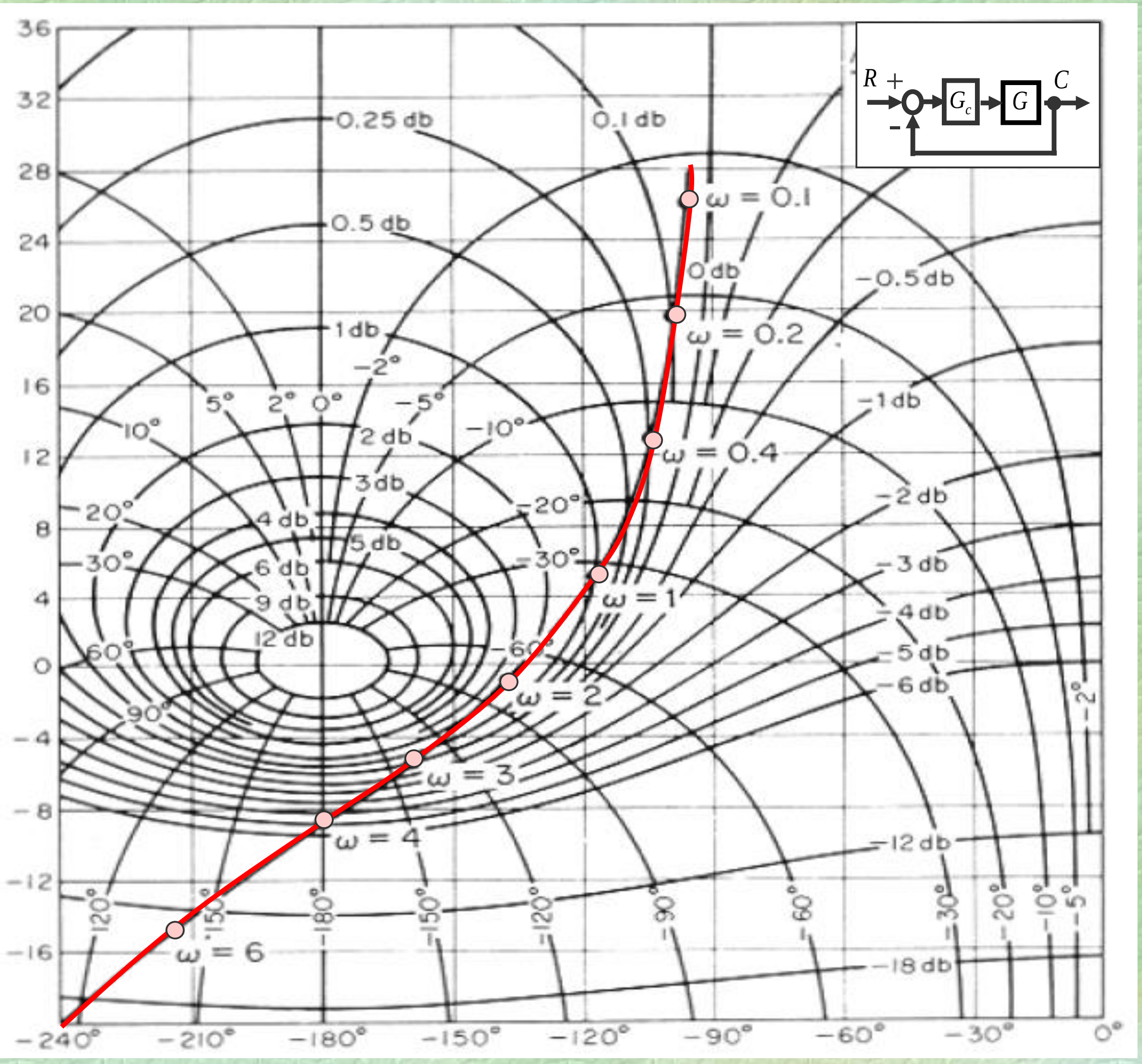
2- System 2 is faster than 1.

3- Intersection frequency with imaginary axis shows the frequency of damped response.

4- All three expression.



Nichols chart specification



How to derive gain crossover frequencies?

How to derive open loop bandwidth?

Deriving T ?

How to derive M_p?

How to derive ω_p?

How to derive closed loop bandwidth?

Type of system?

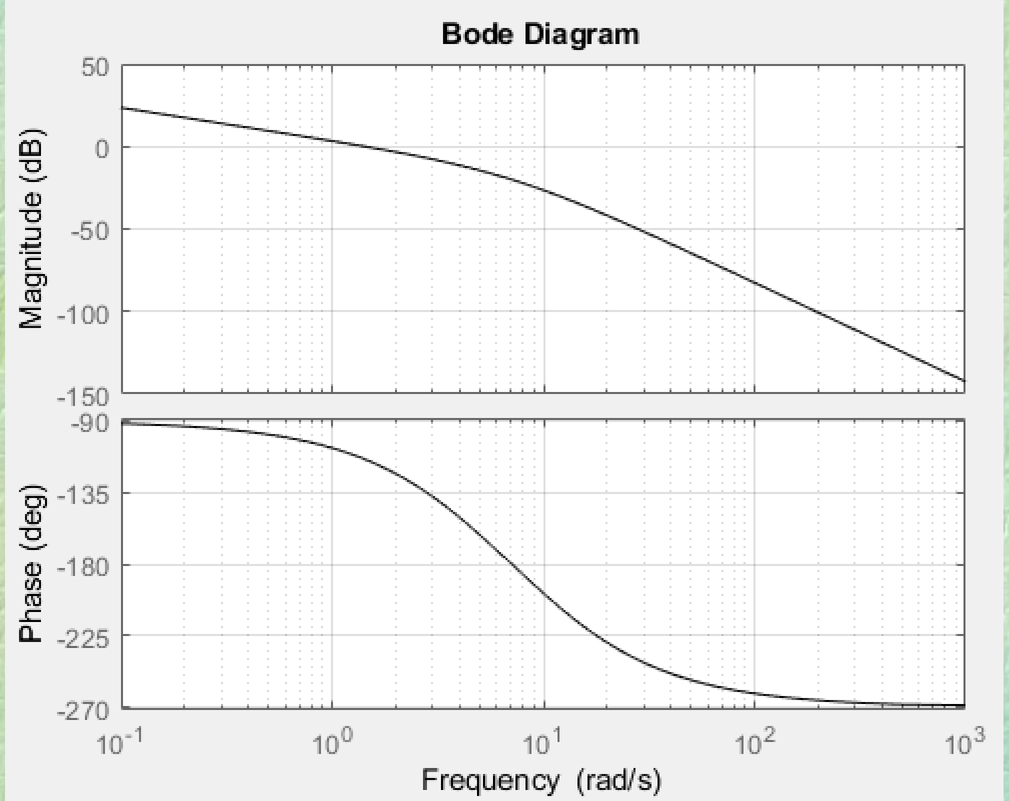
How to derive error constants?

How to derive φ_m and GM?

Stability Analysis Using the Bode Diagram

Exercise 1: The following requirements need to be determined for the given Bode plot:

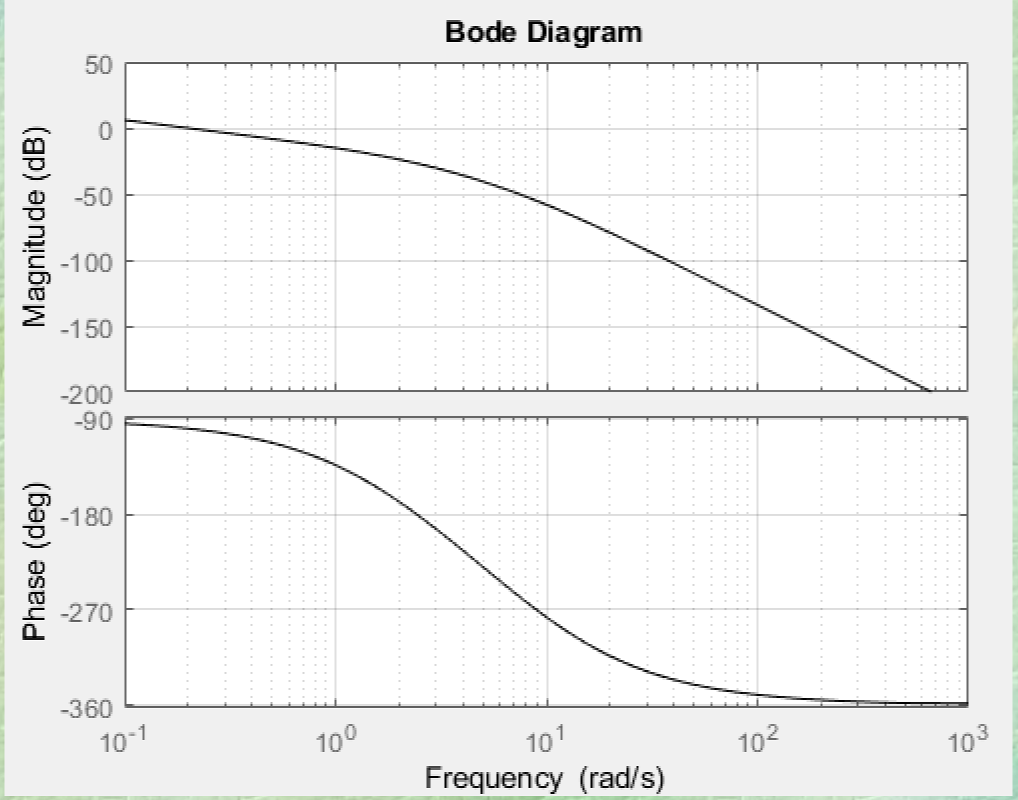
- Gain crossover frequency.
- Phase crossover frequency.
- Gain margin(GM).
- Phase margin(PM).
- Type of system.
- Stability of system.
- Is it minimum-phase?



Stability Analysis Using the Bode Diagram

Exercise 2: The following requirements need to be determined for the given Bode plot:

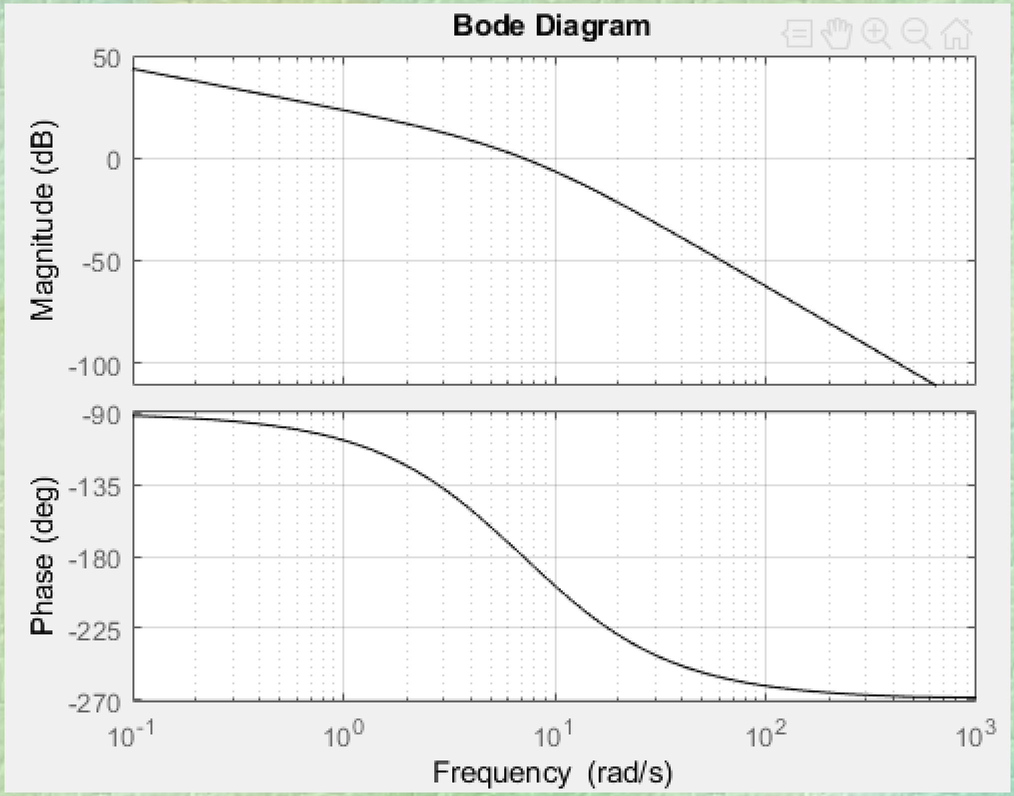
- Gain crossover frequency.
- Phase crossover frequency.
- Gain margin(GM).
- Phase margin(PM).
- Type of system.
- Stability of system.
- Is it minimum-phase?



Stability Analysis Using the Bode Diagram

Exercise 3: The following requirements need to be determined for the given Bode plot:

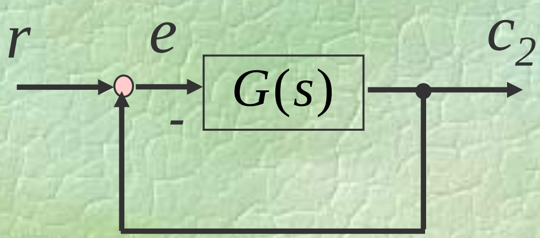
- Gain crossover frequency.
- Phase crossover frequency.
- Gain margin(GM).
- Phase margin(PM).
- Type of system.
- Stability of system.
- Is it minimum-phase?

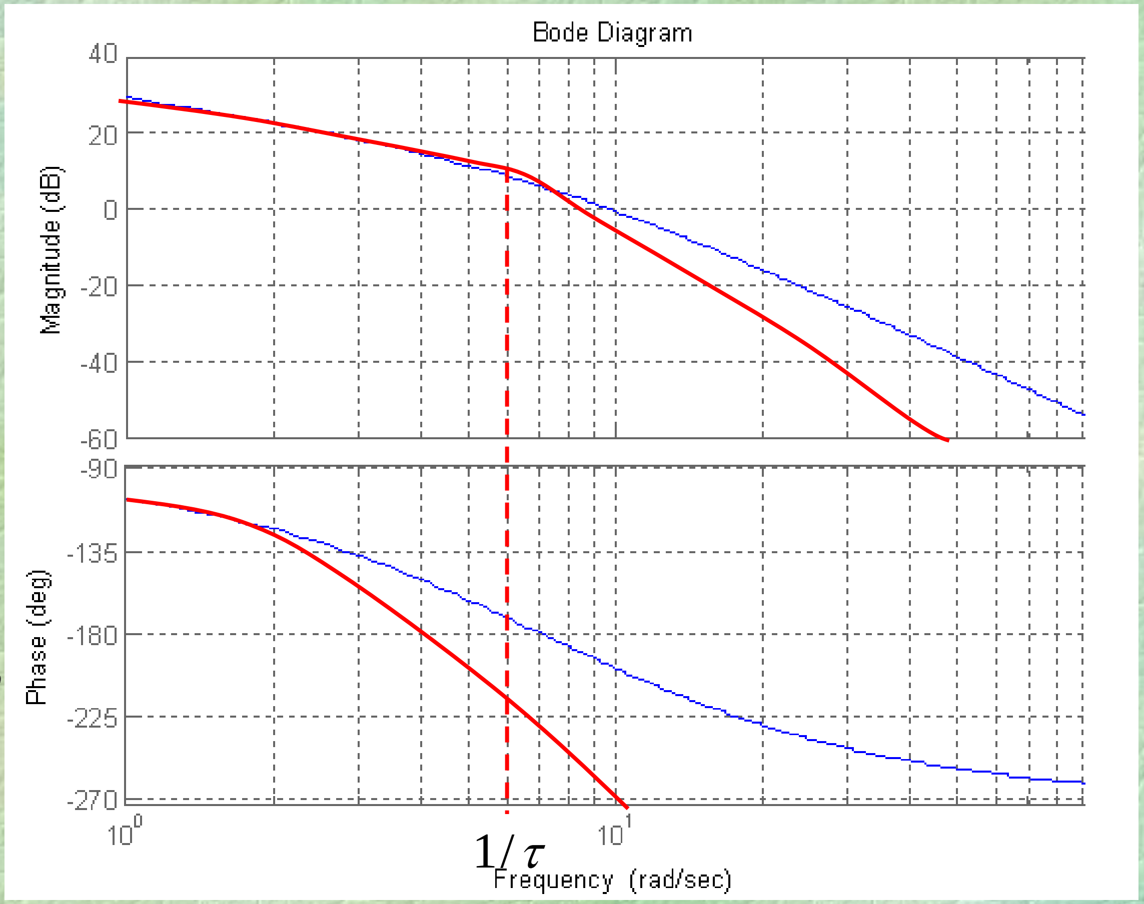


Frequency domain analysis

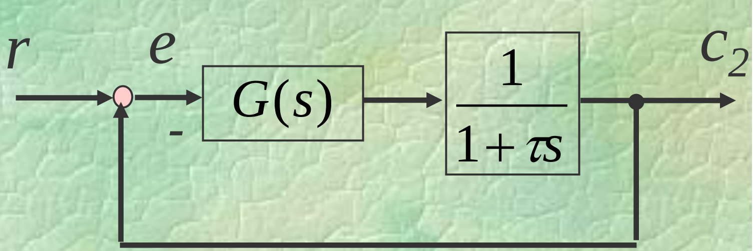
- Introduction.
- Frequency domain charts.
  - Bode plot.
  - Nichols chart.
  - Polar plot.
- Stability analysis.
  - Gain margin.
  - Phase margin.
  - Crossover frequencies.
- Effect of adding poles and zeros on loop transfer function.

Effect of adding poles on Bode plot.





Adding poles



BW ↓ System speed ↓ t_r ↑

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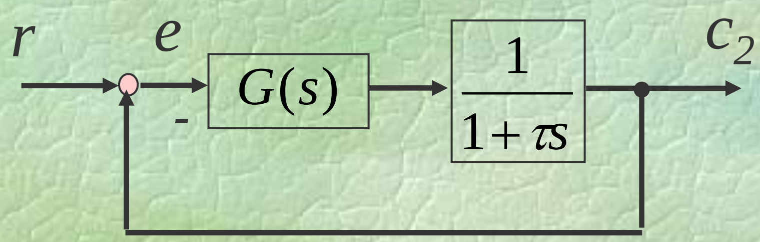
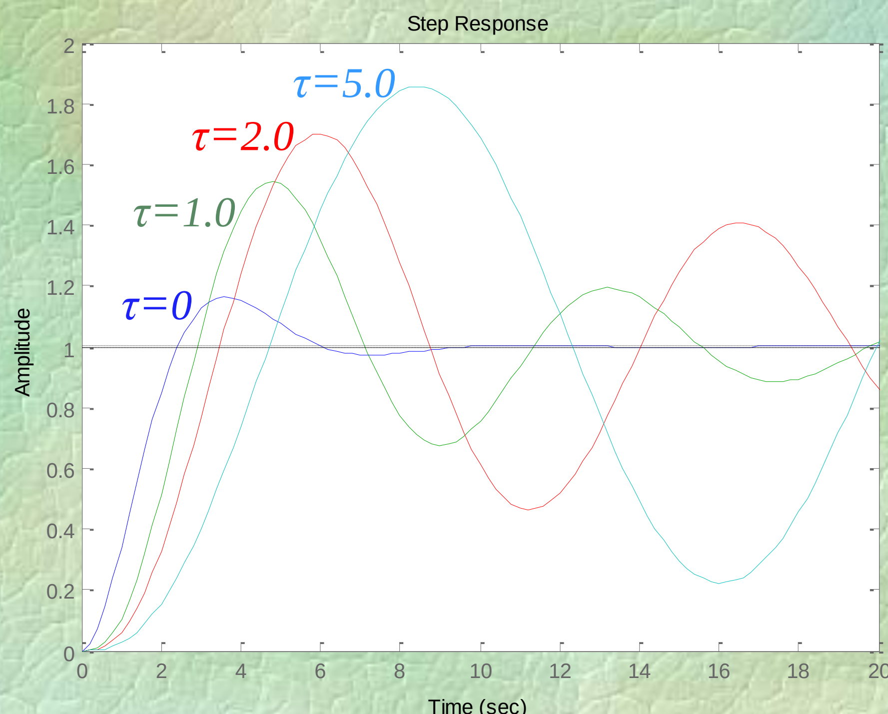
Adding poles to open loop transfer functions



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{\omega_n^2}{\tau s^3+(1+2\zeta\omega_n\tau)s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\omega_n=1 \quad \zeta=0.5 \quad \tau=0,1,2,5$$



P.O. ↑

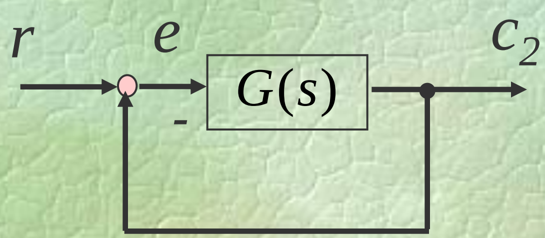
t_r ↑

System speed ↓

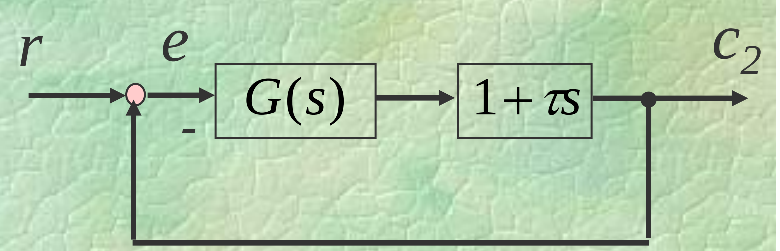
BW ↓

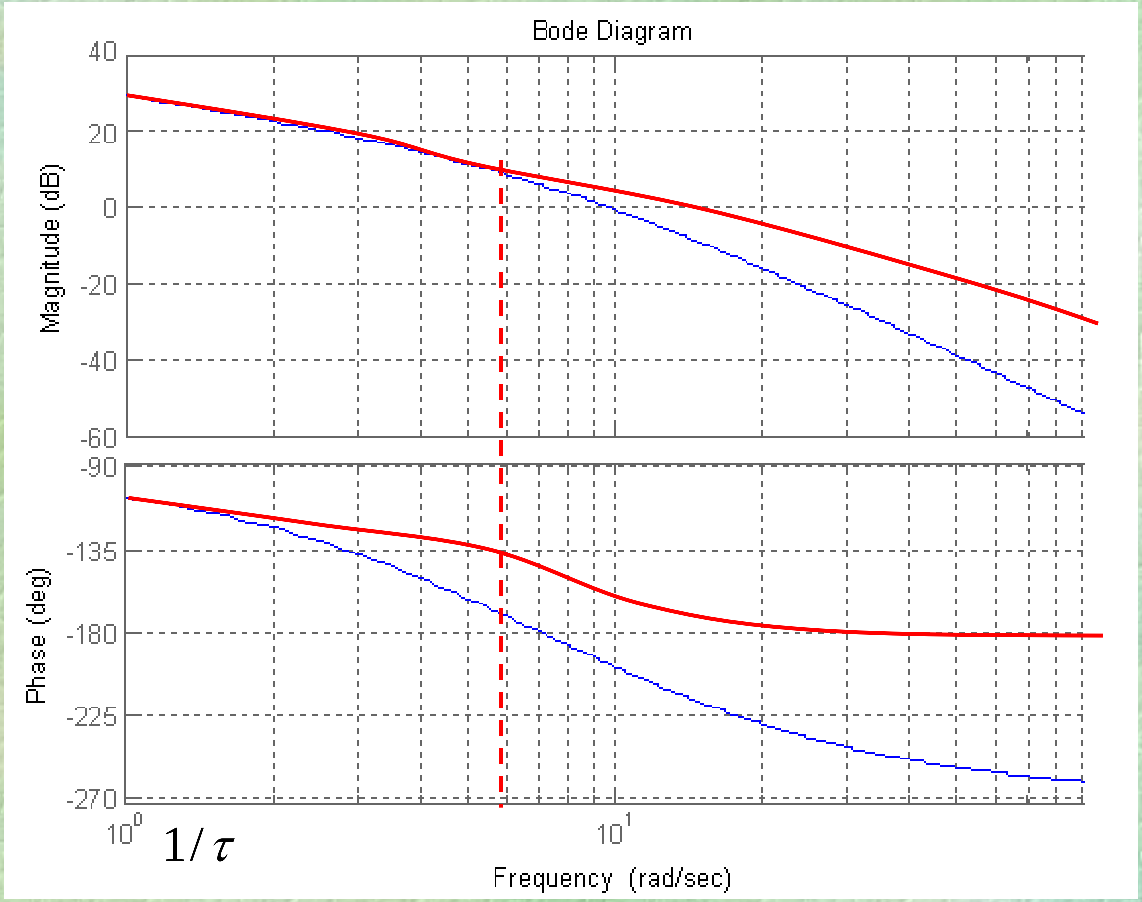
More problem as poles go to ??

Effect of adding zeros on Bode plot.



Adding zeros





BW ↑   System speed ↑ t_r ↓

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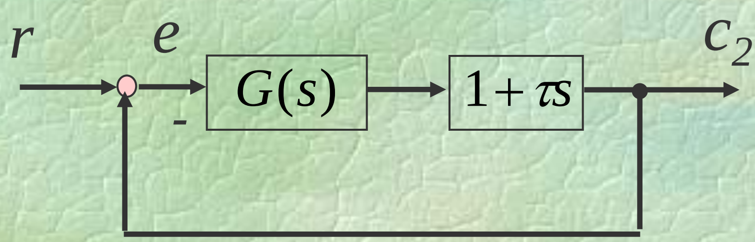
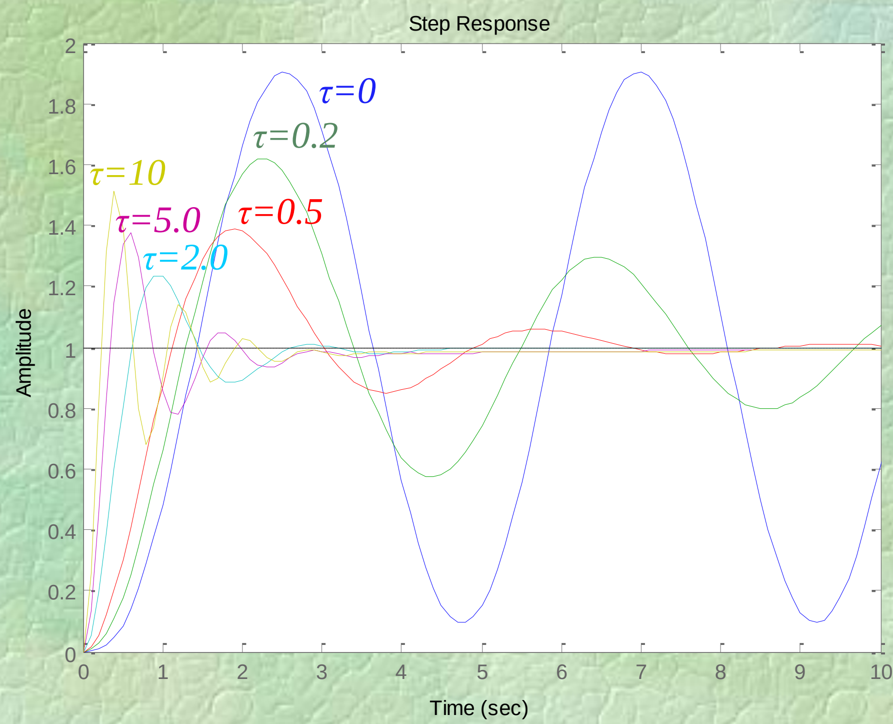
Adding zeros to open loop transfer functions



$$M_2(s)=\frac{C_2(s)}{R(s)}=\frac{6(1+\tau s)}{s^3+3s^2+(2+6\tau)s+6}$$

$$\tau=0,0.2,0.5,2,5,10$$



P.O. ↓ ↑

t_r ↓

System speed ↑

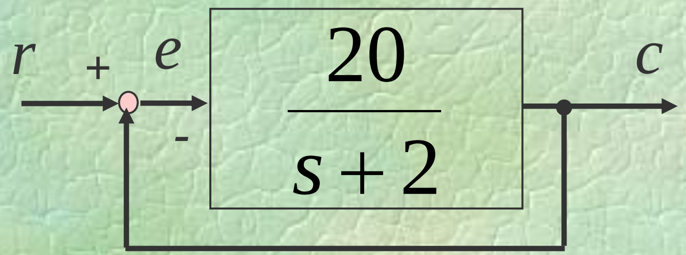
BW ↑

Note: For τ<0 system is unstable. Why?

Exercises

Exercise 4: In the following system

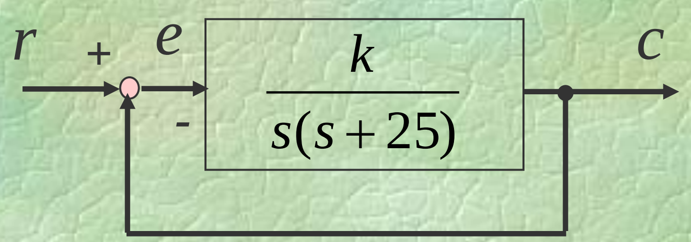
a) Derive Bode plot for L (Don't use MATLAB).
b) Derive Bode plot for T and S (Don't use MATLAB).
c) Derive peak of resonance and resonance frequency from part "b".
d) Derive step response of system and find P.O. from step response.



Exercises

Exercise 5: In the following system let k=200.

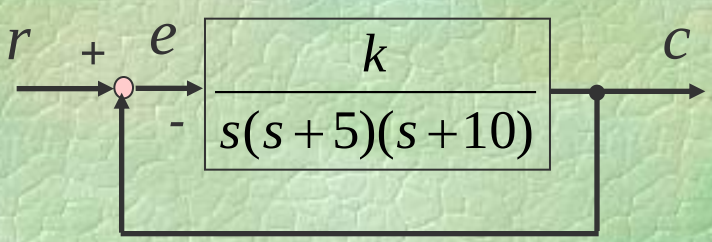
a) Derive Bode plot for L (Don't use MATLAB).
b) Derive Bode plot for T and S (Don't use MATLAB).
c) Derive peak of resonance and resonance frequency from part "b".
d) Derive step response of system and find P.O. from step response.
e) Try part "a" till "f" with k=2000 (Don't use MATLAB).
f) Sketch peak of resonance versus k, and P.O. versus k. $1<k<10000$
g) Discuss about the effect of k on P.O. and peak of resonance.



Exercises

Exercise 6: In the following system let *k*=10.

a) Derive Bode plot for L (Don't use MATLAB).
b) Derive Bode plot for T and S (Don't use MATLAB).
c) Derive peak of resonance and resonance frequency from part "b".
d) Derive step response of system and find P.O. from step response.
e) Try part "a" till "f" with *k*=200 (Don't use MATLAB).
f) Sketch peak of resonance versus *k*, and P.O. versus k. 1<k<1000
g) Discuss about the effect of *k* on P.O. and peak of resonance.



Exercises

Exercise 7: In the figure you will find bode plot of g(s)

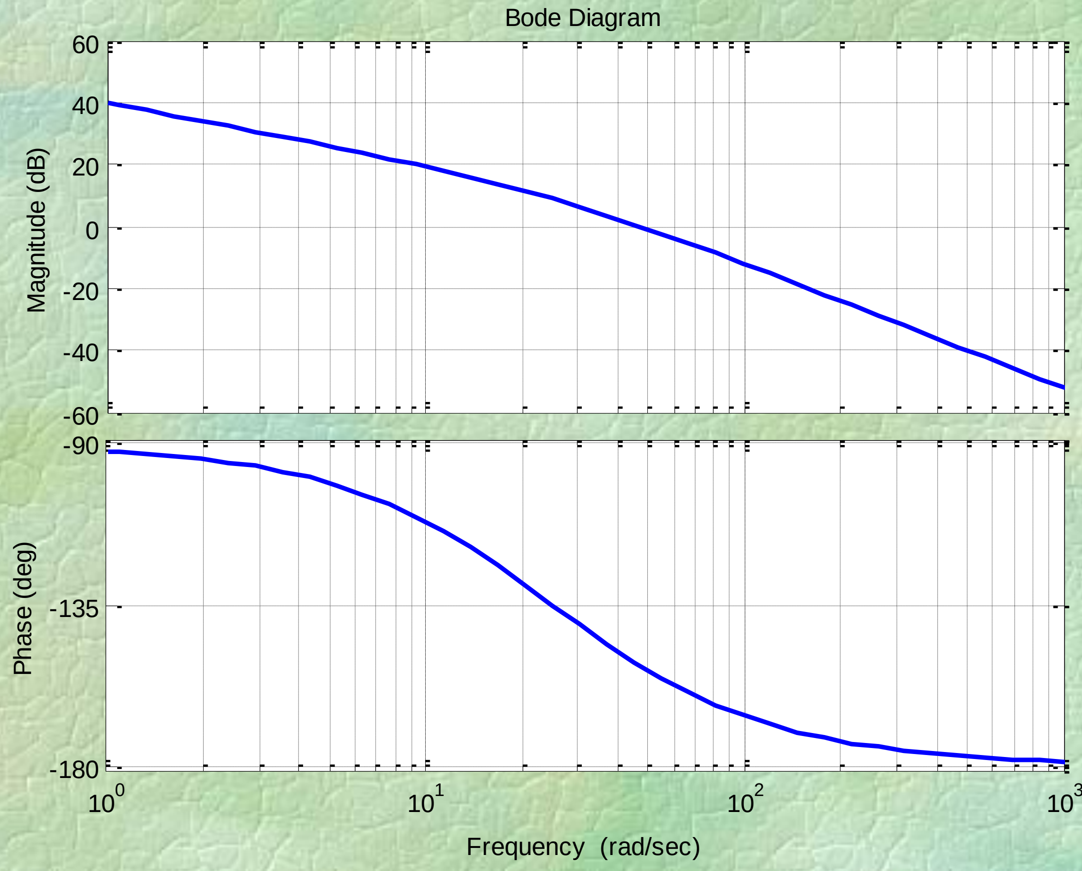
a) Derive type of g.

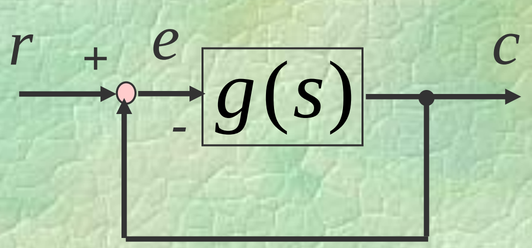
b) Derive k_p, k_v and k_a.

c) Derive number of zeros of g(s).

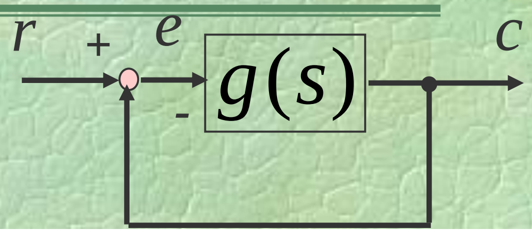
d) Derive number of poles of g(s).

e) Derive g(s).





Exercises



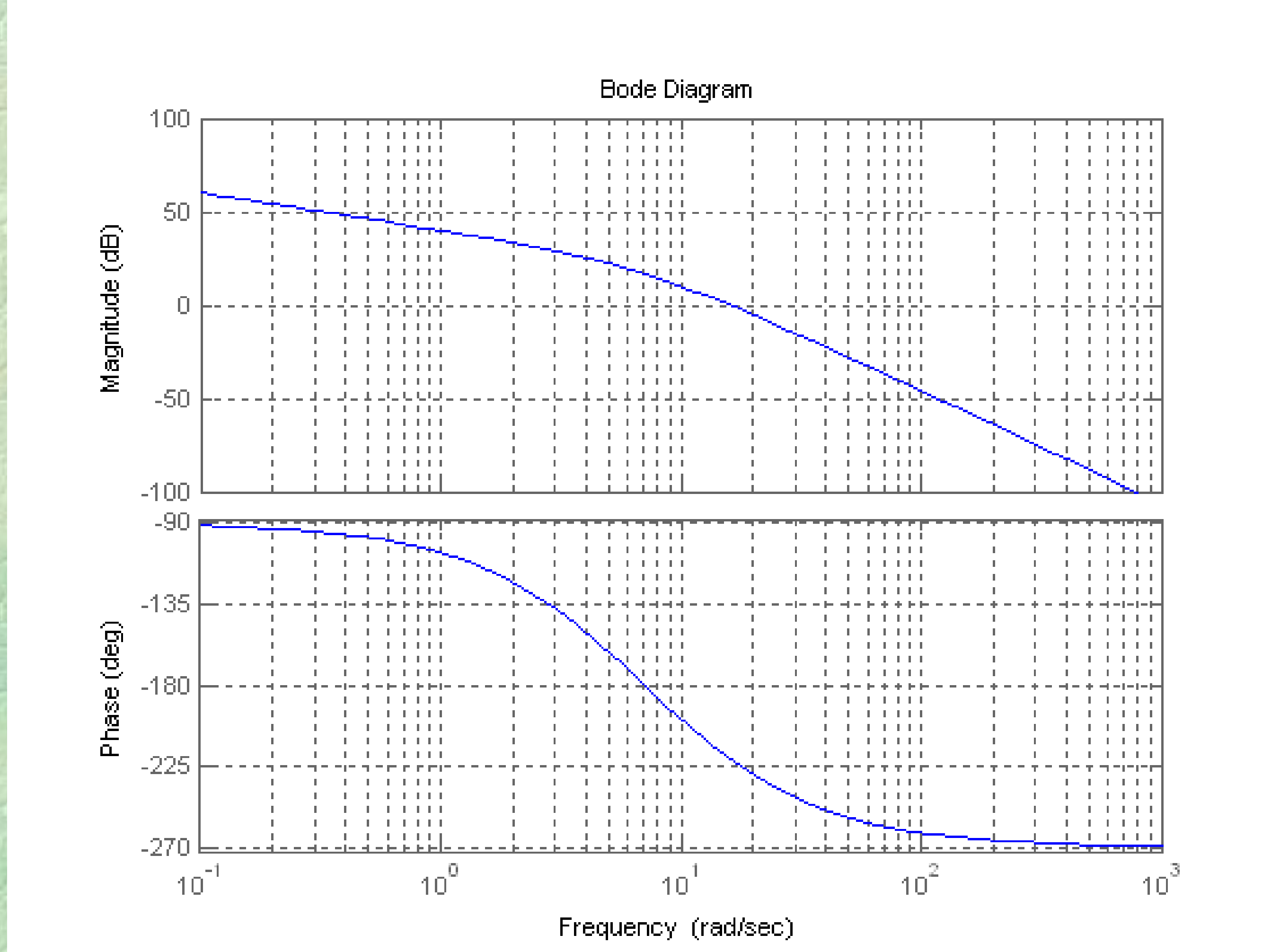
Exercise 8: In the figure you will find bode plot of g(s)

a) Derive type of g.

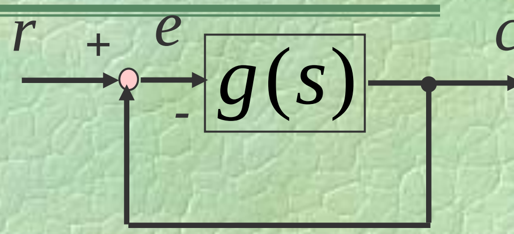
b) Derive k_p, k_v and k_a.

c) Derive number of zeros of g(s).

d) Derive number of poles of g(s).



Exercises



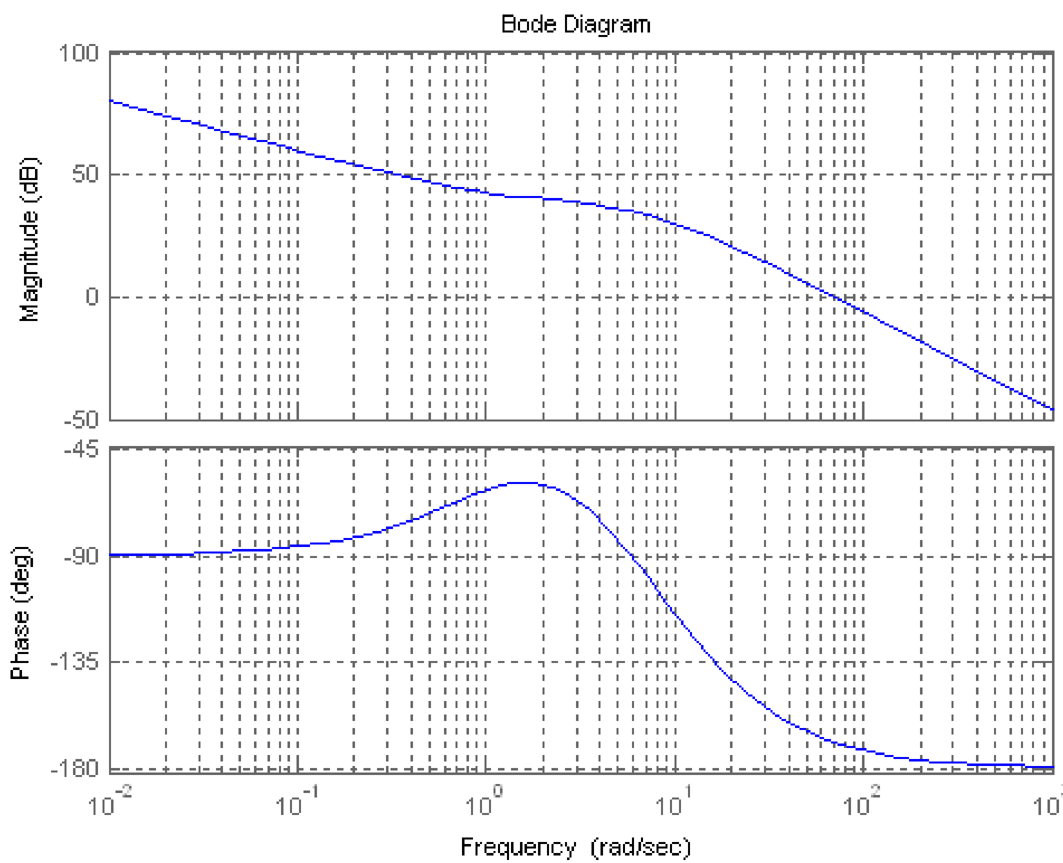
Exercise 9: In the figure you will find bode plot of g(s)

a) Derive type of g.

b) Derive k_p, k_v and k_a.

c) Derive number of zeros of g(s).

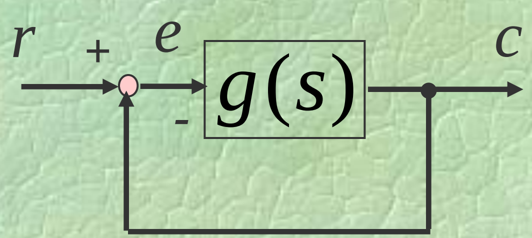
d) Derive number of poles of g(s).

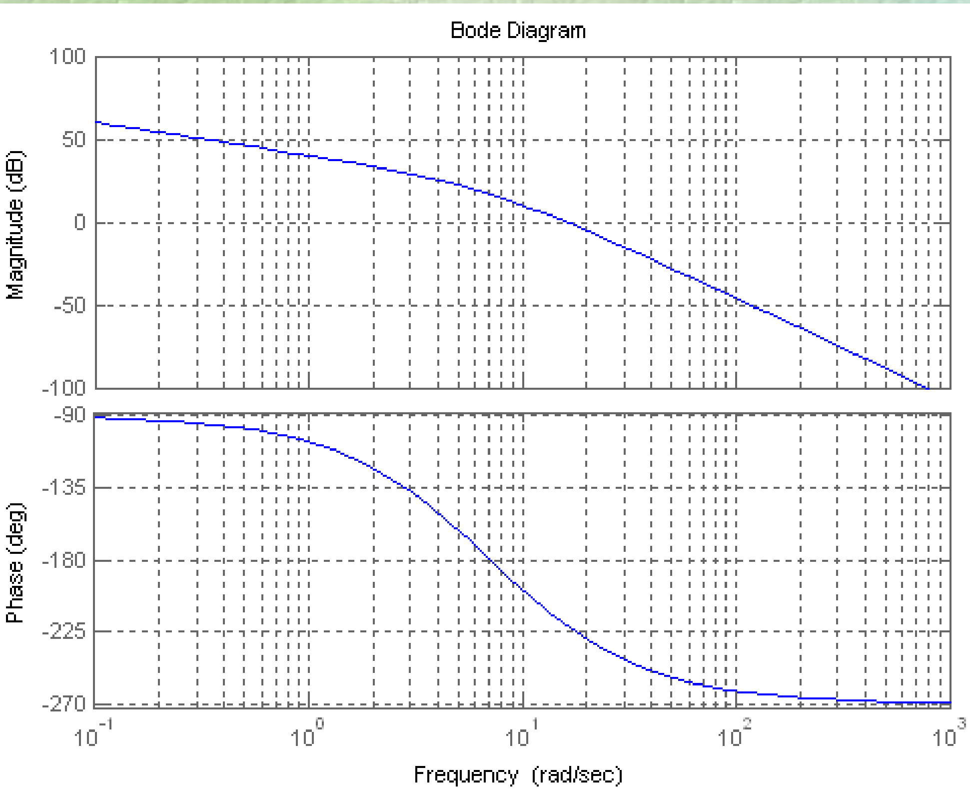


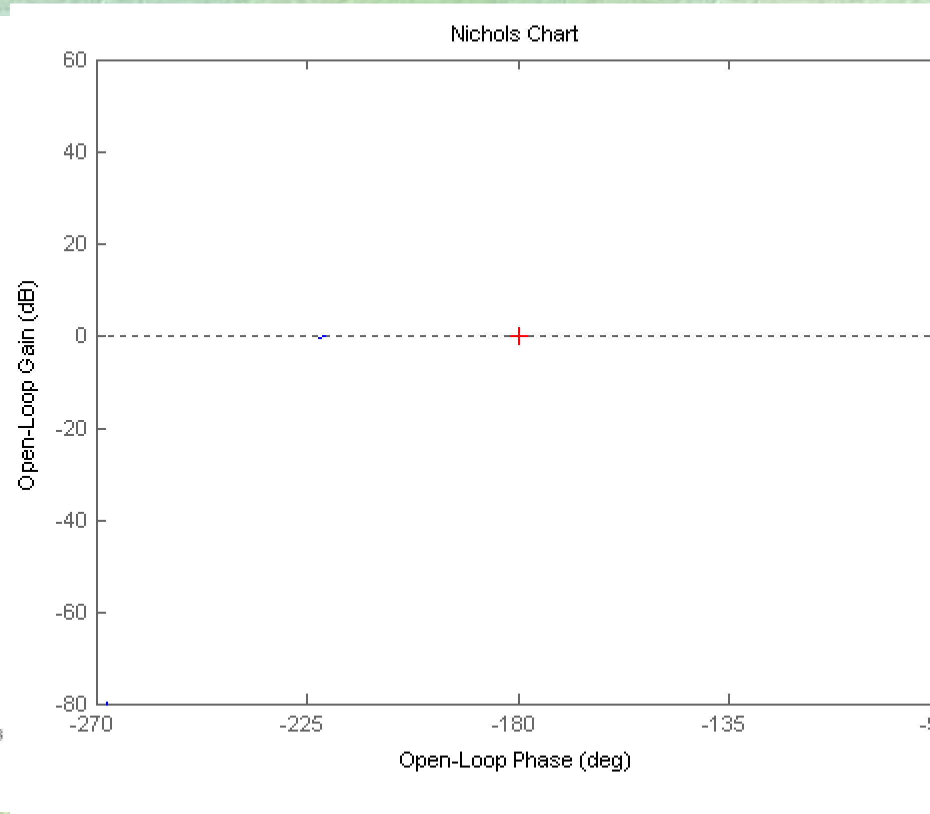
Exercises

Exercise 10: In the figure you will find bode plot of g(s)

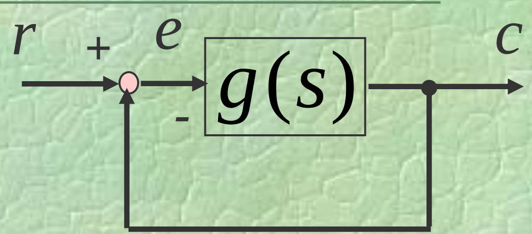
a) Derive plot on the right figure.







Exercises

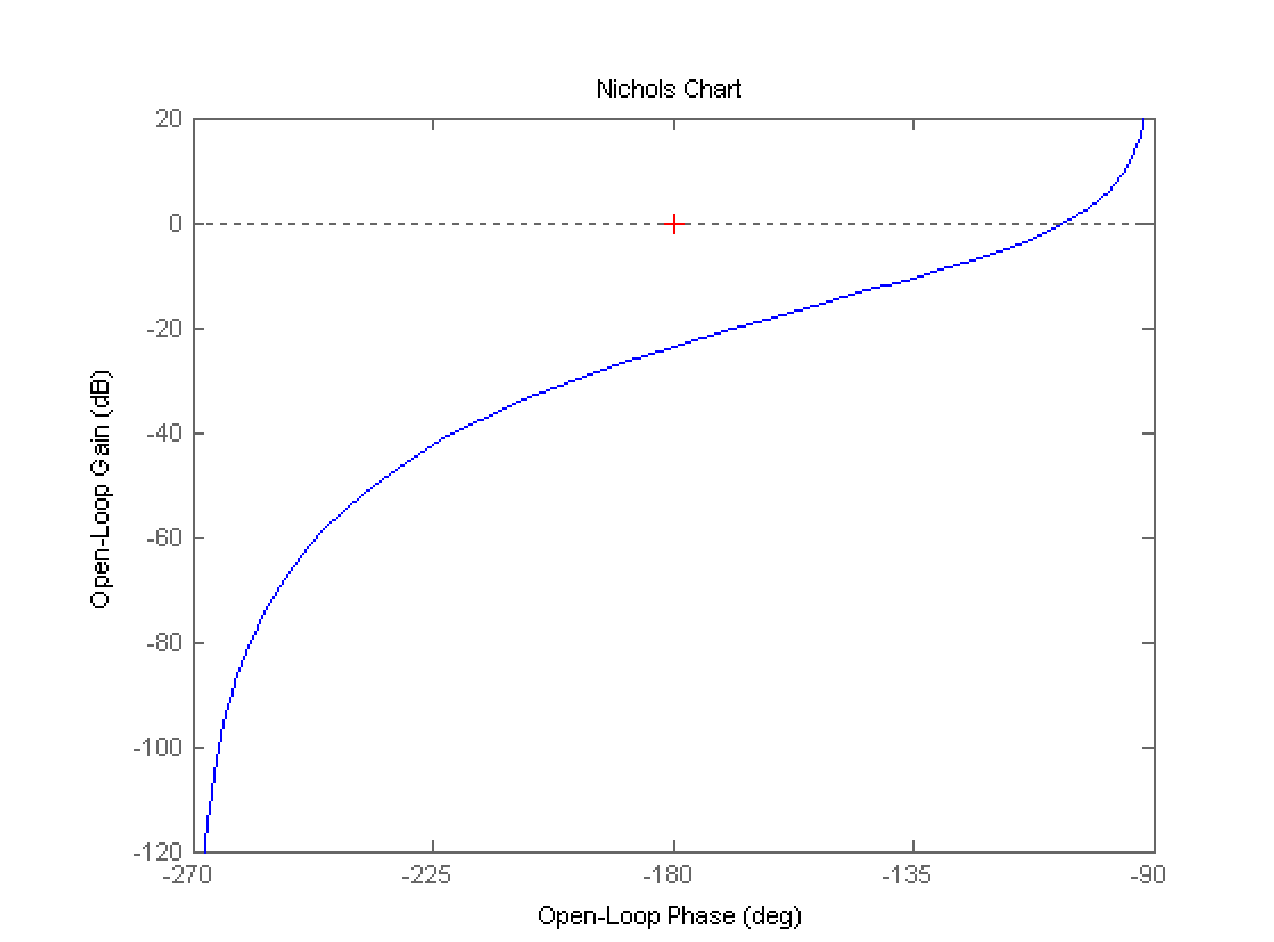


Exercise 11: In the figure you will find Nichols chart of g(s)

a) Derive type of g.

b) Derive number of zeros of g(s).

c) Derive number of poles of g(s).



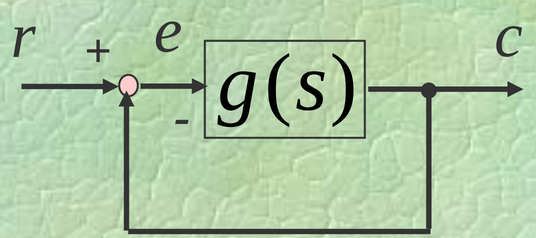
Exercises

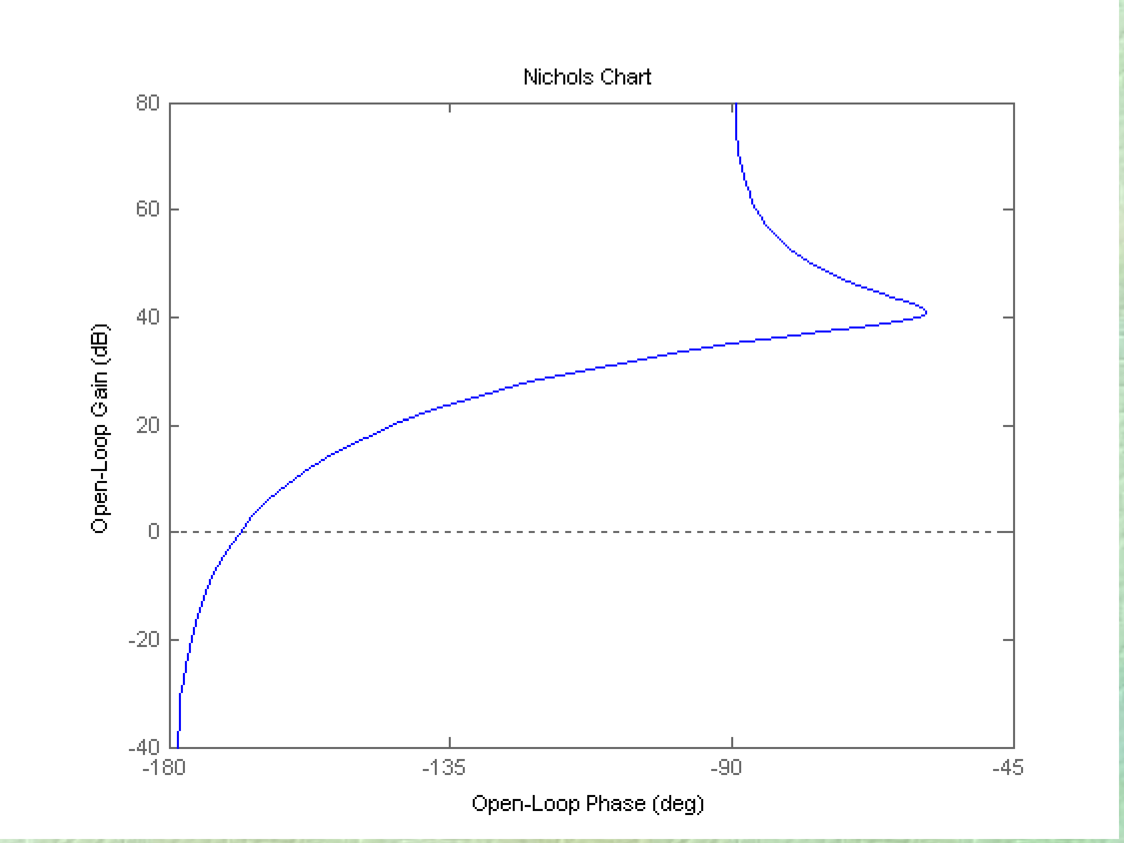
Exercise 12: In the figure you will find Nichols chart of g(s)

a) Derive type of g.

b) Is there any zero in g(s).

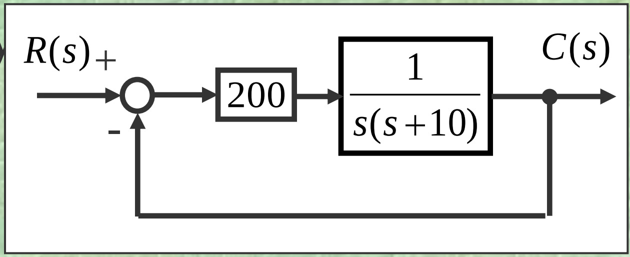
c) What is the difference of number of poles and zeros of g(s)





Exercises

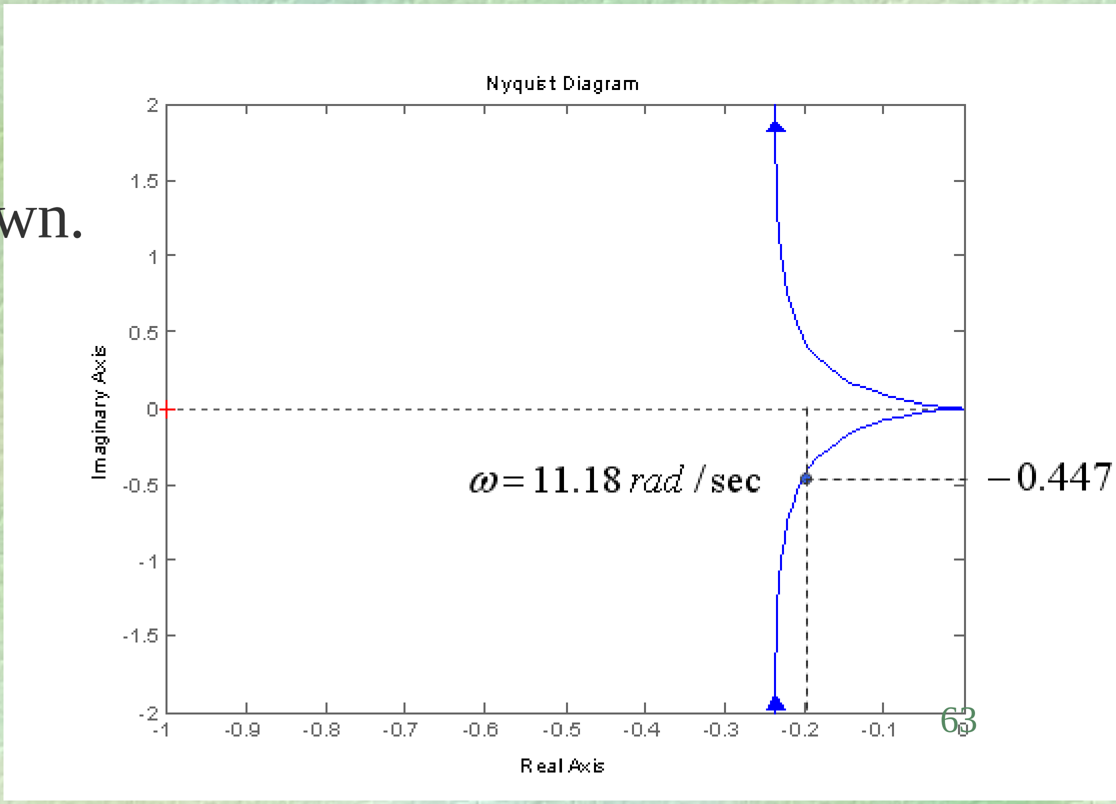
Exercise 13: Derive the gain crossover frequency, phase crossover frequency, GM and PM of following system by use of Bode plot.



$$Answer: \omega_c = 12.5\ ,\ \omega_{180} = \infty, GM = \infty\ and\ \varphi_m = 38^\circ$$

Exercise 14: The polar plot of an openloop system with negativeunit feedback is shown.

a) Find the open loop
b) transfer function.
c) Find the closed loop
d) transfer function.



$$answer\ a: \frac{150}{s(s+25)}\quad b: \frac{150}{s^2+25s+150}$$

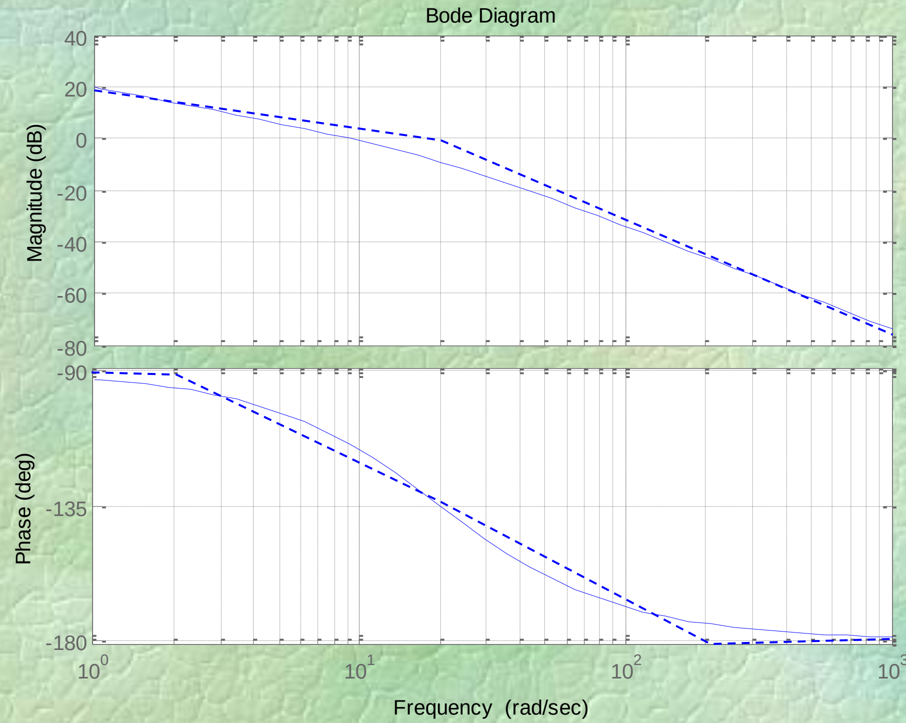
Exercises

Exercise 15: Bode plot of an open loop system with negative unit feedback is shown.

a) Find the open loop transfer function.

b) Find the closed loop transfer function.

$$answer\ a: \frac{200}{s(s+20)} \quad b: \frac{200}{s^2+20s+200}$$

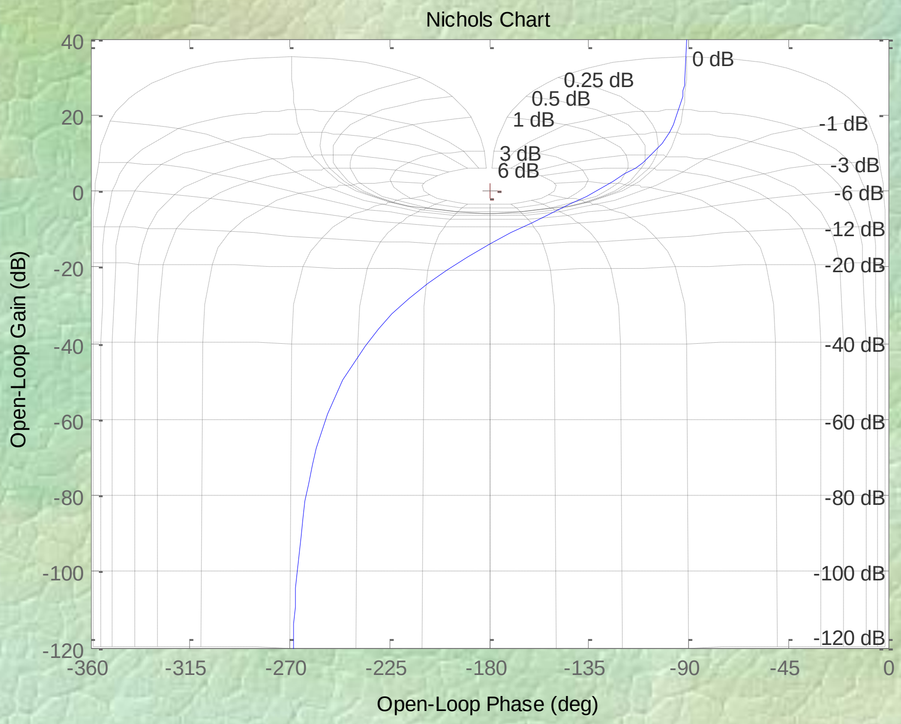


Exercises

Exercise 16: The Nichols chart of an open loop system with negative unit feedback is shown.

a) Find the GM and PM.

b) Find M_P.



$answer\ a: GM = 14\,db, PM = 45^{\circ} \quad b: M_p = 1.8\,db$

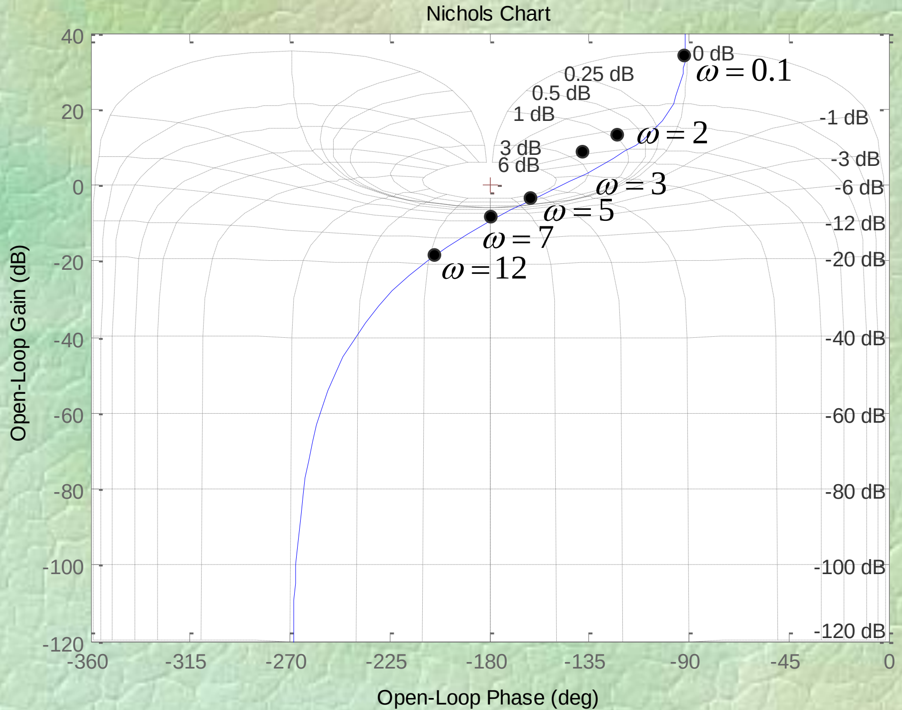
Exercises

Exercise 17: The Nichols chart of a open loop system with negative unit feedback is shown.

a) Find the error constants

b) Find the GM and PM and gain crossover frequency and phase crossover frequency.

c) Find M_P , open loop bandwidth and closed loop bandwidth.



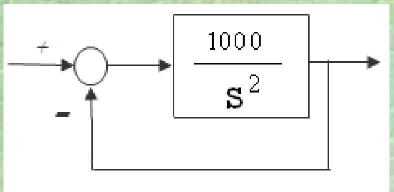
$answer\ a: k_p = \infty, k_v = 5, k_a = 0$

$b: GM = 10\,db, PM = 32^{\circ}, \omega_c = 3.75\,rad/\sec, \omega_{180} = 7\,rad/\sec$

$c: M_p = 5.3\,db, BW_{openloop} = 4.7\,rad/\sec, BW_{closedloop} = 6.3\,rad/\sec$

Exercises

Exercise 18: Draw Nichols chart of following system (Final exam).



Exercise 19: Draw gain-phase plot of a minimum phase type one system with no zero and three poles and GM=2 db and PM=45° (Final exam).

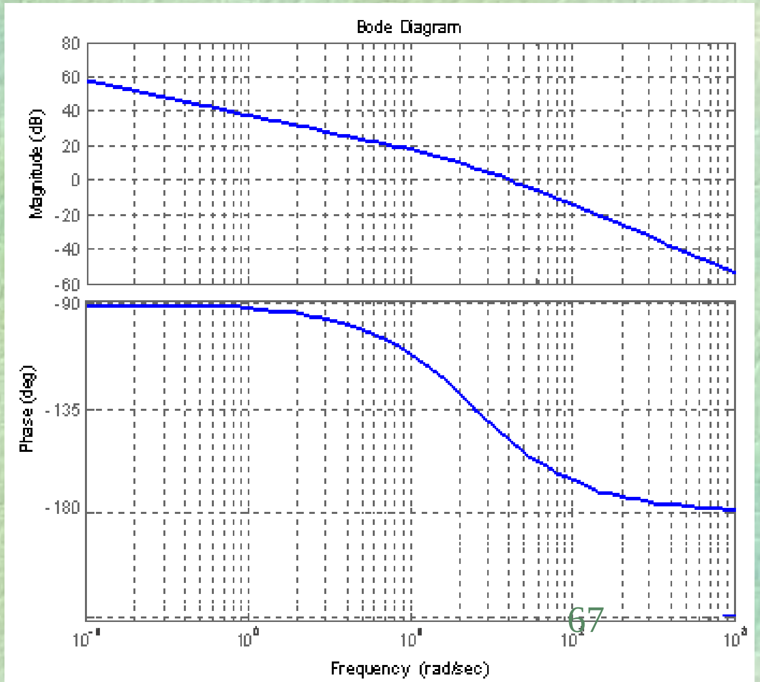
Exercise 20: Bode plot of a minimum phase system is: (Final exam).

a- Derive phase and gain crossover Frequency, Gm and PM.

b- Determine the nonzero error constant.

c- If 0.01 sec delay added inside the feedback loop, derive new Bode plot in the same figure.

d- Derive phase and gain crossover Frequency, Gm and PM of new system.



Exercises

Exercise 21: Nichols chart of a system is given, determine

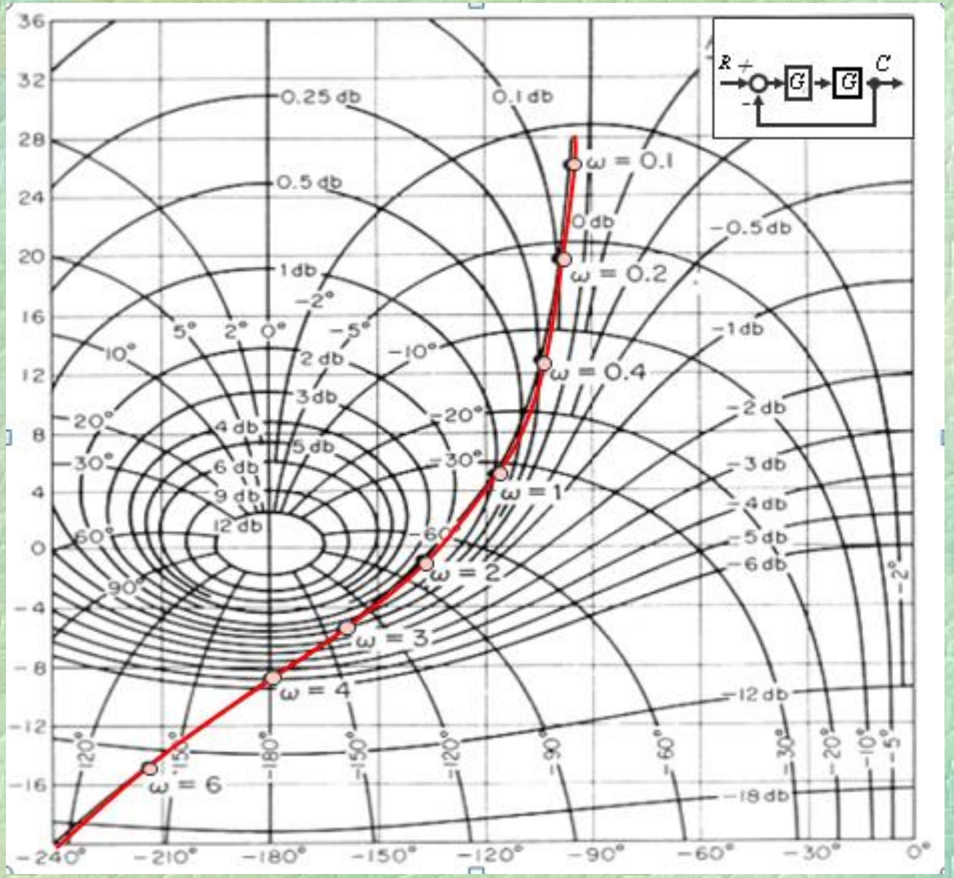
a- Gain and phase cross over frequency.

b- GM and PM.

c- Open loop and closed loop BW.

d- Type of system.

e- All error constant.



Dr. Ali Karimpour Aug 2024