Electric Machinery III

Ali Karimpour

Professor

Ferdowsi University of Mashhad, Iran

Text: A. E. Fitzgerald, Charles Kingsley, Jr And Stephen D. Umans, *Electric Machinery*. Mc GrawHill, 7th Edition

Electrical Machine III Syllabus

1. Introduction
   - Iranian Power System Network
2. Introduction to Rotating Machines
   - Voltage Generation in Electrical Machinery
     - AC Machines
     - DC Machines
   - Force and Torque in Electrical Machines
     - Rotating MMF Waves in AC Machines
     - Electromechanical Energy-Conversion Principles
     - Determination of Magnetic Torques Through Magnetic Field View Point
     - Torque Production Condition
     - Determination of Magnetic Torques Through Coupled Circuit View Point
3. Generation of Sinusoidal Voltage in a Synchronous Generator
   - Rotor Winding Factors
   - Stator Winding Factors
     - Stator Distribution Winding Factor
     - Stator Pitch Winding Factor
     - Generator Connections
4. Synchronous-Machine Analyses
   - Synchronous-Machine Inductances; Equivalent Circuits
   - Steady-State Power-Angle Characteristic
   - Steady-State Operating Characteristic
   - Open- and Short-Circuit Characteristics
   - Effects of Salient Poles; Introduction to Direct and Quadrature-Axis Theory
   - Steady-State Power-Angle Characteristic of Salient-Pole Machines
   - Permanent Magnet Synchronous Machine
   - Transient Behavior of Synchronous Machine.

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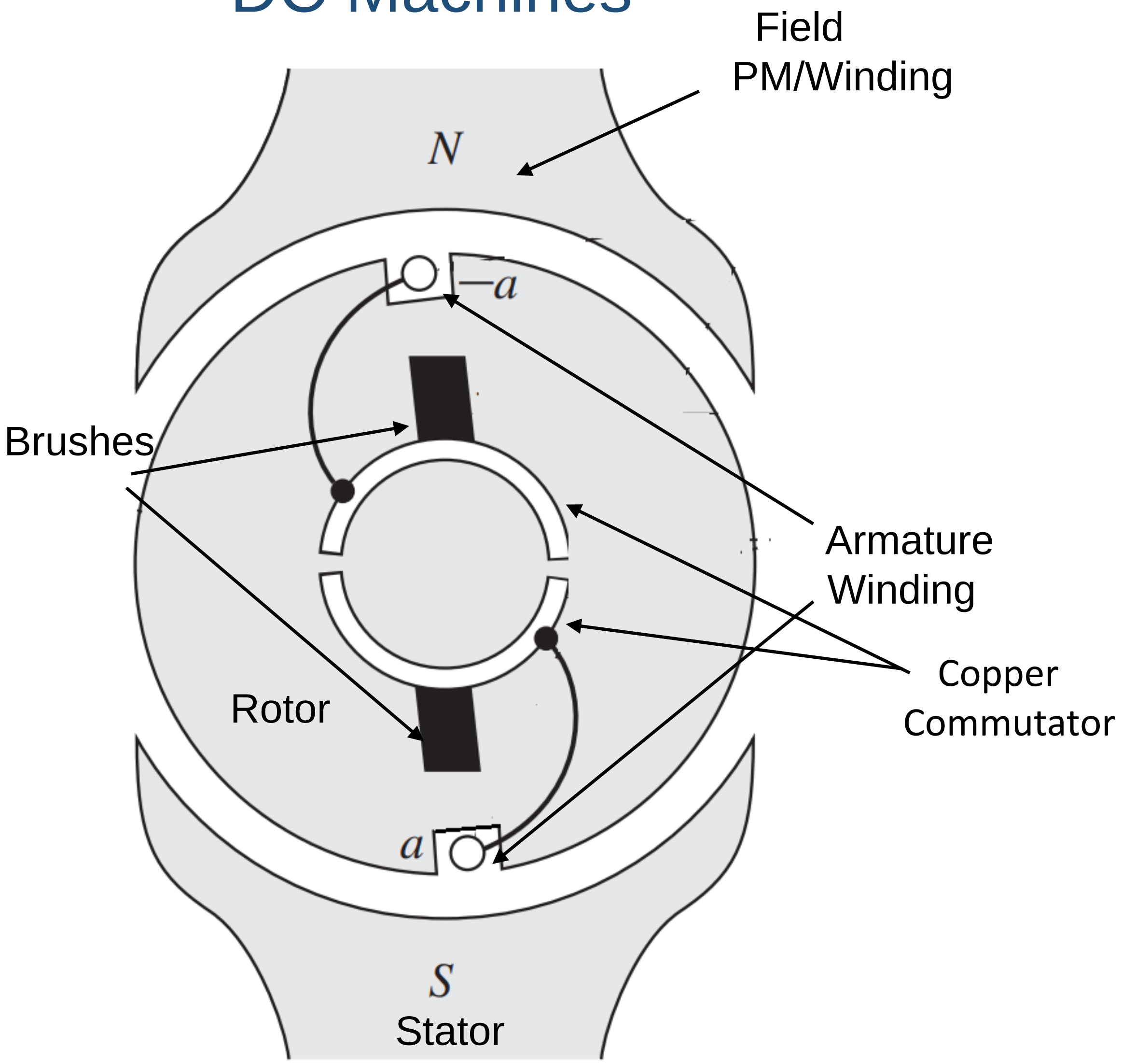
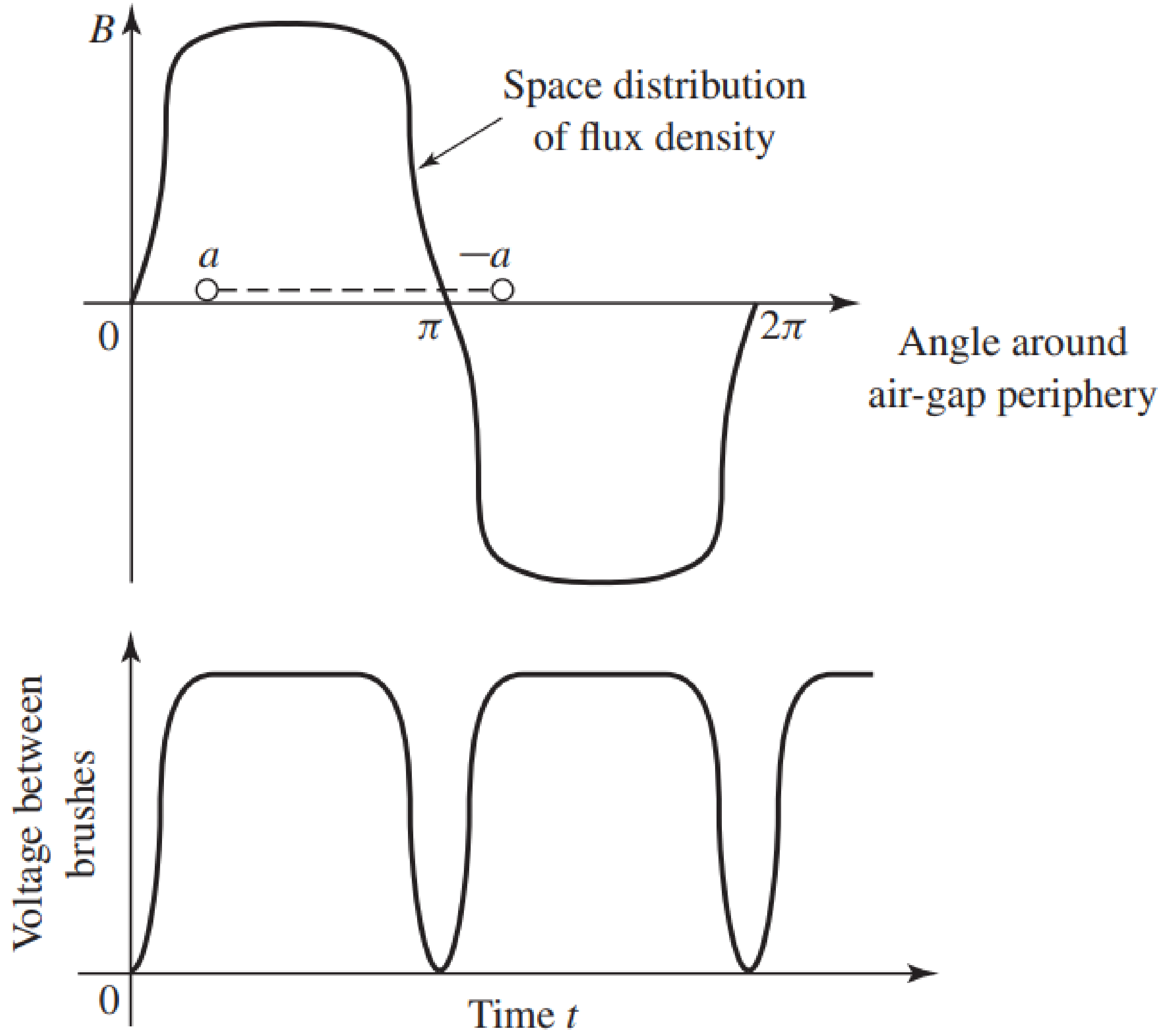
Voltage Generation in Electrical Machinery

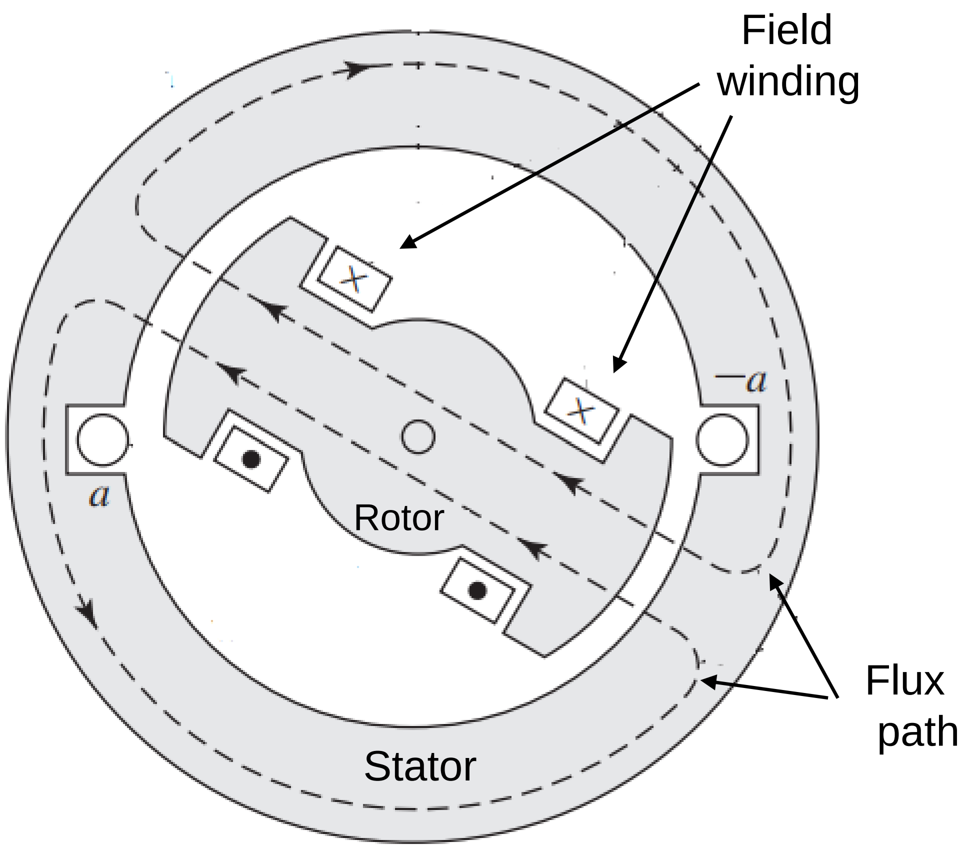
DC Machines

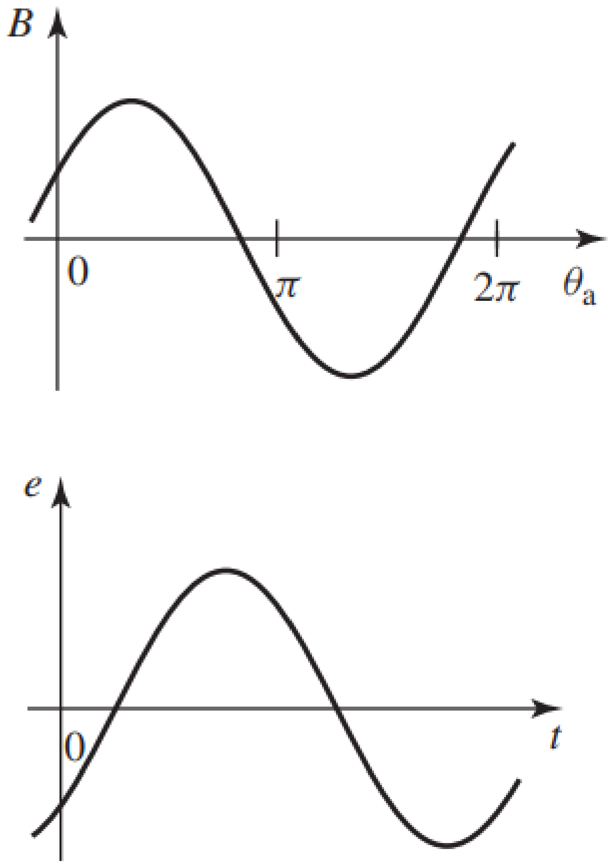




Voltage Generation in Electrical Machinery

AC Machines (Two poles one phase)
Silent pole

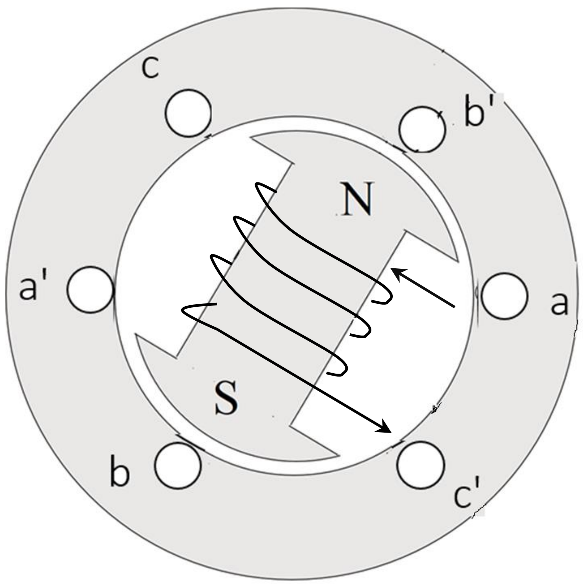


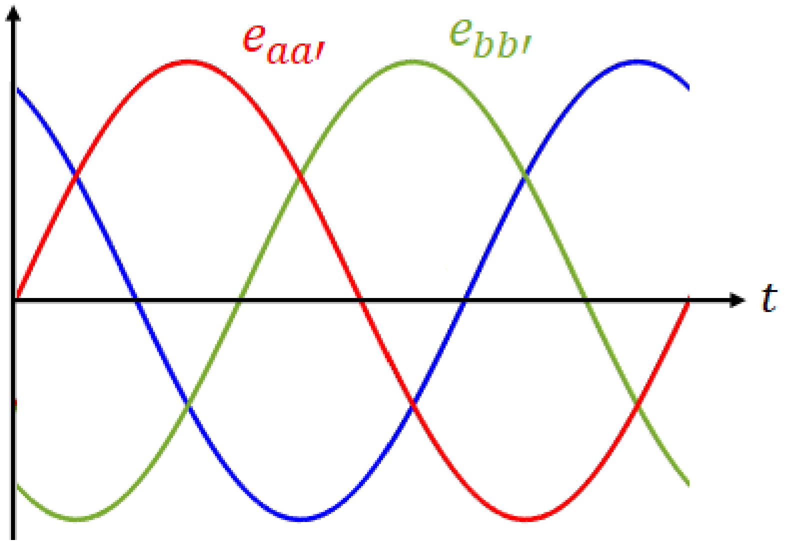


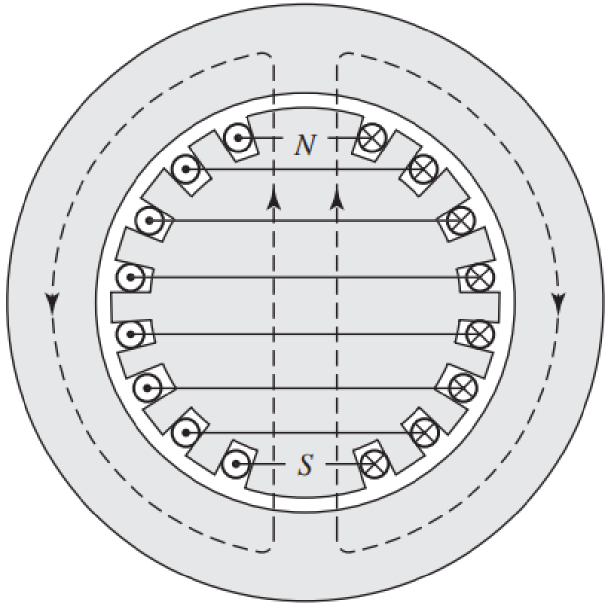
Voltage Generation in Electrical Machinery

AC Machines (Two poles three phases) Silent pole

AC Machines (Two poles) Cylindrical





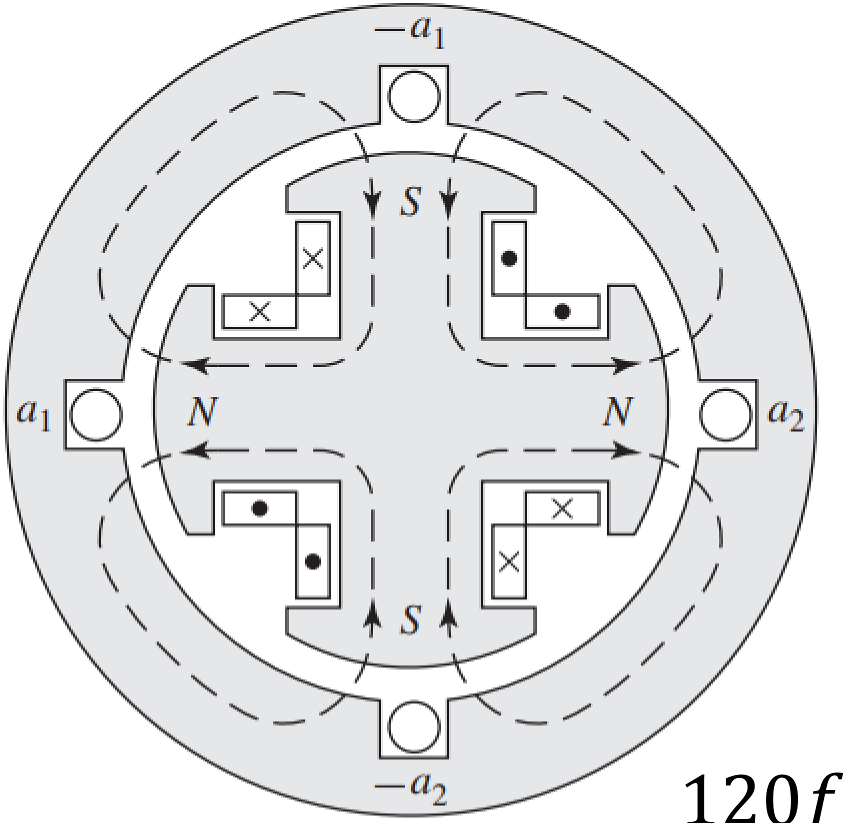


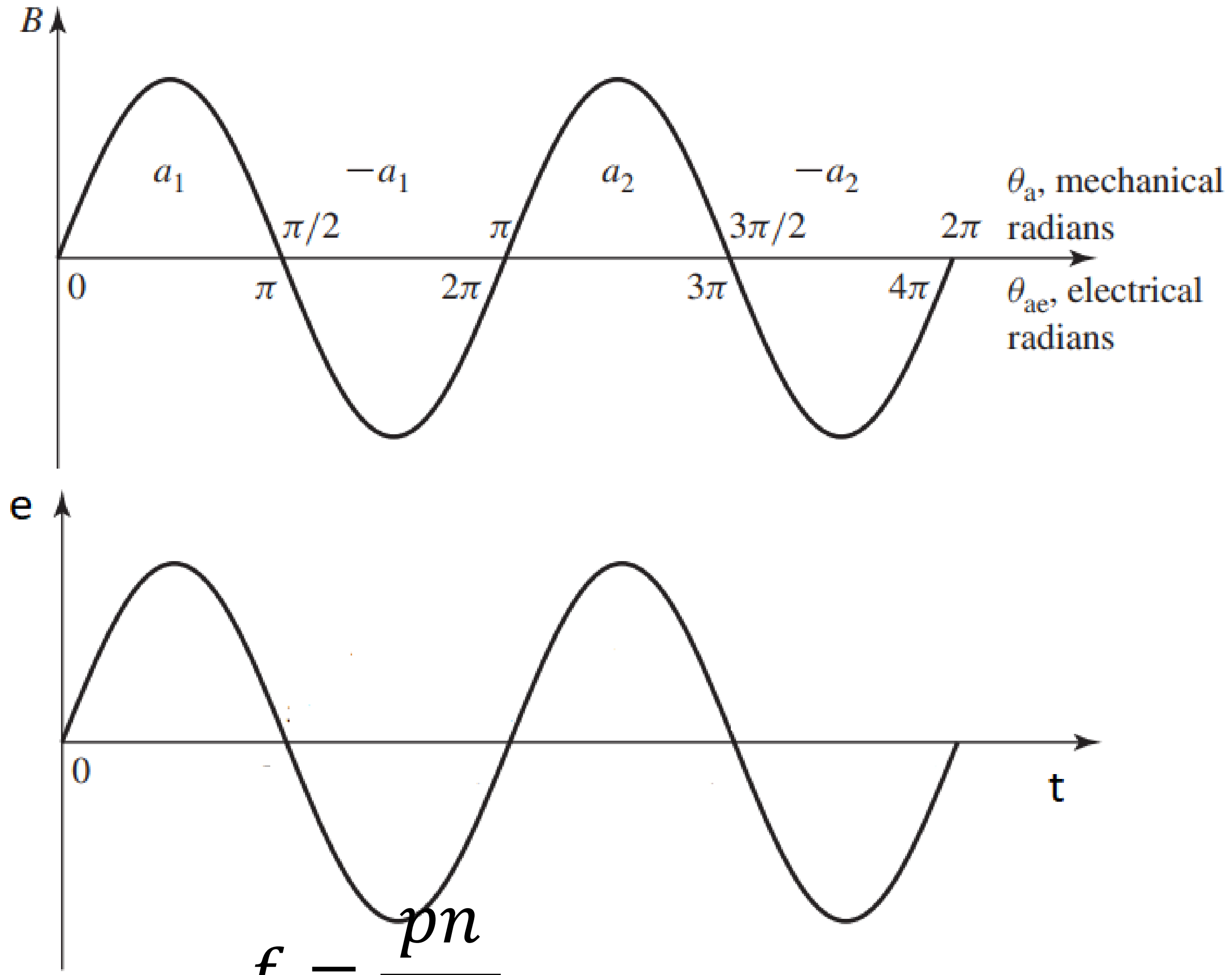
Rotor and stator of a 160 MW, 2-pole generator at the Toos Power Plant, Mashhad, Iran.

Voltage Generation in Electrical Machinery

AC Machines (Four poles one phase)

Silent pole

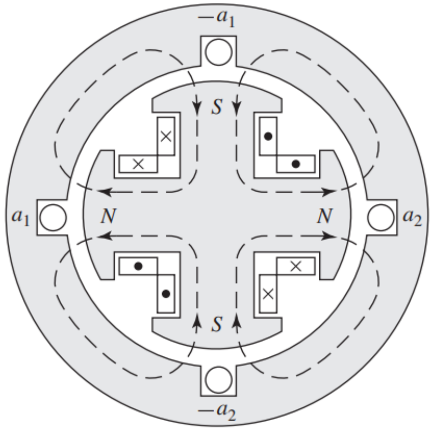


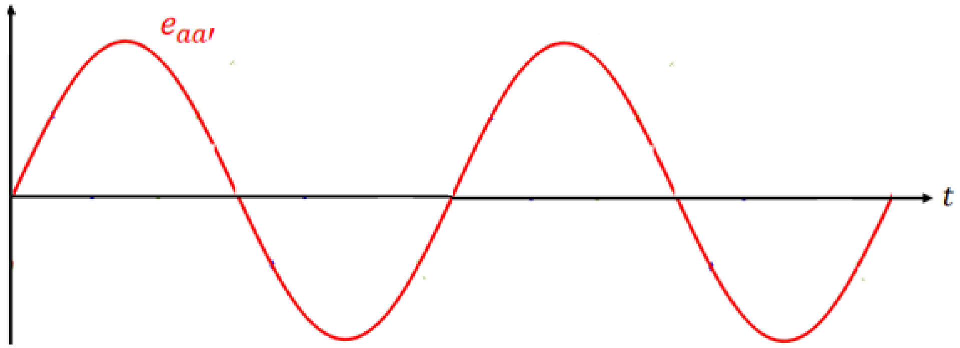


$$n = \frac{120f}{p}$$

$$f = \frac{pn}{120}$$

AC machine basic idea

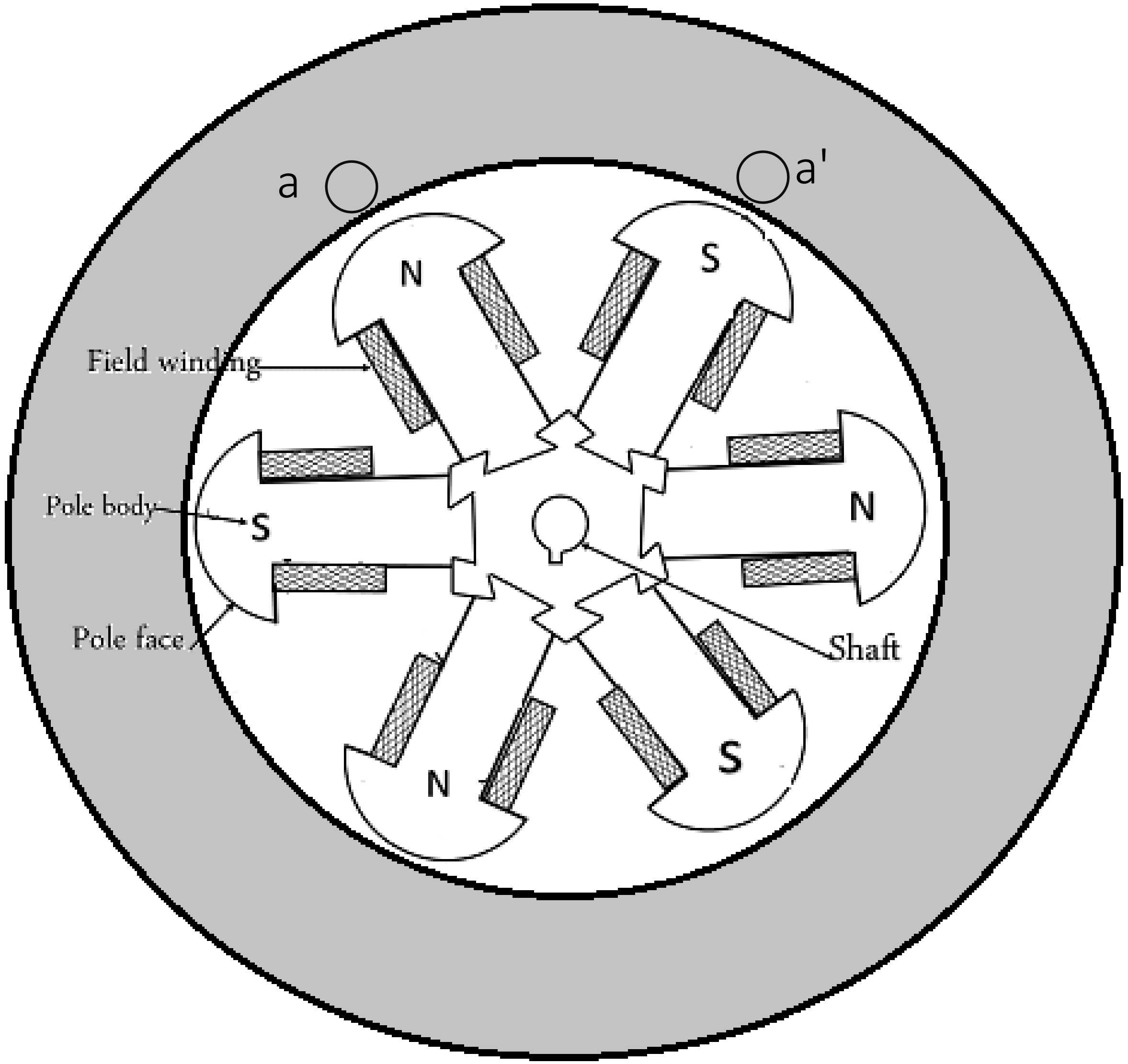




	P=2	P=4	P=6	P=8	P=10	P=12
f=50 Hz	3000 rpm	1500 rpm	1000 rpm	750 rpm	?? rpm	?? rpm
f=60 Hz	3600 rpm	1800 rpm	1200 rpm	900 rpm	?? rpm	?? rpm

$$n = \frac{120f}{p} \qquad f = \frac{pn}{120}$$

Voltage Generation in Electrical Machinery



$$\text{stator pitch} = \pi^{ele} = \frac{\pi^{mec}}{3}$$

$$deg^{ele} = \frac{p}{2} deg^{mec}$$

$$f = \frac{pn}{2 * 60} = \frac{pn}{120}$$

$$n = \frac{120f}{p}$$

Salient pole machine

Rotor and stator of a 90 MW, 22-pole hydro generator at the Dez Power Plant, Dezfool, Iran.

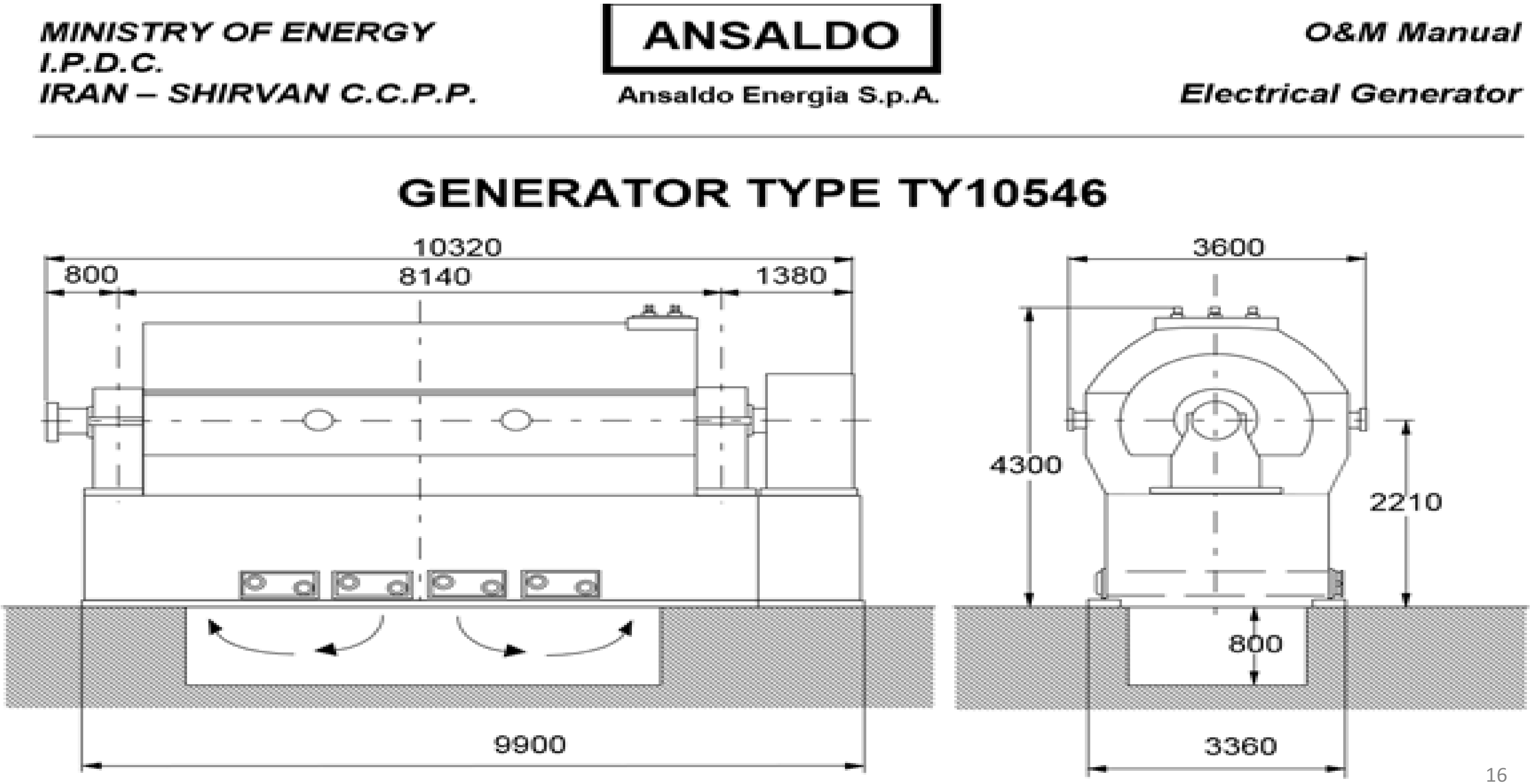
Rotor and stator of a 440 MW, 2-pole turbo generator at the Shahid Salimi Power Plant, Neka, Iran.

Plate of a 160 MW, 2-pole generator at the Toos Power Plant, Mashhad, Iran.

Stator of a 160 MW hydro generator at the Samira Power Plant, located in Iran.

Part of stator winding of a 160 MW, 32-pole hydro generator at the Samira Power Plant, located in Iran.

View of the 200 MVA bipolar generator at the Shirvan power plant, Iran



Dr. Ali Karimpour June 2024

Specifications of the 2-Pole 200 MVA Generator for Shirvan Power Plant, Iran

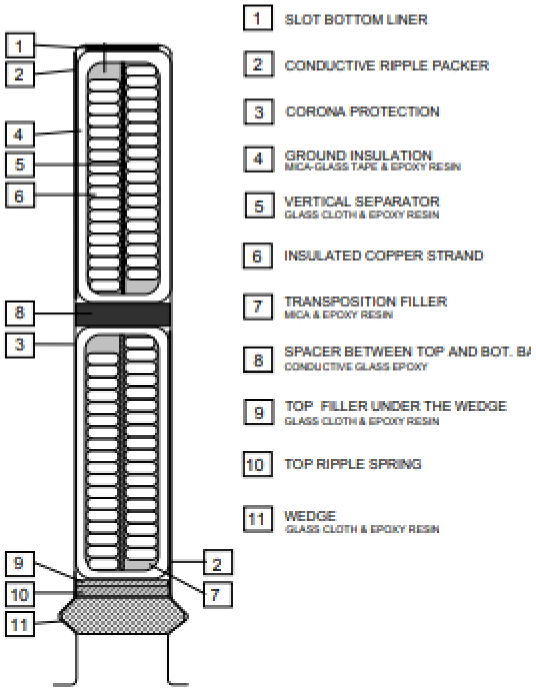
WEIGHTS and Dimensions FOR TRANSPORTATION AND LIFTING		
DESCRIPTION	Unit	Value
Generator height	m	4.3
Generatorm width	m	3.8
Generatorm length	m	10.3
Stator core weight	ton	110
Stator winding weight	ton	15
Stator frame weight	ton	37
Slip rings housing	ton	2.5
Rotor winding weight	ton	5.9
Rotor complete weight	ton	41
Stator complete without bearings and coolers weight	ton	168
Generator complete weight	ton	224

Specifications of the 2-Pole 200 MVA Generator for Shirvan Power Plant, Iran

Shivan Generator Specification		
DESCRIPTION	**Unit**	**Value**
Generator type	-	Air cooled
Rated output at 40 °C	mVA	200
Rated power factor	-	0.8
Rated voltage	V	15750
Rated frequency	Hz	50
Rated current	A	7331
Rated speed	rpm	3000
Excitation system type	-	static type
Excitation current at rated load	A	1417
Excitation voltage at rated load	V	296
Excitation current at no-load rated terminal voltage	A	459
Short circuit ratio (Calculated values)	-	0.47
Synchronous direct axis reactance (Calculated values)	%	238
Transient direct axis reactance unsaturated (Calculated values)	%	23.4
Subtransient direct axis reactance unsaturated (Calculated values)	%	17.9
Conventional efficiencies according to IEC 34 Std. at rated load	%	98.54
Conventional efficiencies according to IEC 34 Std. at 25% rated load	%	96.25

Specifications of the 2-Pole 200 MVA Generator for Shirvan Power Plant, Iran

Shivan Generator Specification (stator & rotor)		
DESCRIPTION	Unit	Value
Core length	cm	462
Inside core diameter(stator)	cm	115
Outside core diameter(stator)	cm	25.1
Number of stator slots	-	60
Height of the stator slot	cm	26
Width of the stator slot	cm	2.6
Lamination thickness	mm	0.5
Stator coil pitch in terms of slot pitch	-	25
Turn per phase in series(stator)	-	2
Number of parallel circuits per each phase(stator)	-	2
Number of conductors per slot(stator)	-	2
Winding connection(stator)	-	Y
Cross section of the stator bar	mm2	1375
Outer diameter(rotor)	cm	105
Number of wound slots	-	36
Effective turns per pole	-	96
Cross section of rotor conductor	mm2	265
Resistance at 75 °C	ohm	0.188



Stator slot cross section

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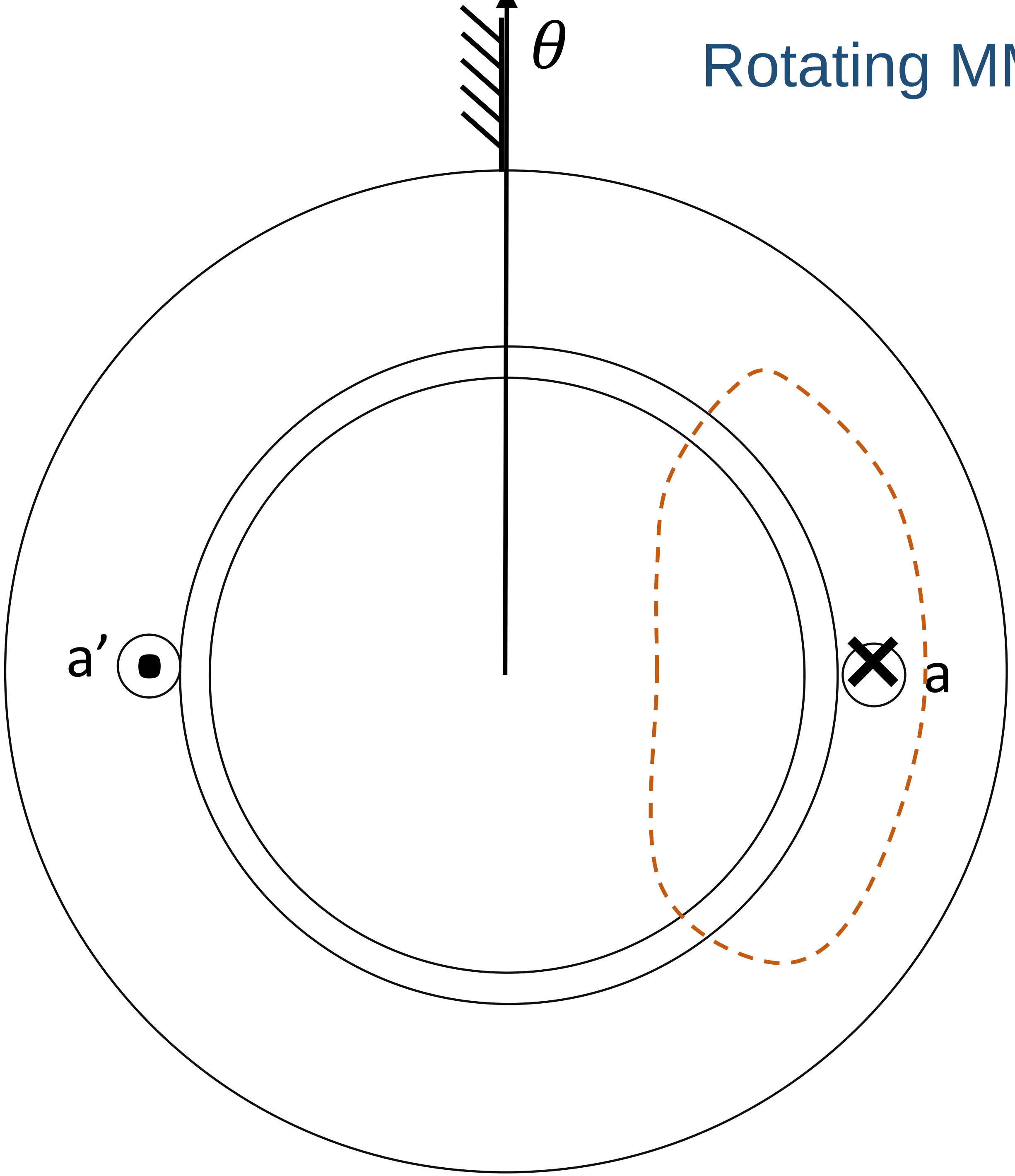
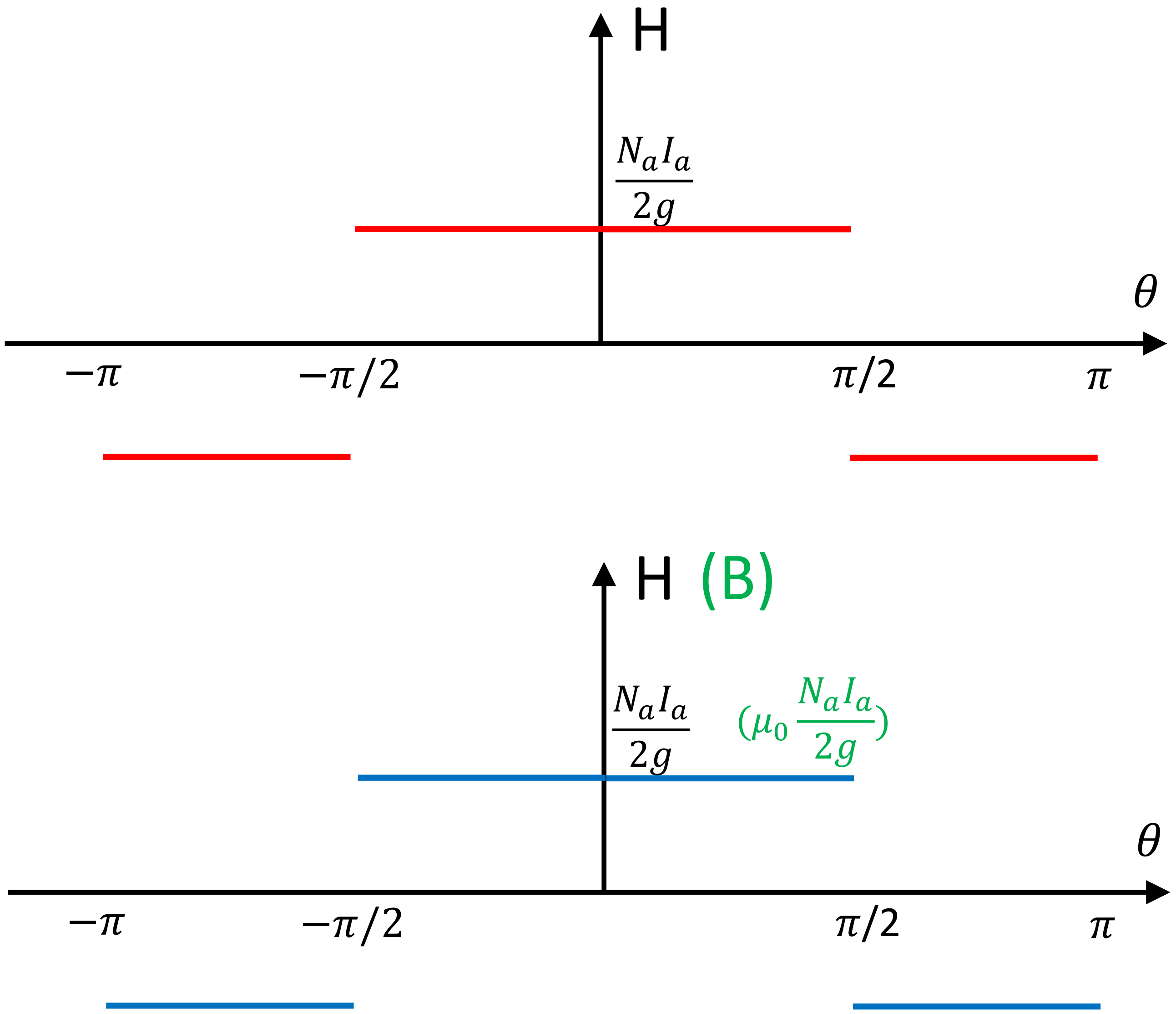
Rotating MMF Waves in AC Machines





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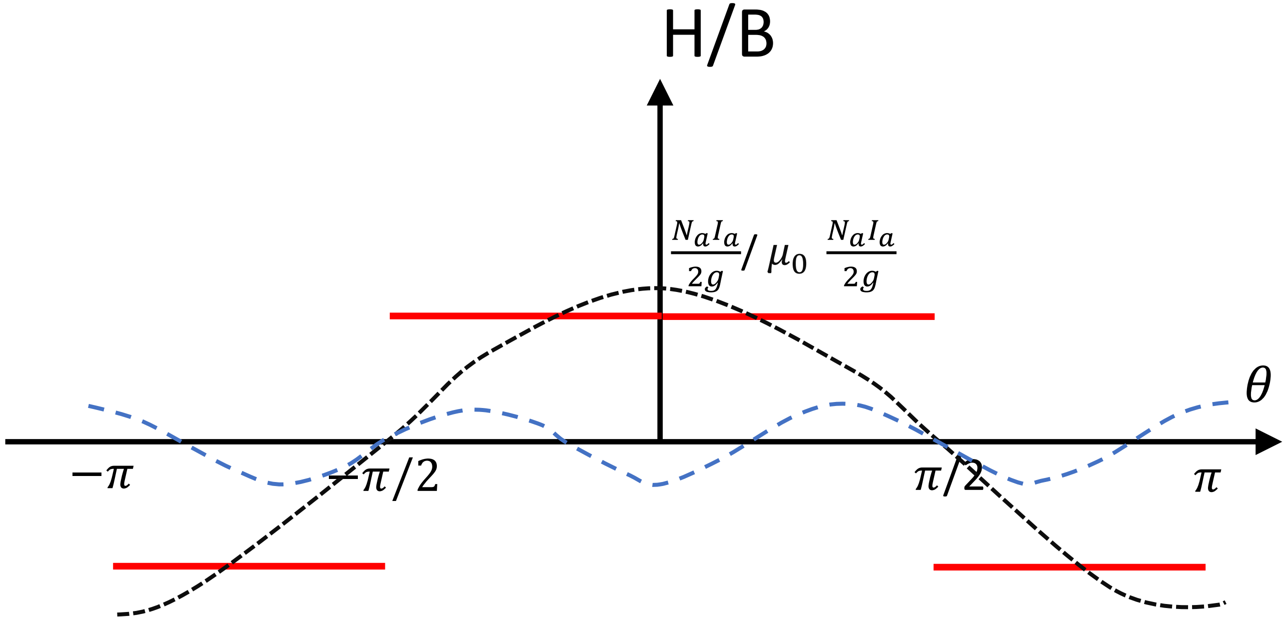
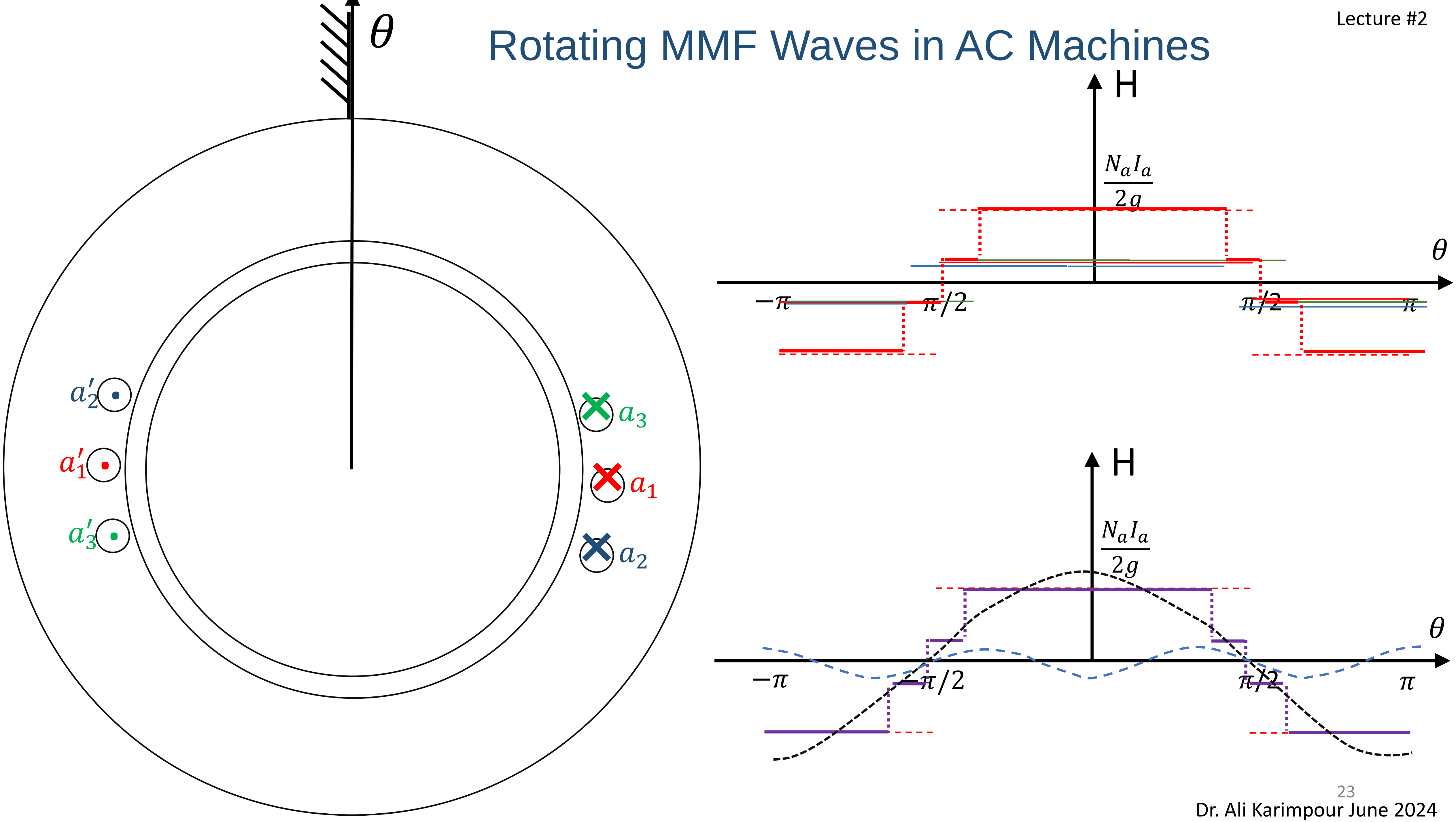


$$H = \frac{4}{\pi}\frac{N_aI_a}{2g}cos\theta - \frac{4}{3\pi}\frac{N_aI_a}{2g}cos3\theta + \frac{4}{5\pi}\frac{N_aI_a}{2g}cos5\theta - \cdots$$

Rotating MMF Waves in AC Machines

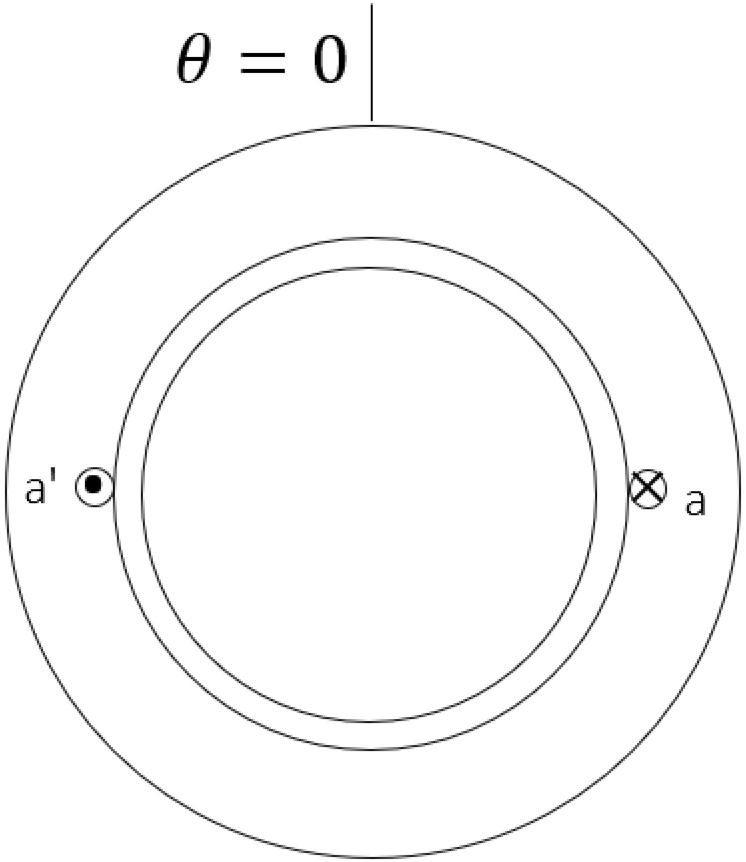


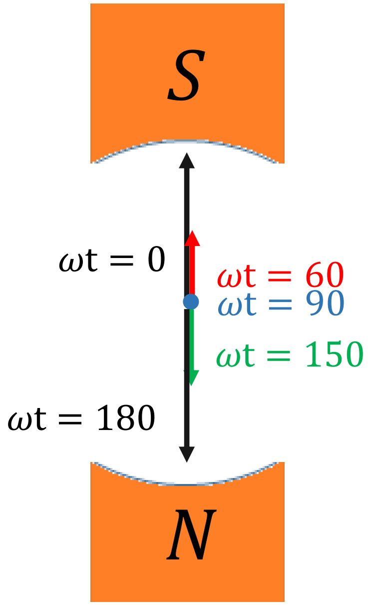
Rotating MMF Waves in AC Machines

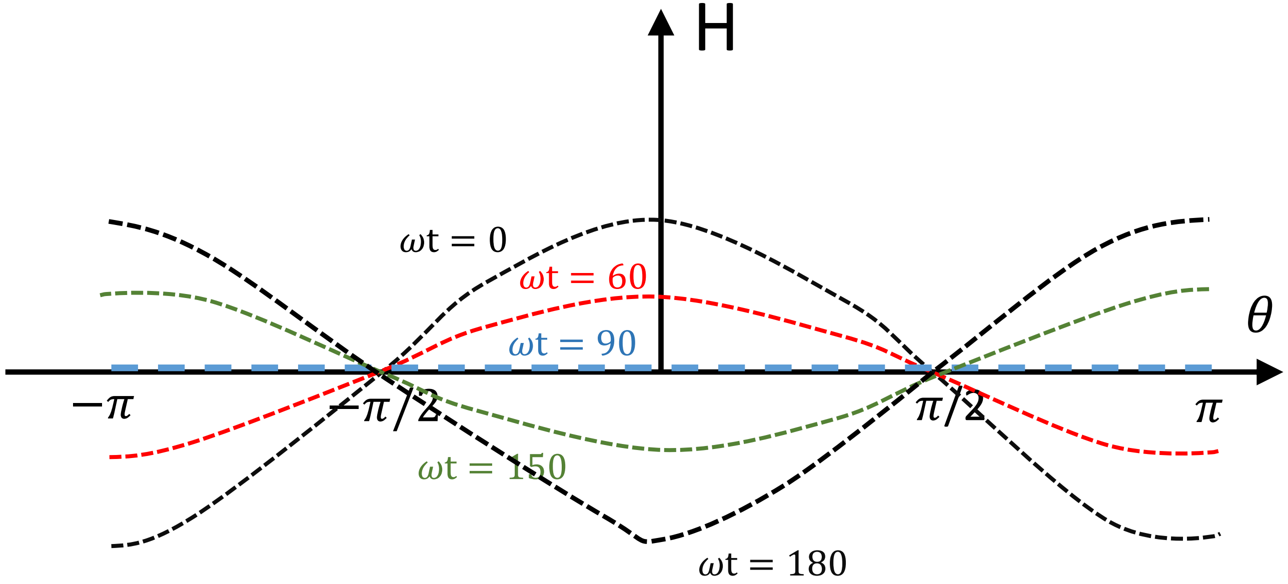
AC machines with one phase

$$I_a = I_m \cos(\omega t)$$

$$\overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a I_a}{2g} cos\theta \qquad \overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \cos(\omega t)\cos(\theta)$$





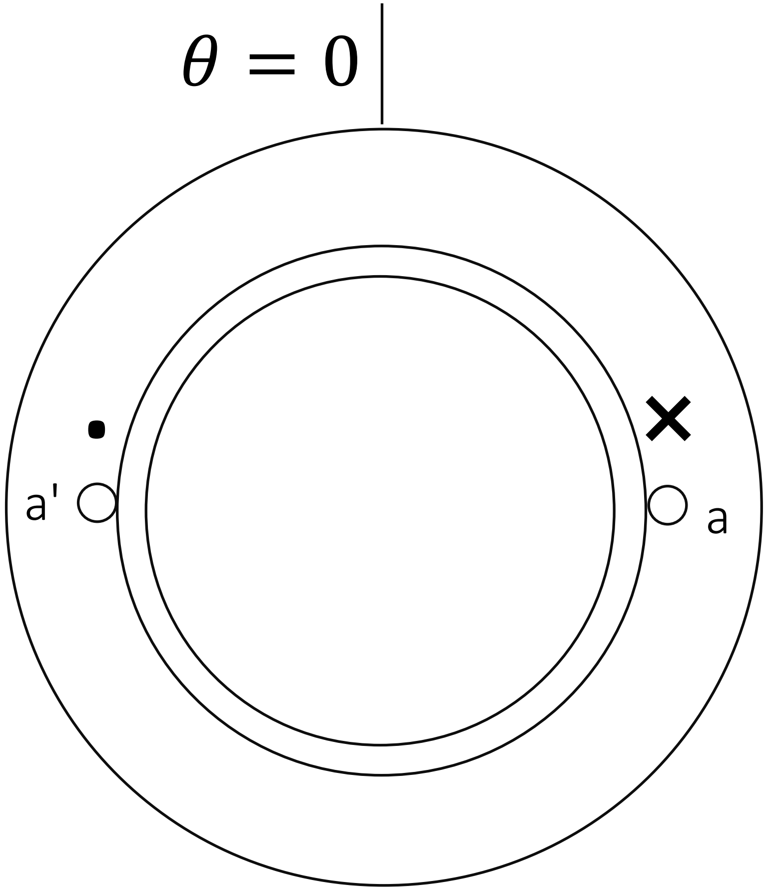


Rotating MMF Waves in AC Machines

AC machines with one phase

$$I_a = I_m \cos(\omega t)$$

$\theta = 0$



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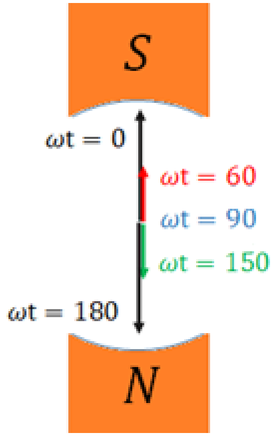
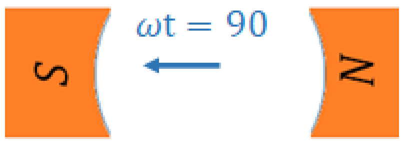
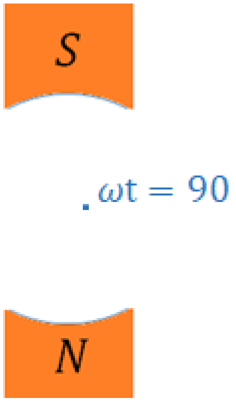
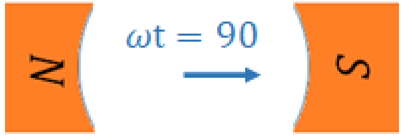


$$\overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t)$$

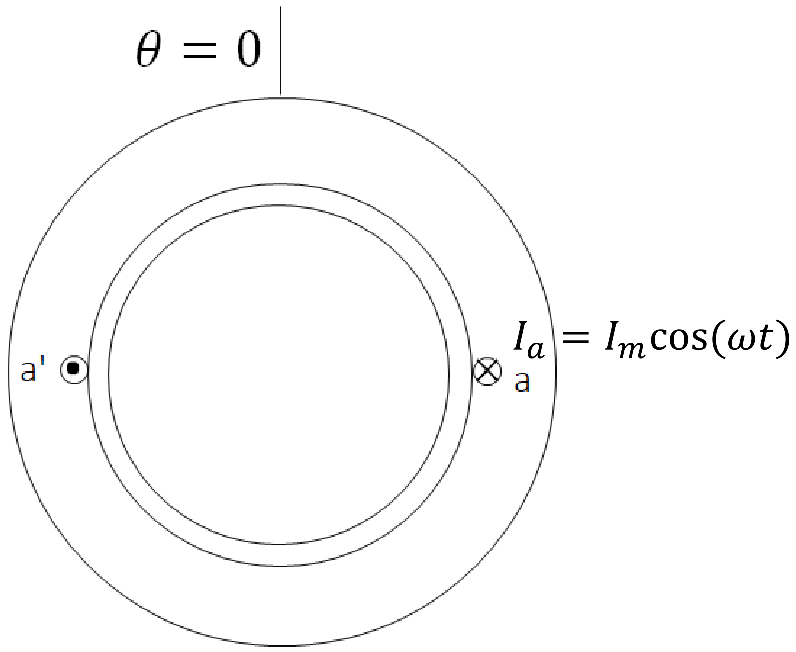


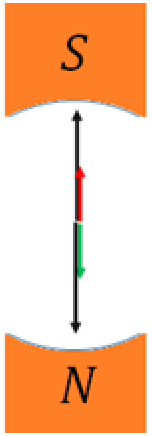


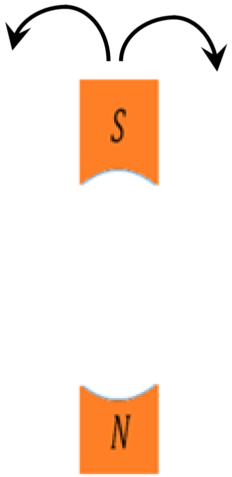


Rotating MMF Waves in AC Machines

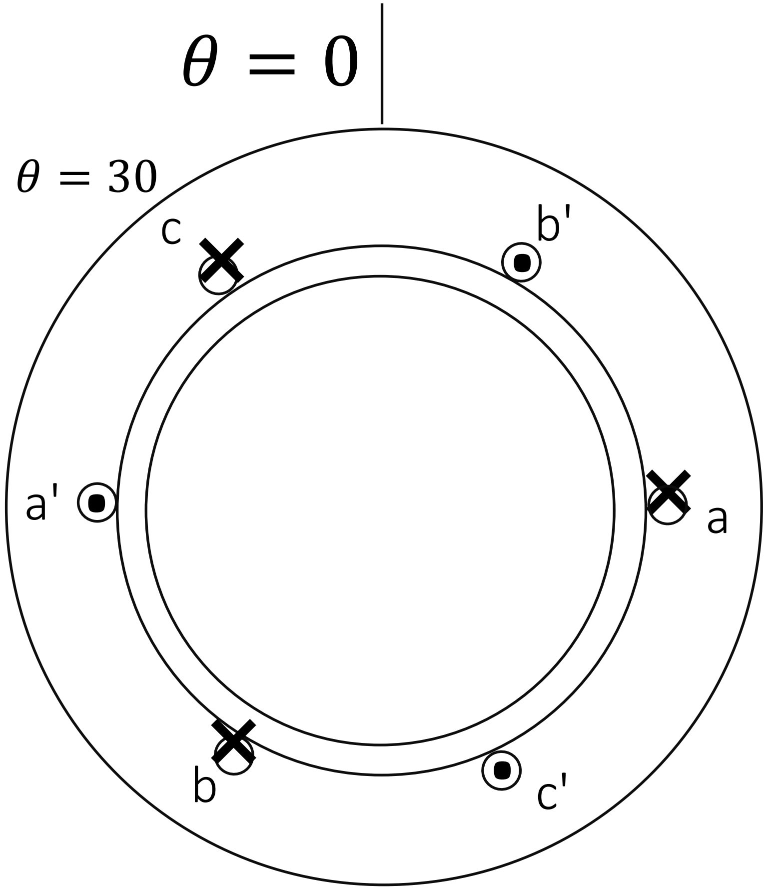
AC machines with one phase







Rotating MMF Waves in AC Machines



$$H_a = \frac{4}{\pi}\frac{k_w N_a I_a}{2g} cos\theta \qquad I_a = I_m \cos(\omega t)$$

$$\overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t)$$

$$\overrightarrow{H_b} = \frac{4}{\pi}\frac{k_w N_a I_b}{2g}\cos(\theta + 120) \qquad I_b = I_m \cos(\omega t + 120)$$

$$\overrightarrow{H_b} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t + 240)$$

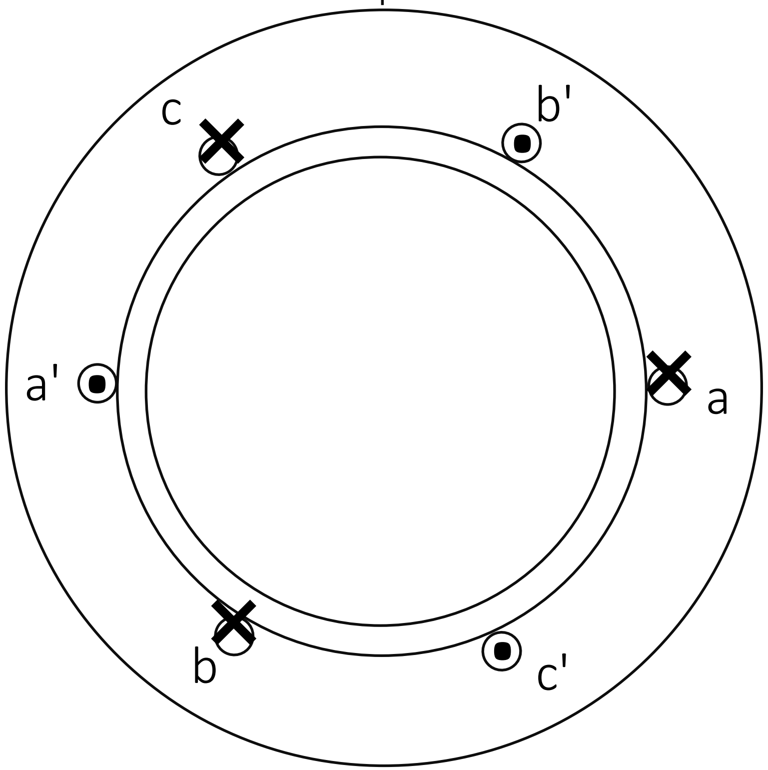
$$\overrightarrow{H_c} = \frac{4}{\pi}\frac{k_w N_a I_c}{2g}\cos(\theta - 120) \qquad I_c = I_m \cos(\omega t - 120)$$

$$\overrightarrow{H_c} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t - 240)$$

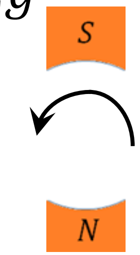
Rotating MMF Waves in AC Machines

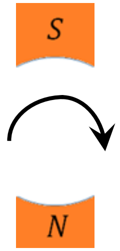
$$I_a = I_m \cos(\omega t) \qquad I_b = I_m \cos(\omega t + 120) \qquad I_c = I_m \cos(\omega t - 120)$$

$\theta = 0$

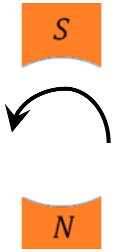


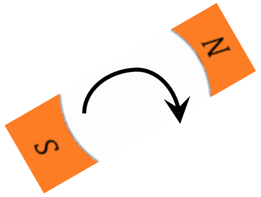
$$\overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t)$$



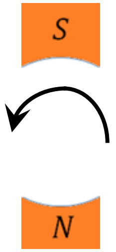


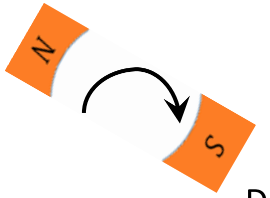
$$\overrightarrow{H_b} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t + 240)$$





$$\overrightarrow{H_c} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t - 240)$$

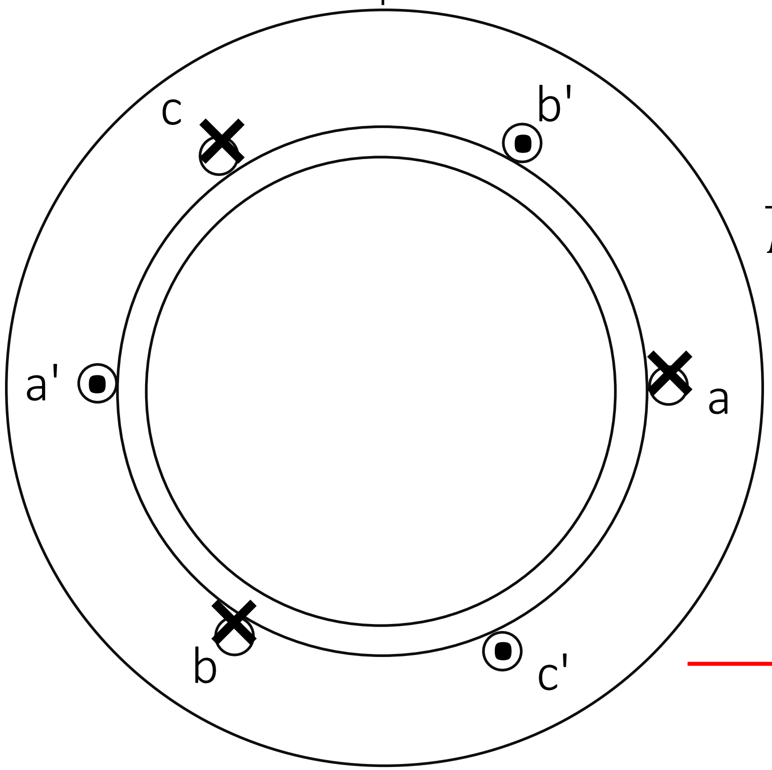




Rotating MMF Waves in AC Machines

$$I_a = I_m\cos(\omega t) \qquad I_b = I_m\cos(\omega t + 120) \qquad I_c = I_m\cos(\omega t - 120)$$

$$\theta = 0$$

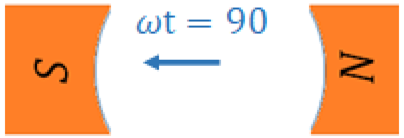


$$\overrightarrow{H_a} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t)$$

$$\overrightarrow{H_b} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t + 240)$$

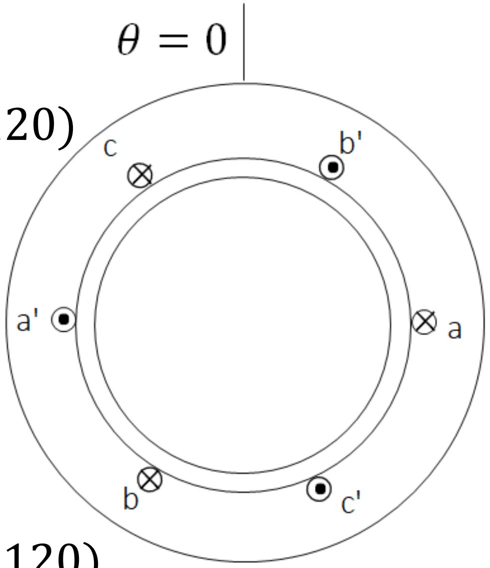
$$\overrightarrow{H_c} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta - \omega t) + \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{1}{2}\cos(\theta + \omega t - 240)$$

$$\overrightarrow{H_s} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{3}{2}\cos(\theta - \omega t)$$



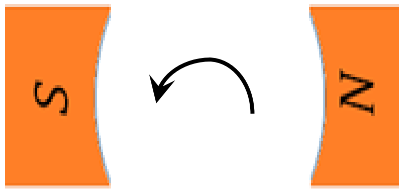
Rotating MMF Waves in AC Machines

$$I_c = I_m \cos(\omega t - 120)$$



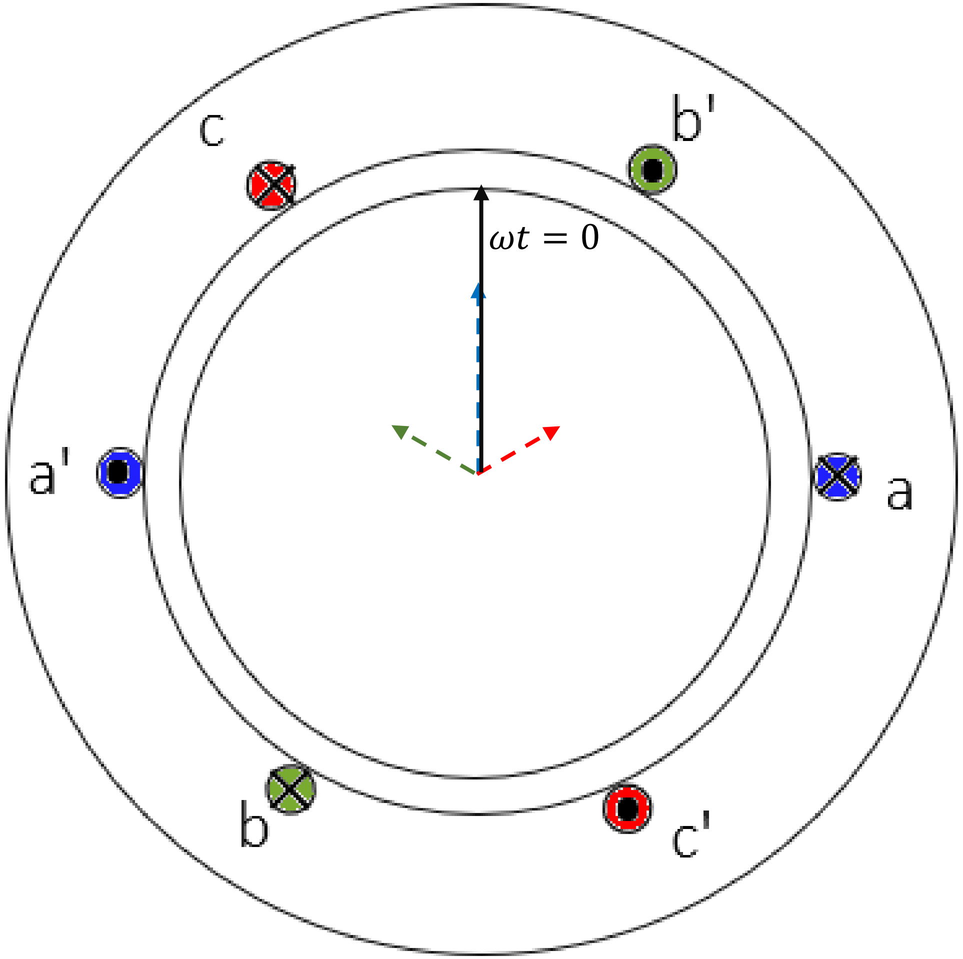
$$I_a = I_m \cos(\omega t)$$

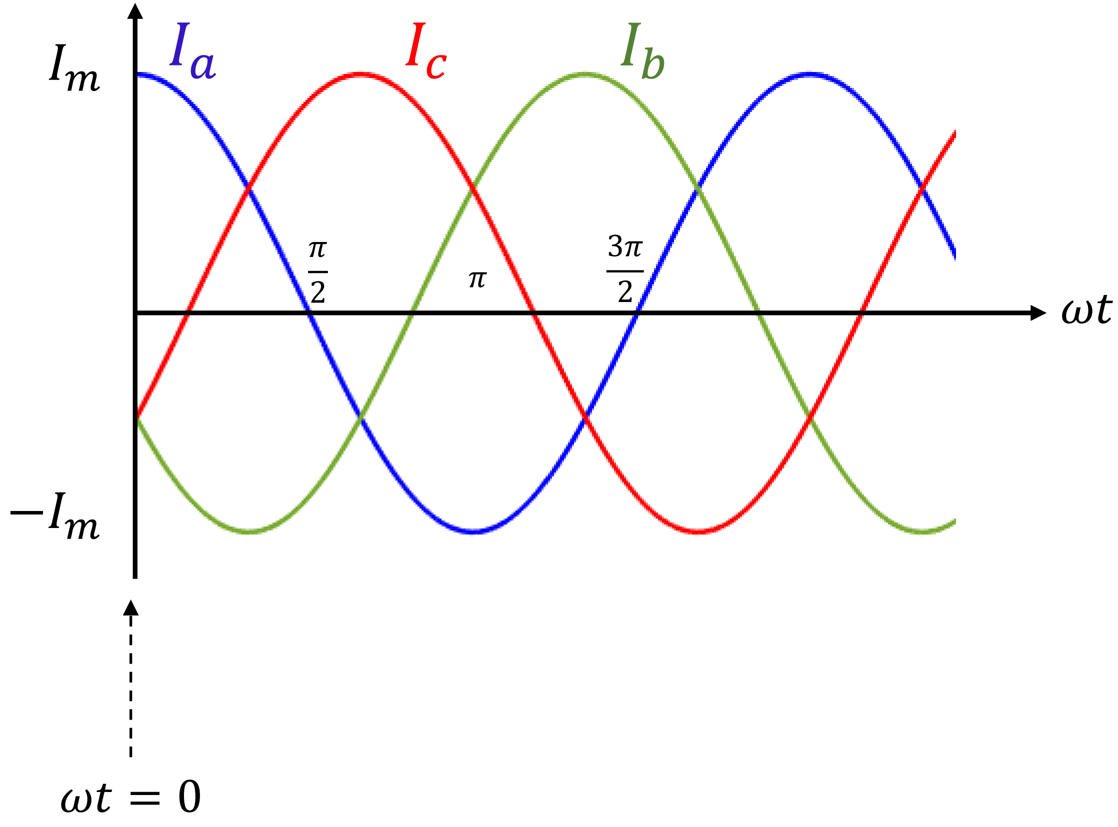
$$I_b = I_m \cos(\omega t + 120)$$



Rotating MMF Waves in AC Machines

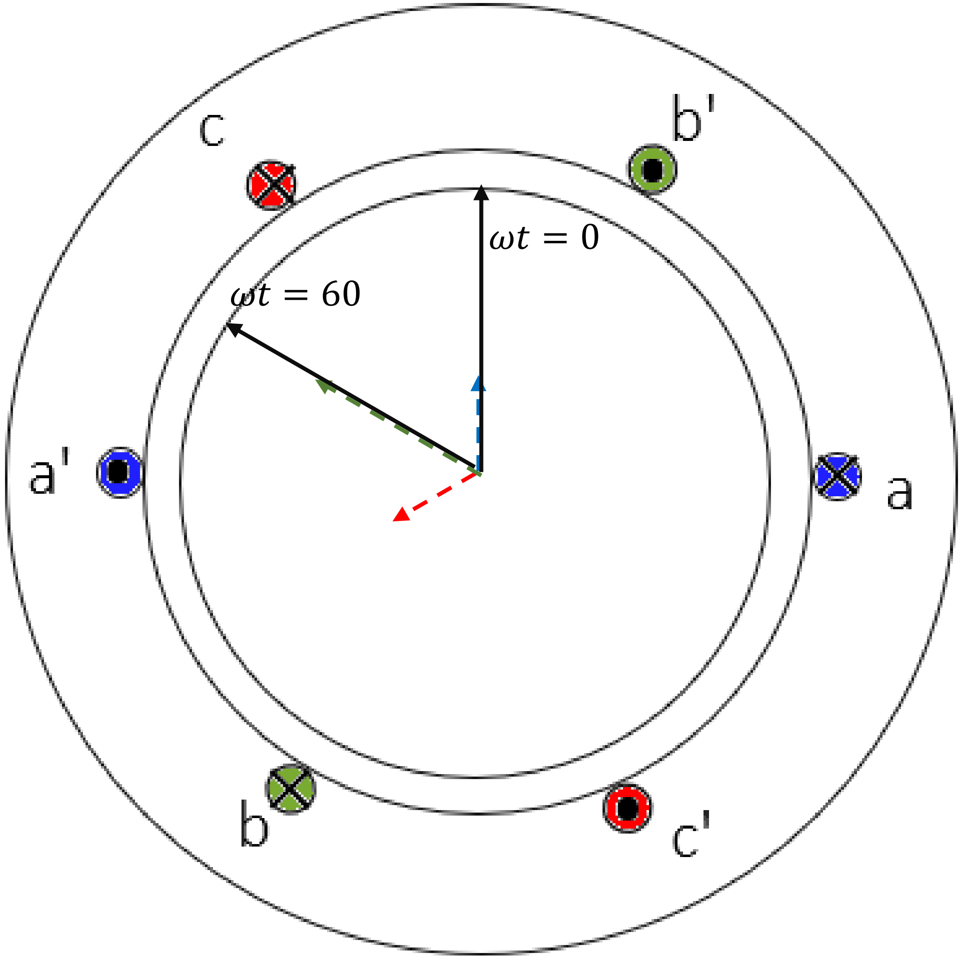
$I_a = I_m \cos(\omega t)$ $I_b = I_m \cos(\omega t + 120)$ $I_c = I_m \cos(\omega t - 120)$

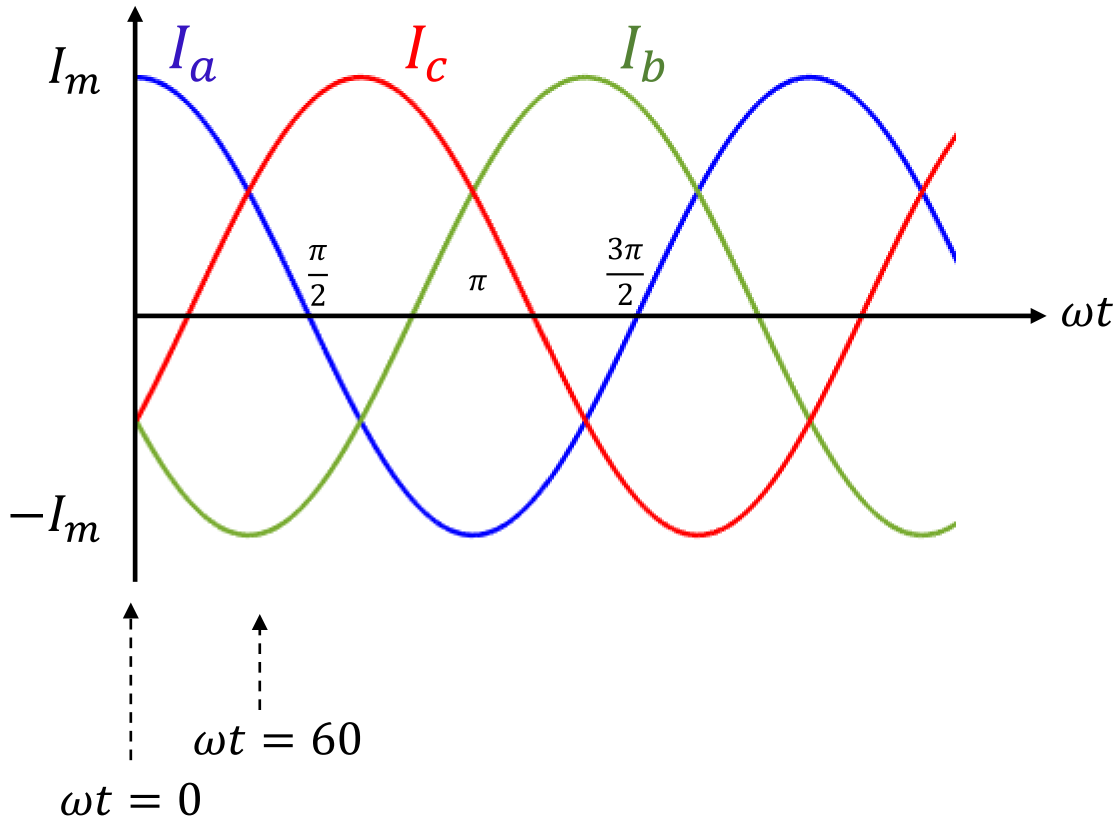




Rotating MMF Waves in AC Machines

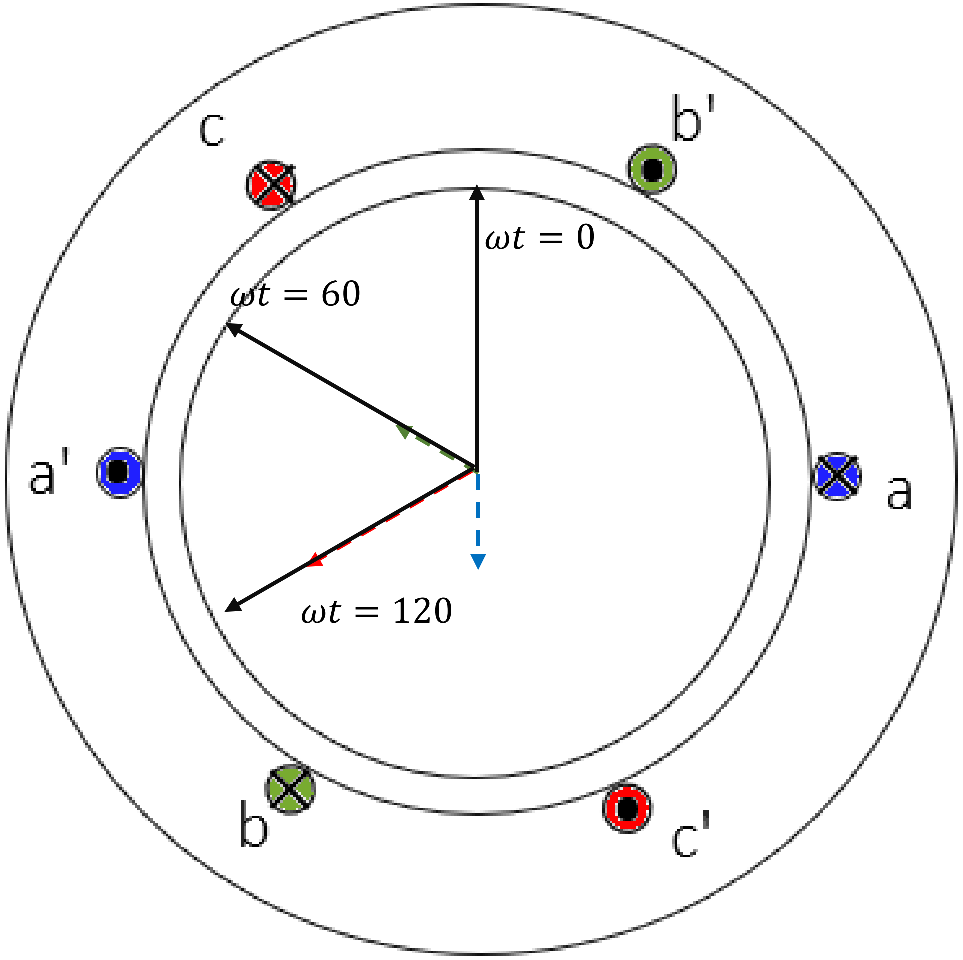
$I_a = I_m \cos(\omega t)$ $\quad$ $I_b = I_m \cos(\omega t + 120)$ $\quad$ $I_c = I_m \cos(\omega t - 120)$

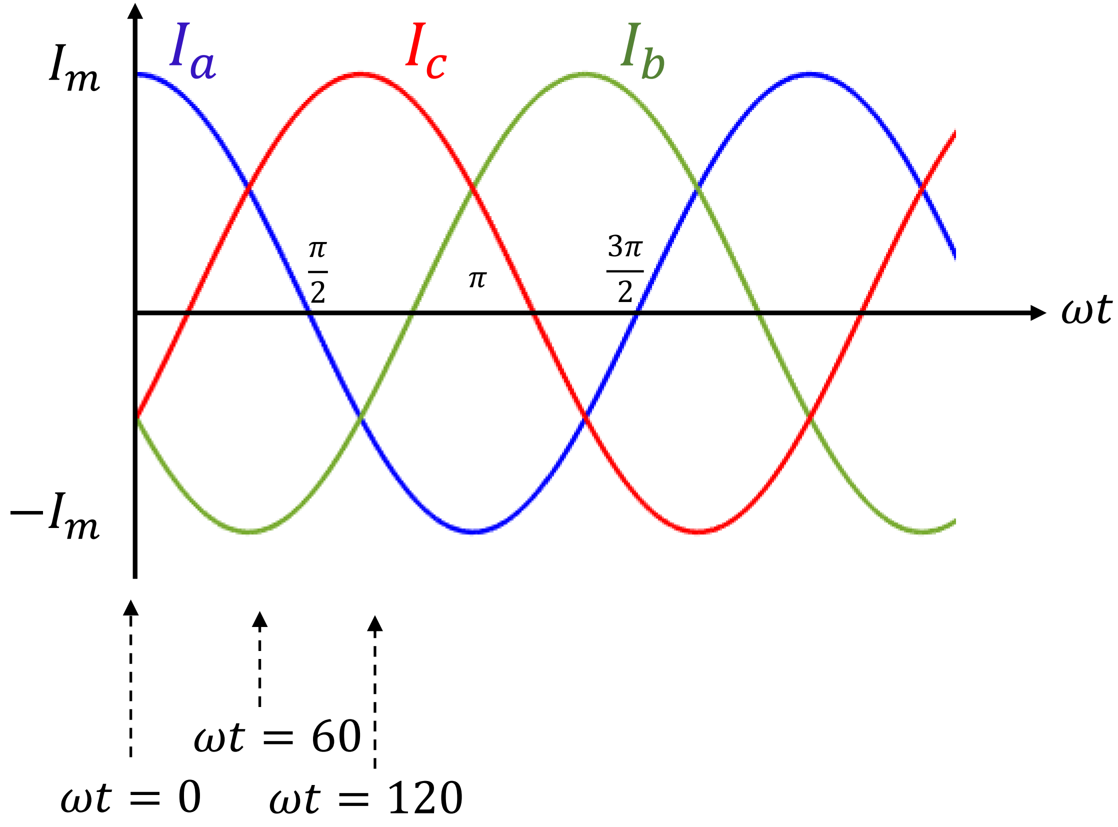




Rotating MMF Waves in AC Machines

$I_a = I_m \cos(\omega t)$ $\quad$ $I_b = I_m \cos(\omega t + 120)$ $\quad$ $I_c = I_m \cos(\omega t - 120)$





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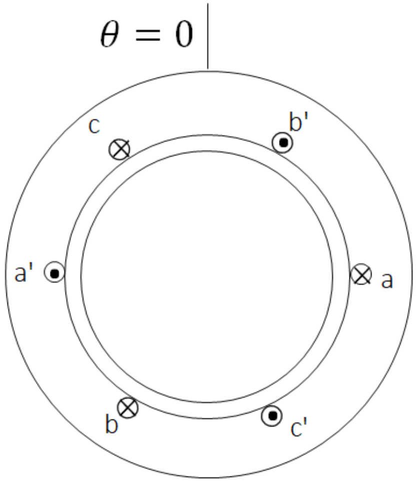
Torque determination in cylindrical AC Machine (Electromagnetic view point)

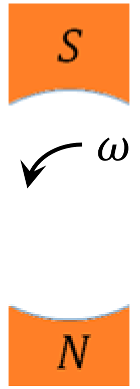


$$I_a = I_m \cos(\omega t)$$

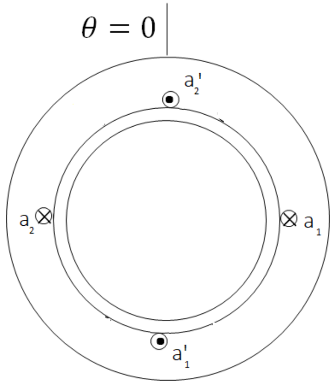
$$I_b = I_m \cos(\omega t + 120)$$

$$I_c = I_m \cos(\omega t - 120)$$

$$\overrightarrow{H_s} = \frac{4}{\pi} \frac{k_w N_a}{2g} I_m \frac{3}{2} \cos(\theta - \omega t)$$

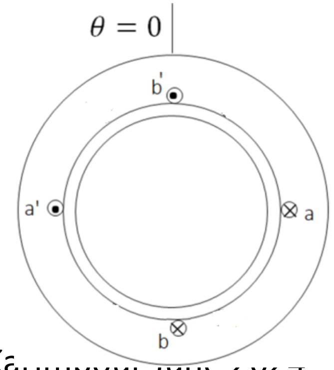


What does happen if we change two currents?



What does happen if we have a three-phase four poles machine?

What does happen if we have a two-phase two poles machine?



What does happen if we apply a capacitance in phase b and then use same voltage for both phase.

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        - Torque Production Condition
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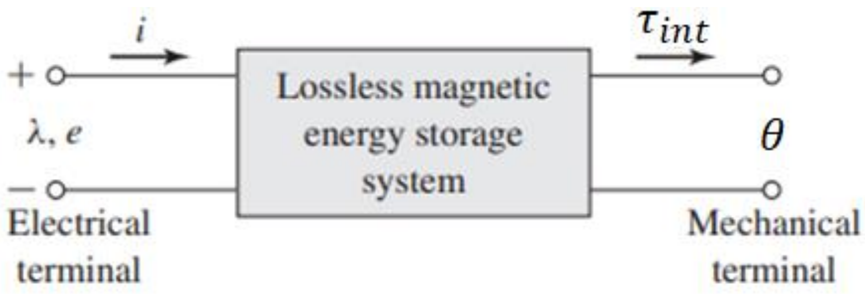
Electromechanical Energy Conversion Principal



$$W_{fld} = \frac{\lambda^2}{2L(\theta)}$$

$$W_{fld} = \int_V \left(\frac{B^2}{2\mu}\right) dV$$

$$\tau_{int} = -\frac{\partial w_{fld}(B,\theta)}{\partial \theta}$$

$$\tau_{int} = -\frac{\partial w_{fld}(\lambda,\theta)}{\partial \theta}$$

$$W'_{fld} = \frac{1}{2}L(\theta)i^2$$

$$W'_{fld} = \int_V \frac{1}{2}\mu H^2 dV$$

$$\tau_{int} = \frac{\partial w'_{fld}(H,\theta)}{\partial \theta}$$

Magnetic field view point

$$\tau_{int} = \frac{\partial w'_{fld}(i,\theta)}{\partial \theta}$$

Coupled circuit view point

Torque determination in AC Machines

- Magnetic field view point (Nonsalient-Pole(Cylindrical) Machines)
- Couple circuit view point (Nonsalient-Pole(Cylindrical) and Salient-Pole Machines)

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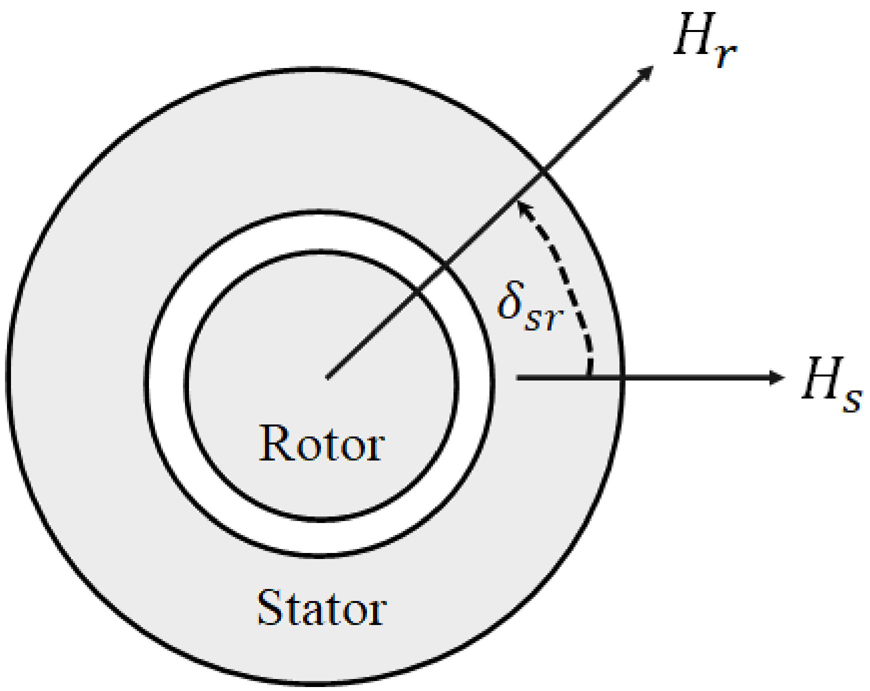
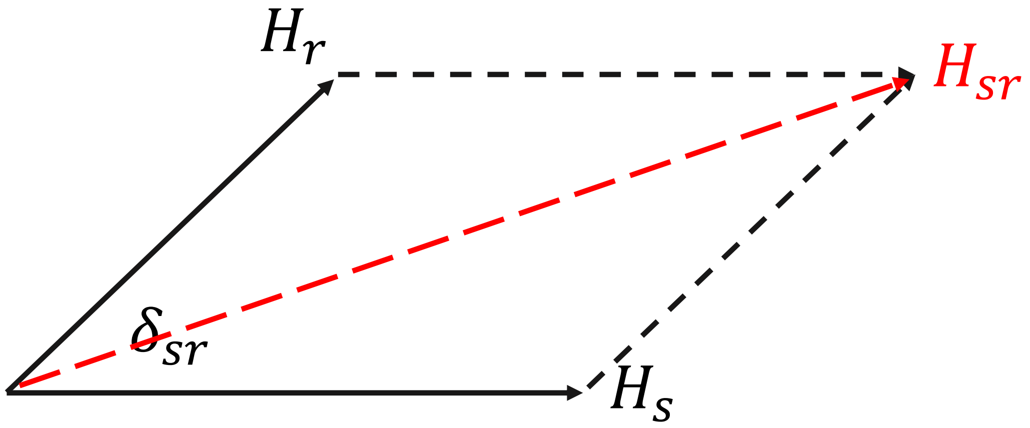
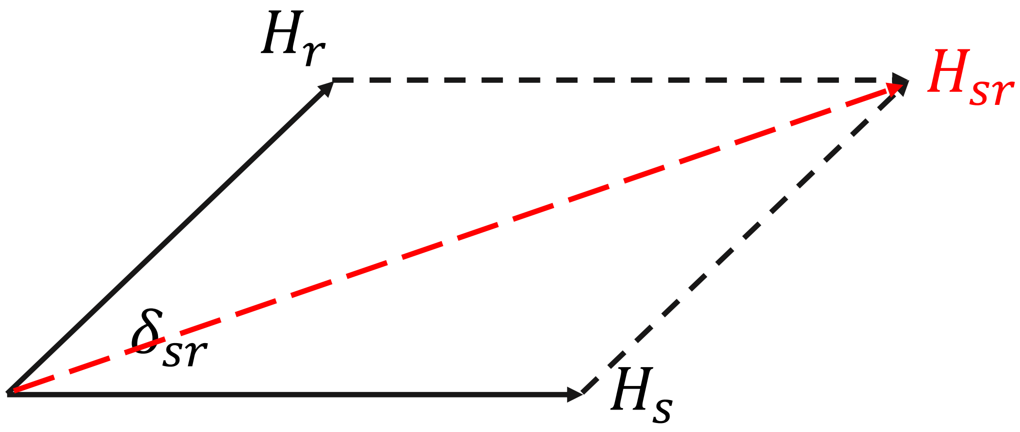
Torque determination in electrical Machines (Magnetic field view point)

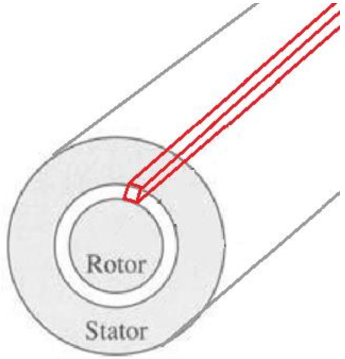




Torque determination in electrical Machines (Magnetic field view point)



$$|H_{sr}|^2 = |H_s|^2 + |H_r|^2 + 2|H_s||H_r|cos\delta_{sr}$$



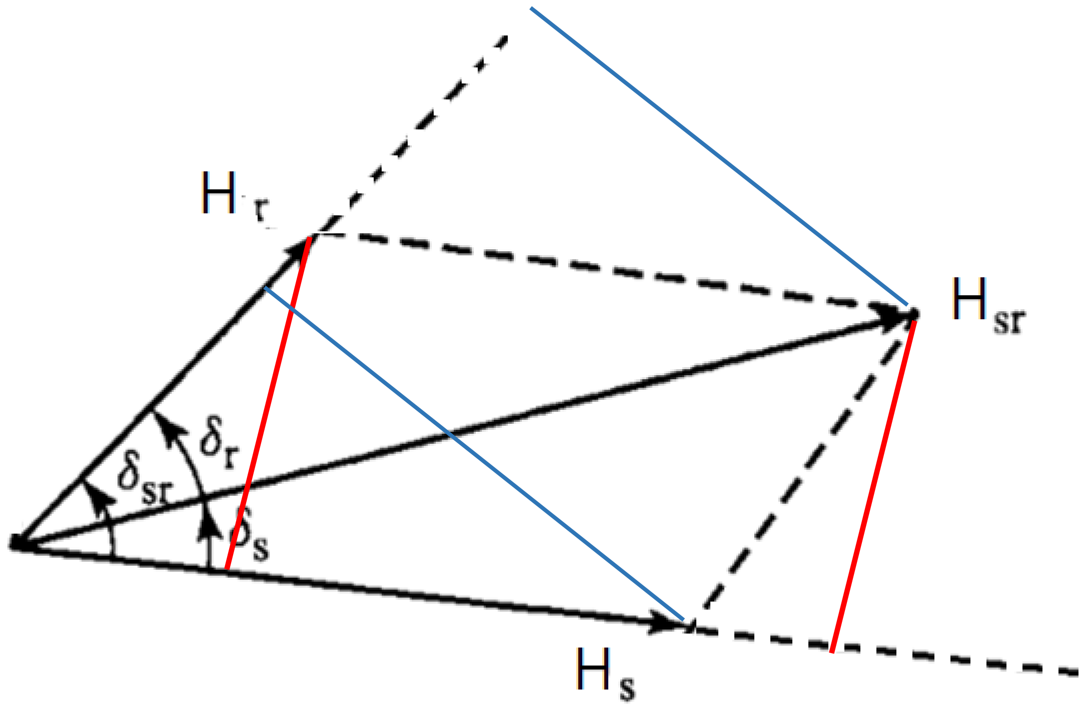
$$\omega'_{fld} = \int_v \frac{1}{2}\mu_0\,|H_{sr}|^2 sin^2(\theta)dv$$

$$\omega'_{fld} = \int_0^{2\pi} \frac{1}{2}\mu_0\,|H_{sr}|^2 sin^2(\theta)lgrd\theta = \frac{\mu_0\pi grl}{2}|H_{sr}|^2$$

$$\omega'_{fld} = \frac{\mu_0\pi grl}{2}|H_{sr}|^2 = \frac{\mu_0\pi grl}{2}\{|H_s|^2 + |H_r|^2 + 2|H_s||H_r|cos\delta_{sr}\}$$

$$\tau = \frac{\partial\omega'_{fld}}{\partial\delta_{sr}} \qquad \tau = -\mu_0\pi grl|H_s||H_r|sin\delta_{sr}$$

Torque determination in electrical Machines (Magnetic field view point)



$$H_s = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{3}{2} \qquad H_r = \frac{4}{\pi}\frac{k_{wr} N_r}{2g} I_r$$

$$\tau = -\mu_0 \pi g r l |H_s||H_r| sin\delta_{sr}$$

$$\tau = -\mu_0 \pi g r l |H_s||H_r| sin\delta_{sr} \qquad \tau = -\mu_0 \pi g r l |H_s||H_{sr}| sin\delta_s \qquad \tau = -\mu_0 \pi g r l |H_r||H_{sr}| sin\delta_r$$

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Electrical Machine III Syllabus

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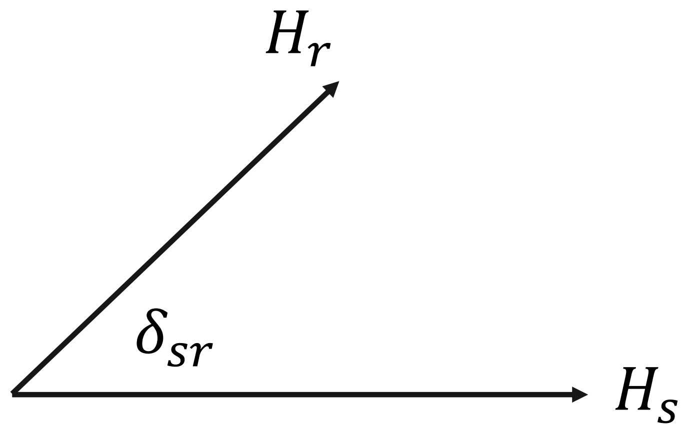
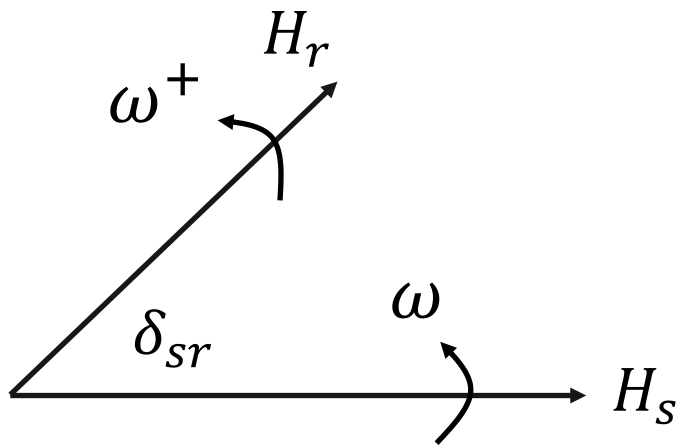
Torque Production Condition



$$\tau = -\mu_0 \pi g r l |H_s||H_r| sin\delta_{sr}$$

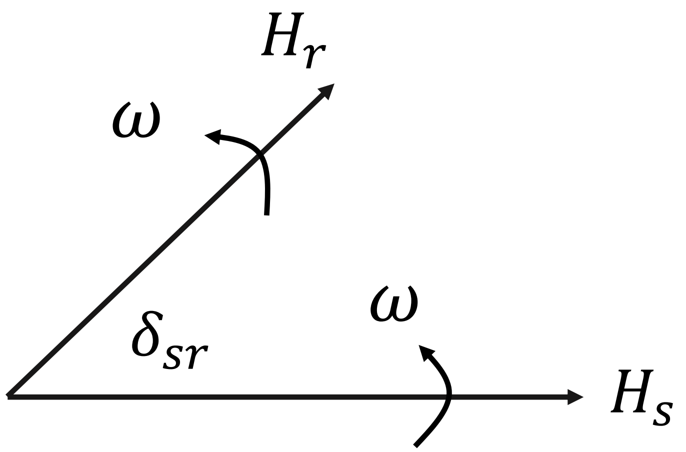
Now let suppose that H_s rotates at speed of ω and H_r rotates at speed of ω^+.



$$\tau = -\mu_0 \pi g r l |H_s||H_r| \sin(\dots)$$

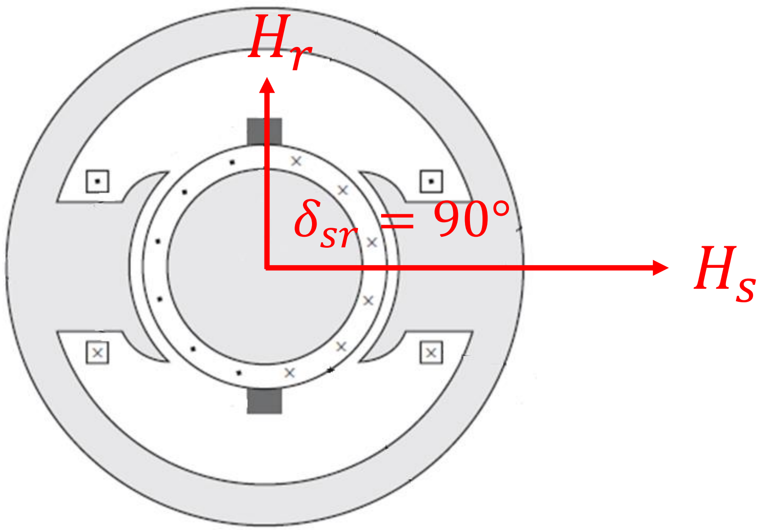
Electric machines **can produce torque at steady state** if ………………………………

Torque Production Condition

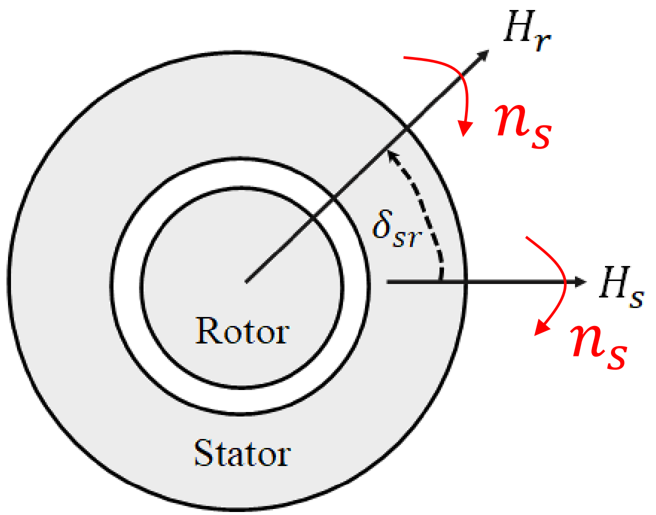


$$\tau = -\mu_0 \pi g r l |H_s||H_r| sin\delta_{sr}$$

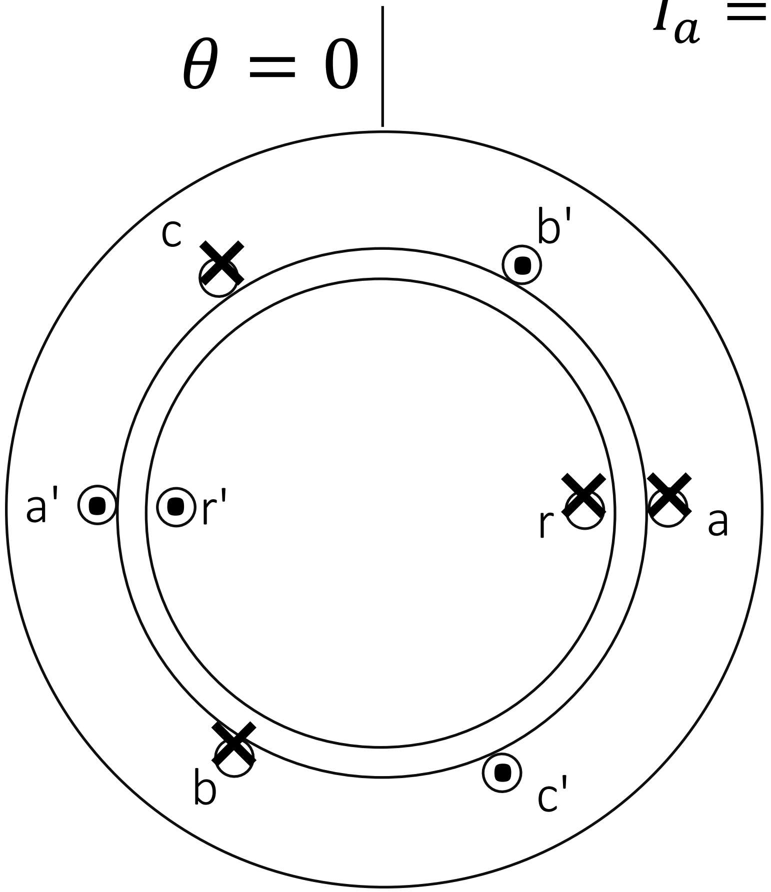
DC Machines:



AC Machines ??



Torque Production Condition

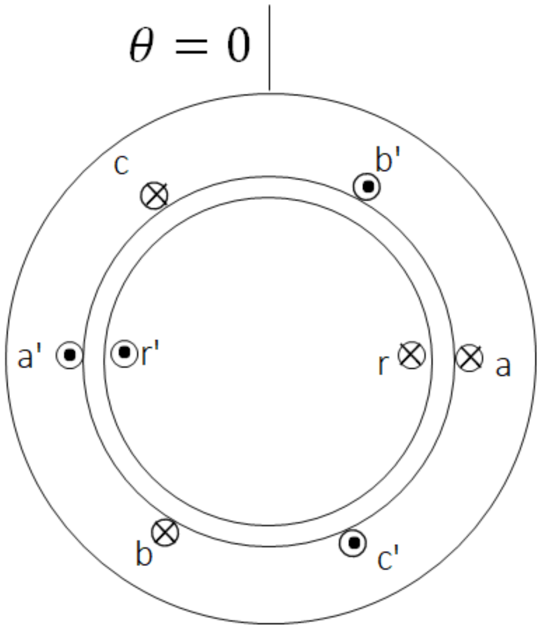


$$I_a = I_m \cos(\omega t) \qquad I_b = I_m \cos(\omega t + 120) \qquad I_c = I_m \cos(\omega t - 120)$$

$$\overrightarrow{H_s} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{3}{2}\cos(\theta - \omega t)$$

$$\overrightarrow{H_r} = \frac{4}{\pi}\frac{k_{wr} N_r}{2g} I_r \cos(\theta)$$

Torque Production Condition

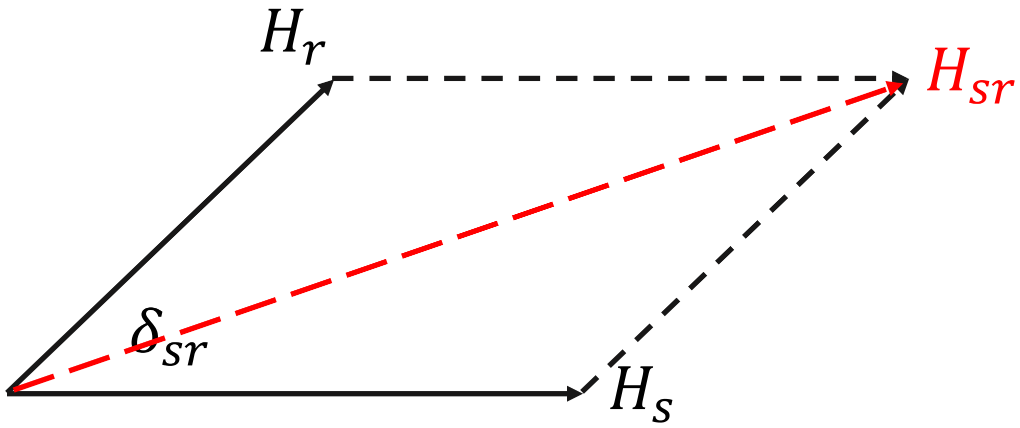


$$I_a = I_m\cos(\omega t) \qquad I_b = I_m\cos(\omega t + 120) \quad I_c = I_m\cos(\omega t - 120)$$

$$\overrightarrow{H_s} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{3}{2}\cos(\theta - \omega t)$$

ω

$$\overrightarrow{H_r} = \frac{4}{\pi}\frac{k_{wr} N_r}{2g} I_r \cos(\theta) \qquad I_r = cte.$$

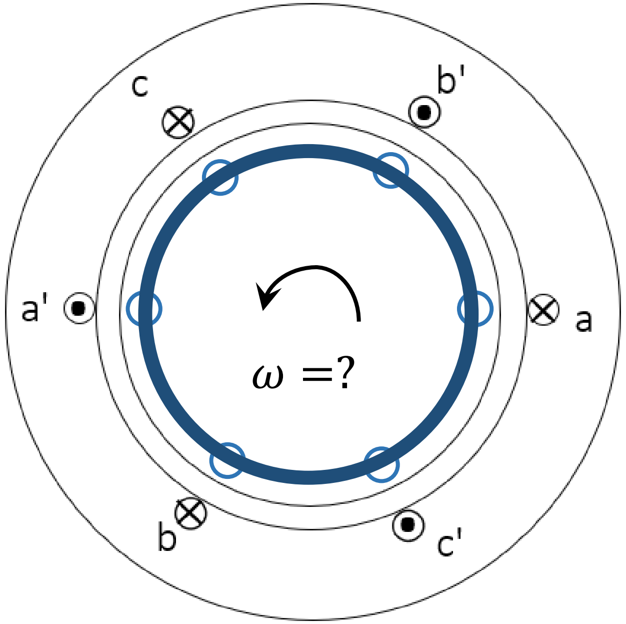


$$\tau = -\frac{\mu_0 \pi g D l}{2} |H_s||H_r| sin\delta_{sr}$$

Synchronous machine

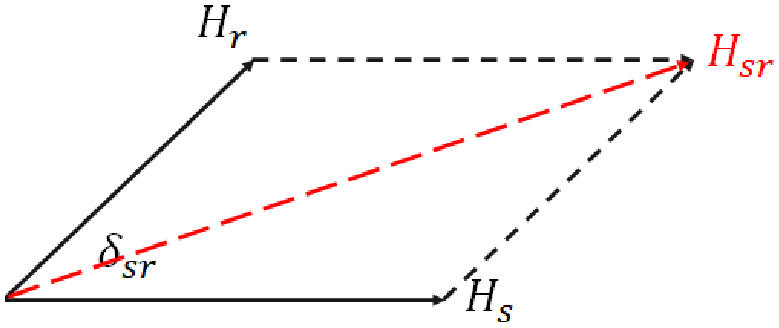
Torque Production Condition

$I_a = I_m \cos(\omega t)$ $\qquad$ $I_b = I_m \cos(\omega t + 120)$ $\qquad$ $I_c = I_m \cos(\omega t - 120)$



$$\overrightarrow{H_s} = \frac{4}{\pi}\frac{k_w N_a}{2g} I_m \frac{3}{2}\cos(\theta - \omega t)$$

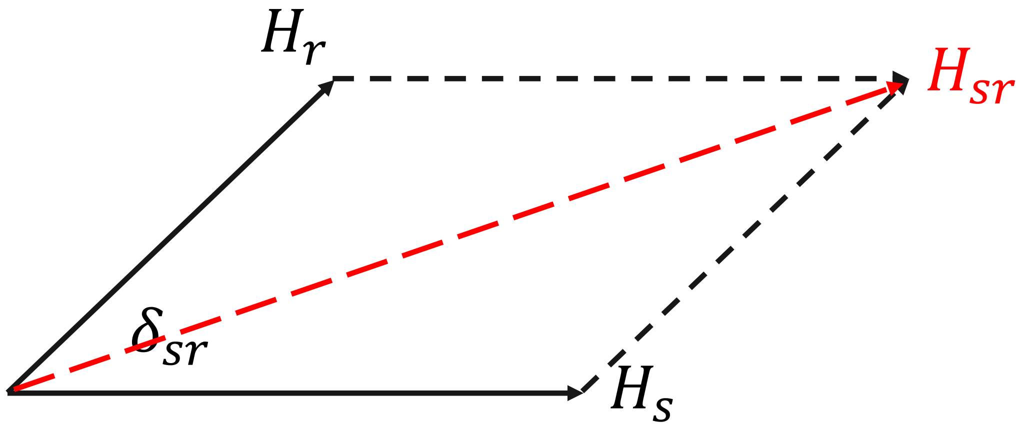
ω



$$\tau = -\frac{\mu_0 \pi g D l}{2}|H_s||H_r| sin\delta_{sr}$$

Asynchronous machine/Induction machine

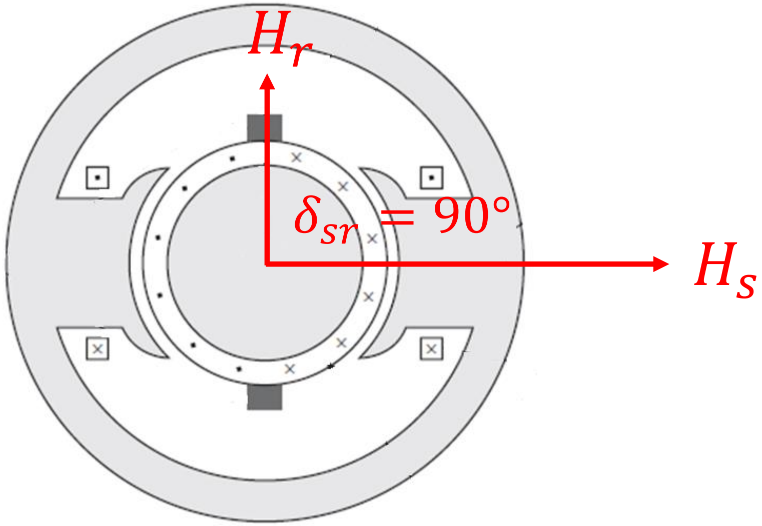
Torque Production Condition

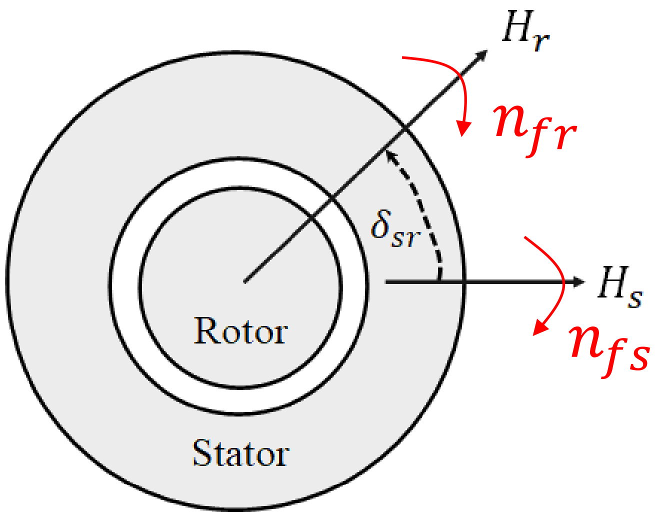


$$\tau = -\frac{\mu_0 \pi g D l}{2} |H_s||H_r| sin\delta_{sr}$$

DC Machines:

AC Machines ??



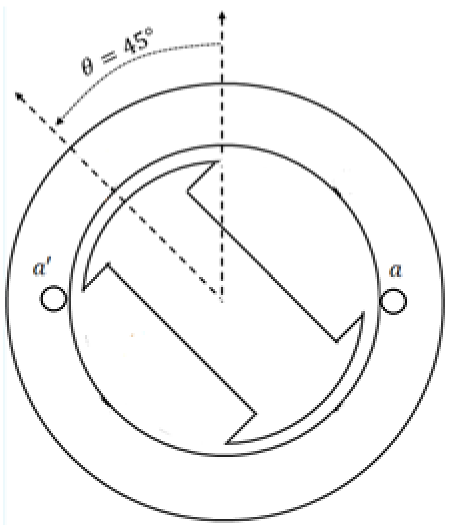


Torque Production Condition

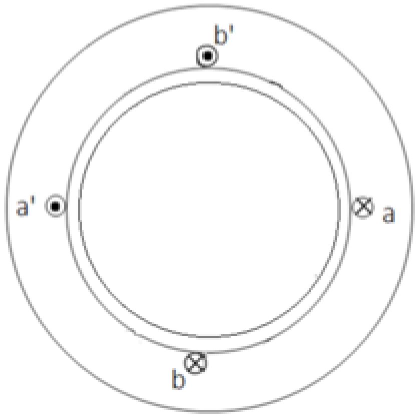
Example 1 (Final Exam, December 2022): In the figure below, θ is the angle between the stator coil axis and the rotor axis. The coil a carries a constant current of 20.

Guidance: To solve this problem, you should rely on physical understanding; mathematical relationships are not required. Determine the direction of the torque in terms of clockwise (cw) or counterclockwise (ccw).

a) For θ = 90°, determine the existence and direction of the torque.
b) For θ = 0°, determine the existence and direction of the torque.
a) For θ = 45°, determine the existence and direction of the torque.
b) For θ = -45°, determine the existence and direction of the torque.

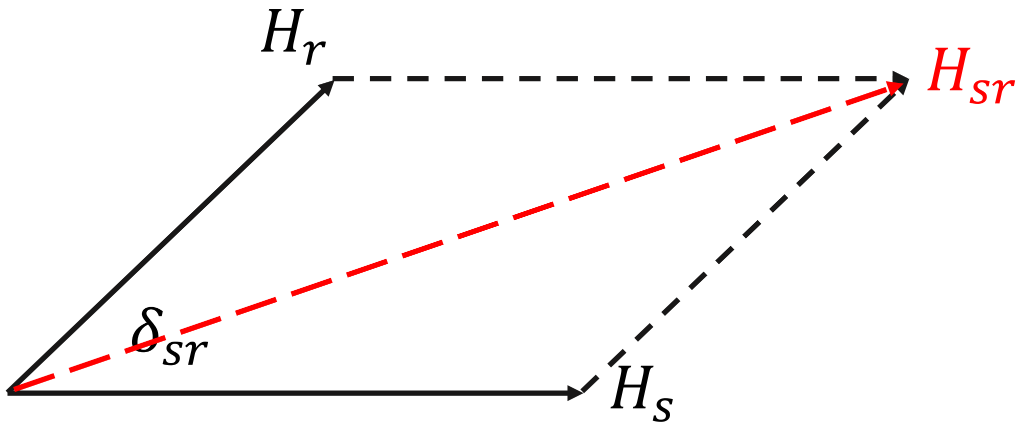


Example 2 (Final Exam, January 2023): Consider the diagram below. Assume that the positive current enters from the side marked with a cross and exits from the other side.
Given the applied currents, comment on the speed and direction of the rotating magnetic field in the stator.



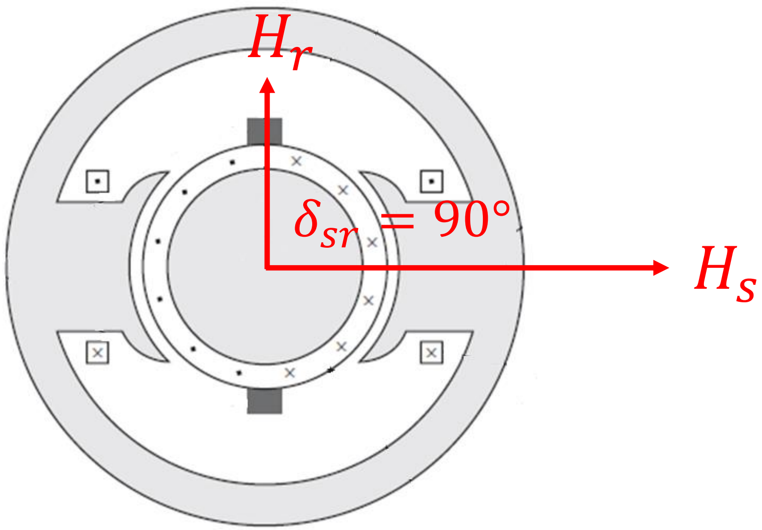
$$i_a = 5\sin(100\pi t + 33)\,, i_b = 5\sin(100\pi t + 123)$$

Torque Production Condition

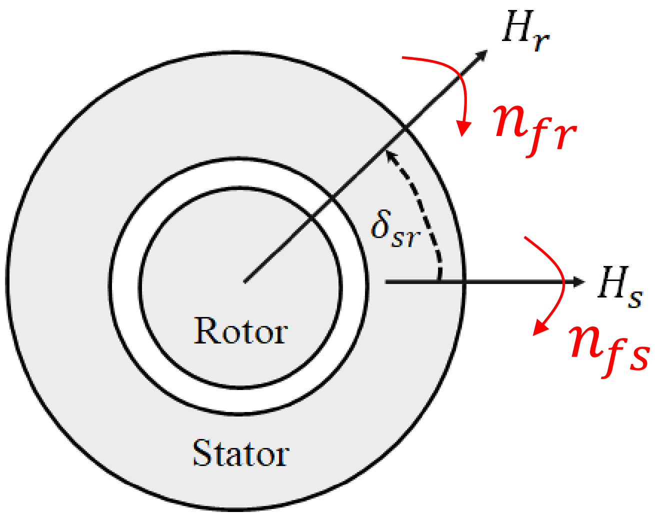


$$\tau = -\frac{\mu_0 \pi g D l}{2} |H_s||H_r| sin\delta_{sr}$$

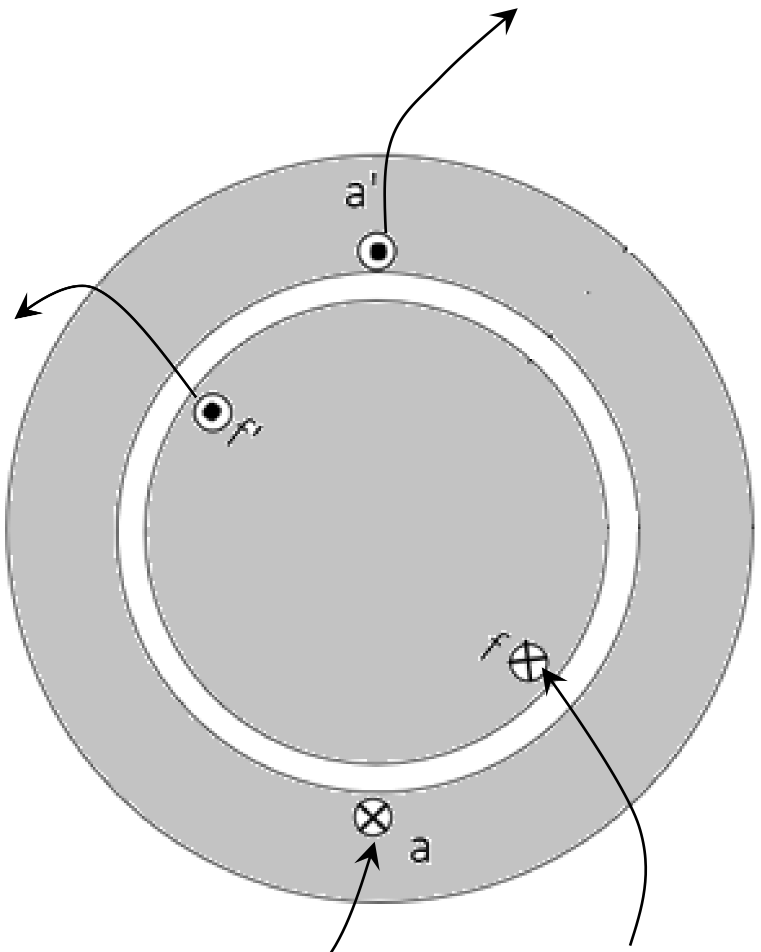
DC Machines:



AC Machines ??



Torque Production Condition

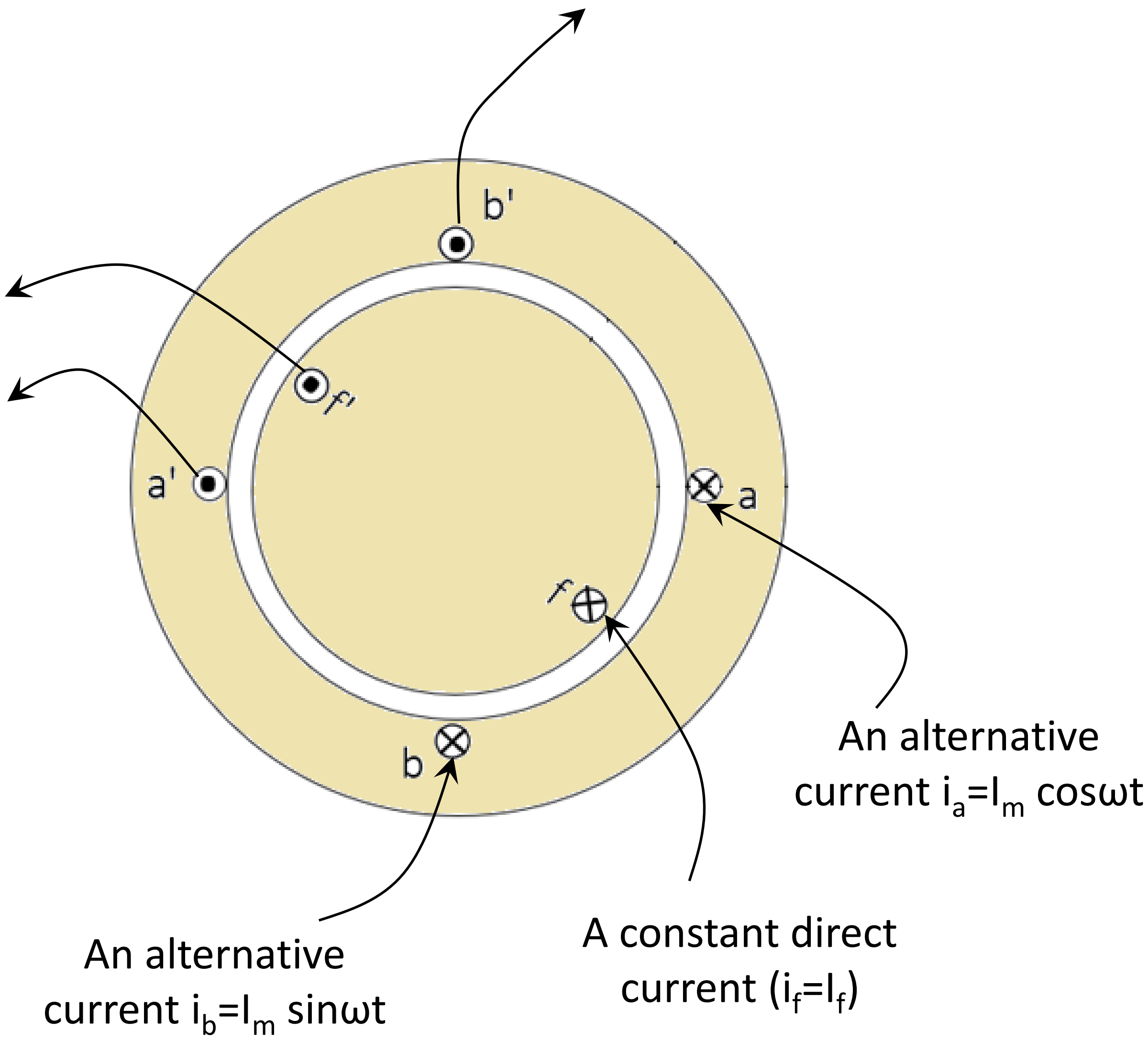


An alternative current $i_a=I_m \sin\omega t$

A constant direct current ($i_f=I_f$)

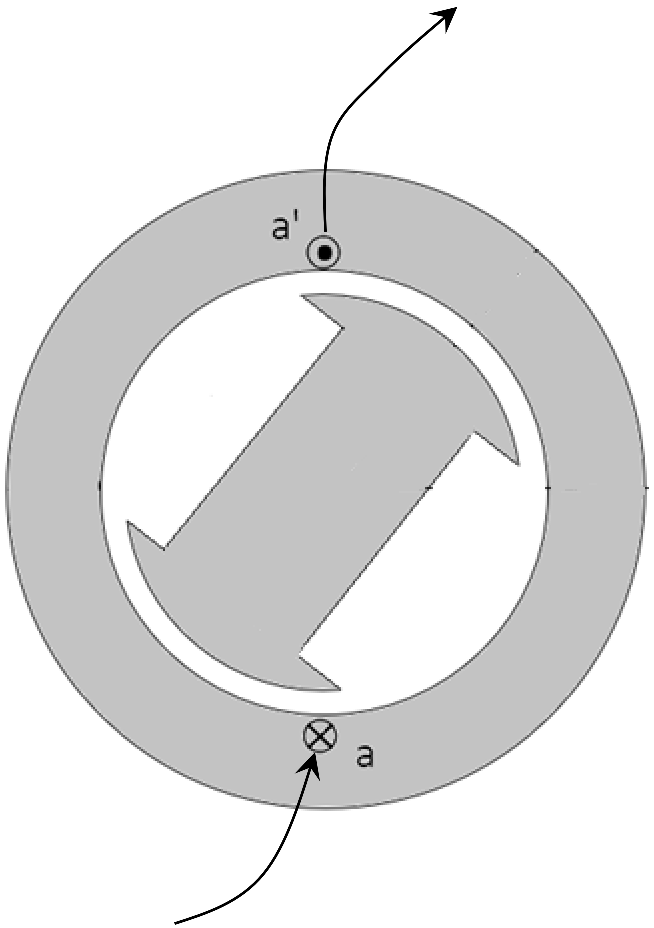
The motor can just work at the speed of …... in ….. or …...

Torque Production Condition



The motor can just work at the speed of …... in …..

Torque Production Condition



An alternative current $i_a = I_m \sin\omega t$

The motor can just work at the speed of …... in ….. or …...

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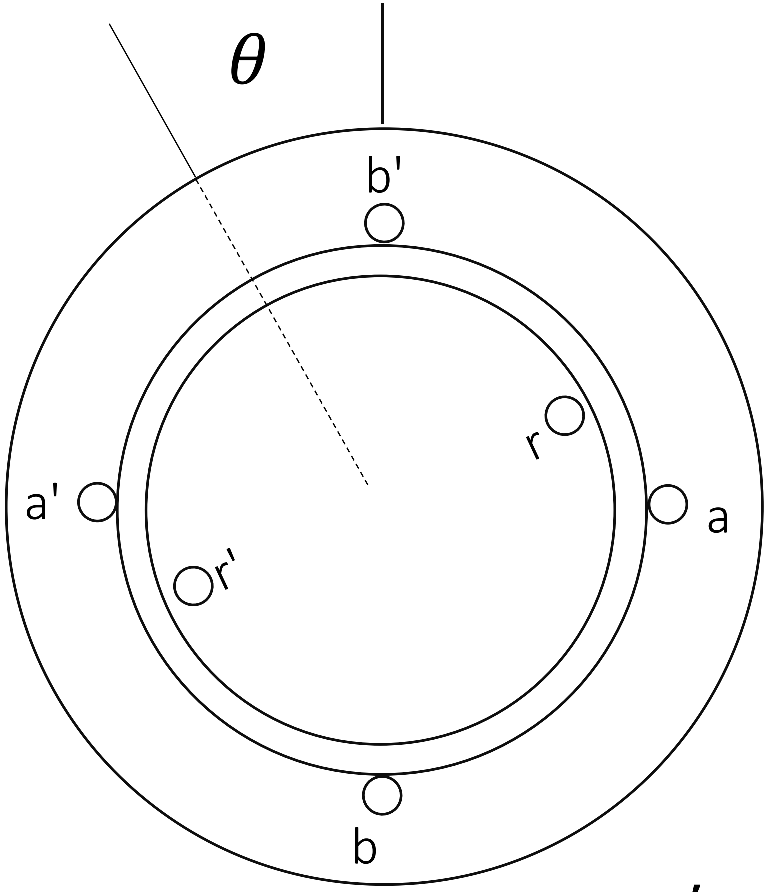
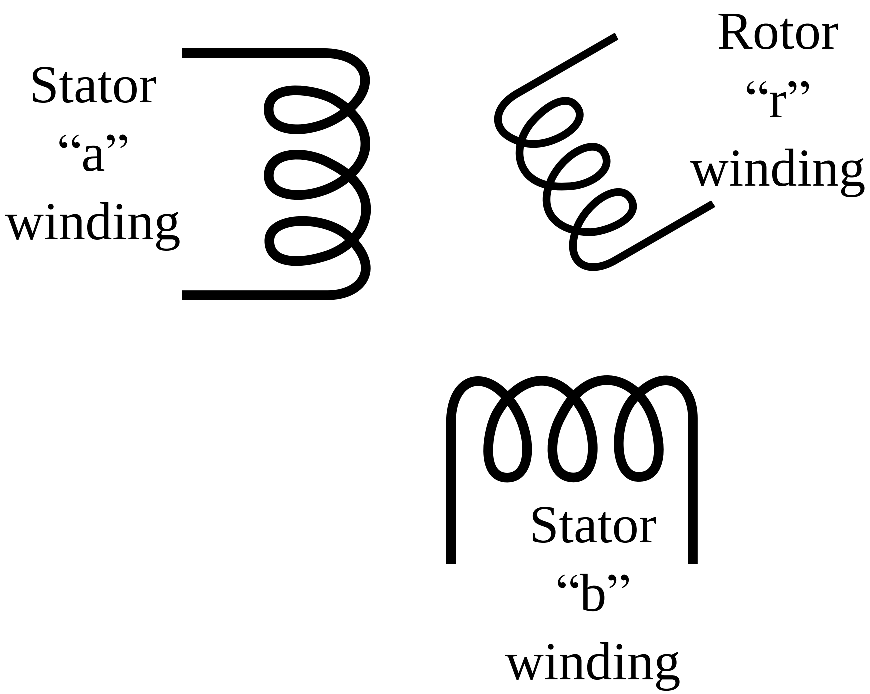
Torque determination in cylindrical two phases AC Machine (Coupled circuit view point)

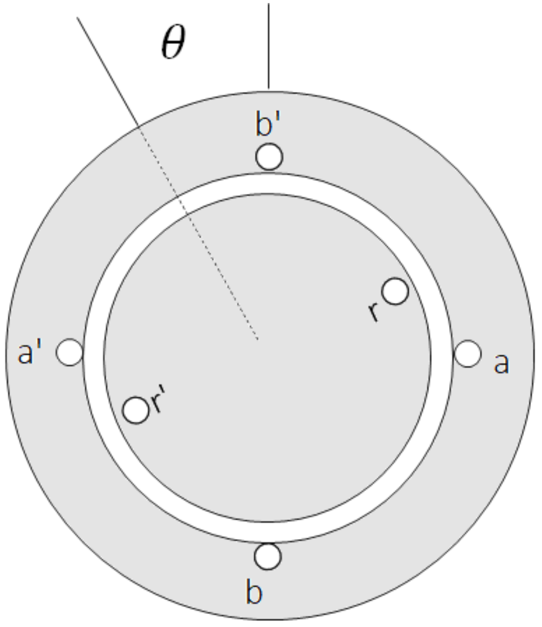




$$\omega'_{fld} = \frac{1}{2} L_{aa} i_a^2 + \frac{1}{2} L_{bb} i_b^2 + \frac{1}{2} L_{rr} i_r^2 + L_{ab} i_a i_b + L_{ar} i_a i_r + L_{br} i_b i_r$$

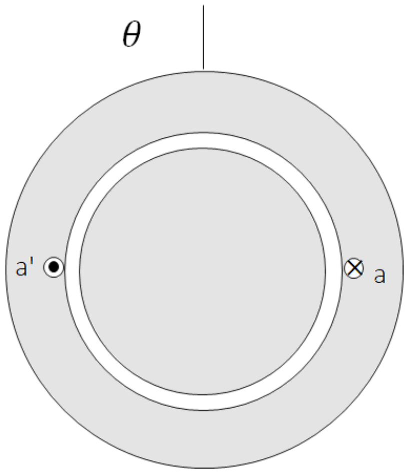
$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta}$$

Torque determination in cylindrical two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2} L_{aa} i_a^2 + \frac{1}{2} L_{bb} i_b^2 + \frac{1}{2} L_{rr} i_r^2 + L_{ab} i_a i_b + L_{ar} i_a i_r + L_{br} i_b i_r$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ar} \\ L_{ba} & L_{bb} & L_{br} \\ L_{ra} & L_{rb} & L_{rr} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$



$$L_{aa} = \frac{\lambda_a}{i_a}\Big|_{i_b = i_r = \circ} \qquad \lambda_a = k_\omega N_a \varphi_a$$

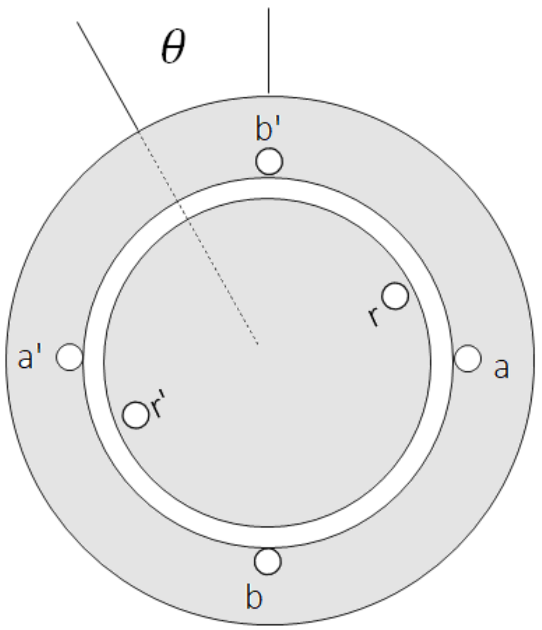
$$B_a = \frac{4}{\pi} \frac{k_\omega N_a i_a}{2g} \mu_0 cos\theta \qquad \varphi_a = \int B_a dA \qquad \varphi_a = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{4}{\pi} \frac{k_\omega N_a i_a}{2g} \mu_0 cos\theta r l d\theta$$

$$L_{aa} = \underbrace{\frac{4k_\omega^2 N_a^2}{\pi g} \mu_0 r l}_{L_{aa0}}$$

$$L_{aa} = L_{bb} = L_{aa0} + l_{al}$$

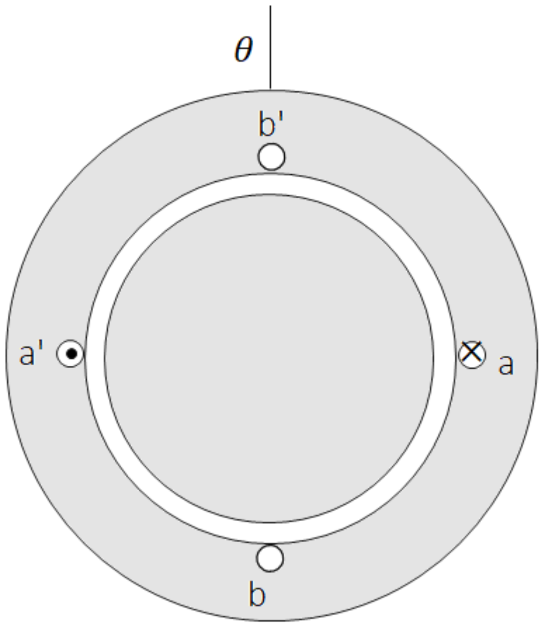
$$L_{rr} = \frac{4k_{\omega r}^2 N_r^2}{\pi g} \mu_0 r l + l_{rl}$$

Torque determination in cylindrical two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2} L_{aa} i_a^2 + \frac{1}{2} L_{bb} i_b^2 + \frac{1}{2} L_{rr} i_r^2 + L_{ab} i_a i_b + L_{ar} i_a i_r + L_{br} i_b i_r$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ar} \\ L_{ba} & L_{bb} & L_{br} \\ L_{ra} & L_{rb} & L_{rr} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$



$$L_{ba} = L_{ab} = \frac{\lambda_b}{i_a}\Big|_{i_b = i_r = 0} \qquad \lambda_b = k_\omega N_b . 0$$

$$L_{ab} = L_{ba} = 0$$

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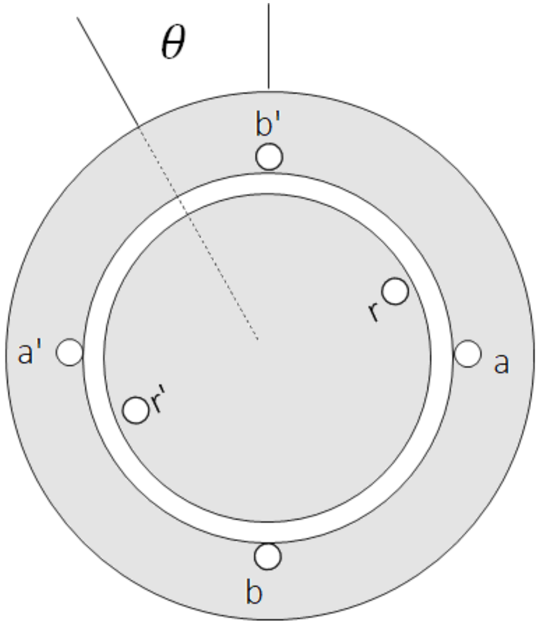
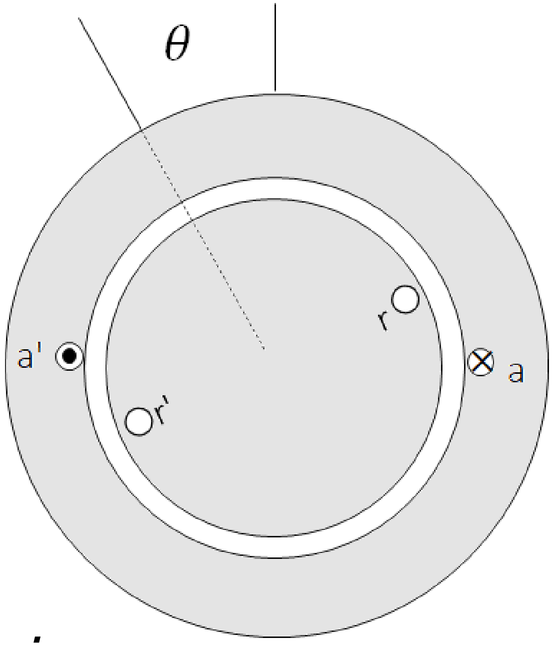
Torque determination in cylindrical two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_ai_b + L_{ar}i_ai_r + L_{br}i_bi_r$$

$$\begin{bmatrix}\lambda_a\\ \lambda_b\\ \lambda_r\end{bmatrix} = \begin{bmatrix}L_{aa} & L_{ab} & L_{ar}\\ L_{ba} & L_{bb} & L_{br}\\ L_{ra} & L_{rb} & L_{rr}\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_r\end{bmatrix}$$

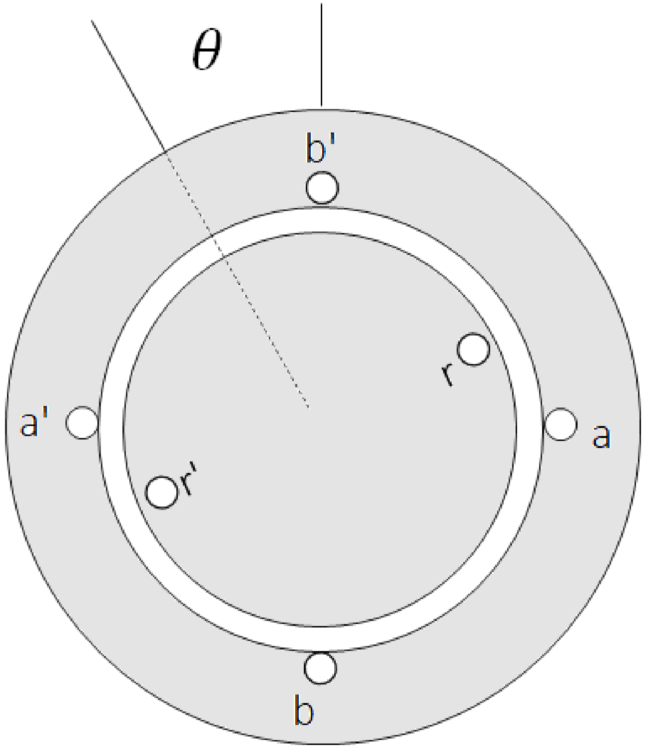


$$L_{ra} = L_{ar} = \frac{\lambda_r}{i_a}\Big|_{i_b = i_r = \circ} \qquad \lambda_r = k_{\omega r}N_r\varphi_a cos\theta$$

$$B_a = \frac{4}{\pi}\frac{k_\omega N_a i_a}{2g}\mu_0 cos\theta \qquad \varphi_a = \int B_a dA \qquad \varphi_a = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \frac{4}{\pi}\frac{k_\omega N_a i_a}{2g}\mu_0 cos\theta r l d\theta$$

$$\varphi_a = \frac{4k_\omega N_a}{\pi g}\mu_0 r l i_a \qquad \boxed{L_{ra} = L_{ar} = \underbrace{\frac{4k_{\omega r}N_r k_\omega N_a}{\pi g}\mu_0 r l}_{M} cos\theta} \qquad L_{rb} = L_{br} = Mcos(\theta + 90)$$

Torque determination in cylindrical two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_a i_b + L_{ar}i_a i_r + L_{br}i_b i_r$$

$$L_{aa} = L_{bb} = L_{aa0} + l_{al} \qquad L_{rr} = L_{rr0} + l_{rl} \qquad L_{ab} = L_{ba} = 0$$

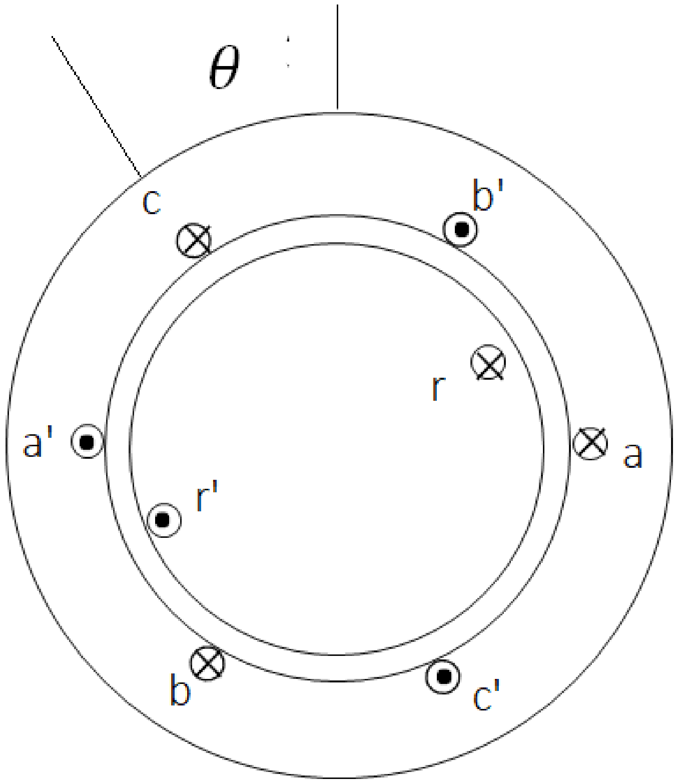
$$L_{ra} = L_{ar} = Mcos\theta \qquad L_{rb} = L_{br} = Mcos(\theta + 90)$$

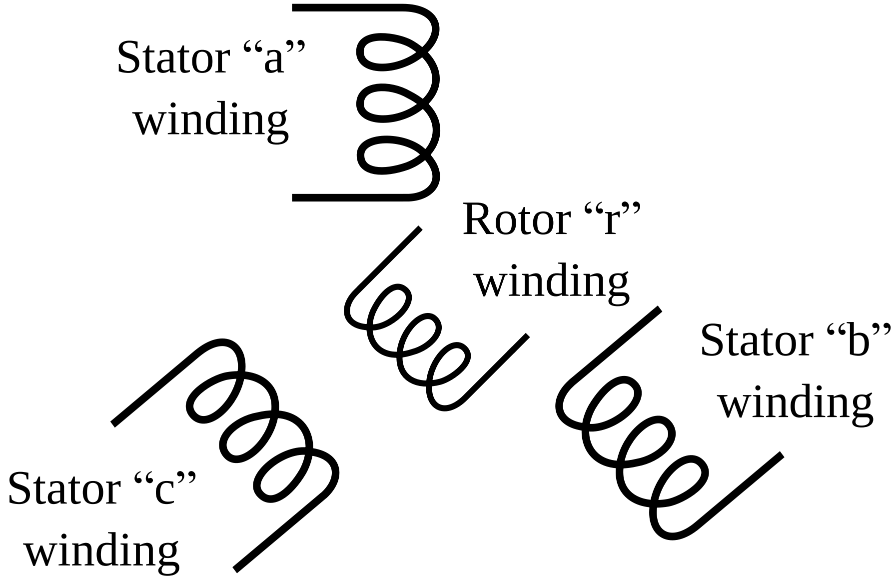
$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta} \qquad \tau = -Mi_a i_r \sin\theta - Mi_a i_r \sin(\theta + 90)$$

$$i_a = I_m \cos(\omega t) \qquad i_b = I_m \cos(\omega t + 90) \qquad i_r = I_r = cte.$$

$$\tau = -MI_m I_r \sin(\theta - \omega t) \qquad \tau_{ss} = -MI_m I_r \sin(\delta)$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)

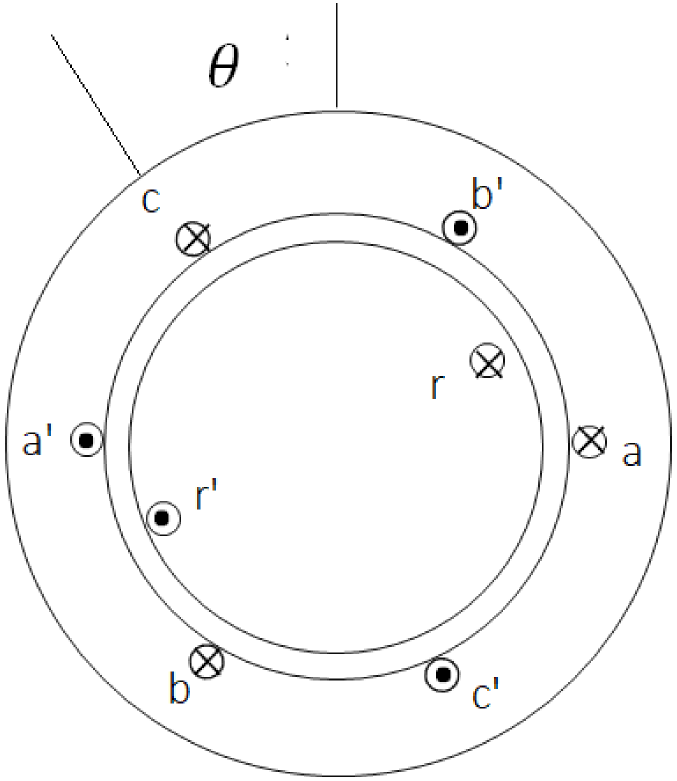




$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{cc}i_c^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_ai_b + L_{ac}i_ai_c + L_{bc}i_bi_c + L_{ar}i_ai_r + L_{br}i_bi_r + L_{cr}i_ci_r$$

$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta}$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{cc}i_c^2 + \frac{1}{2}L_{rr}i_r^2 +$$

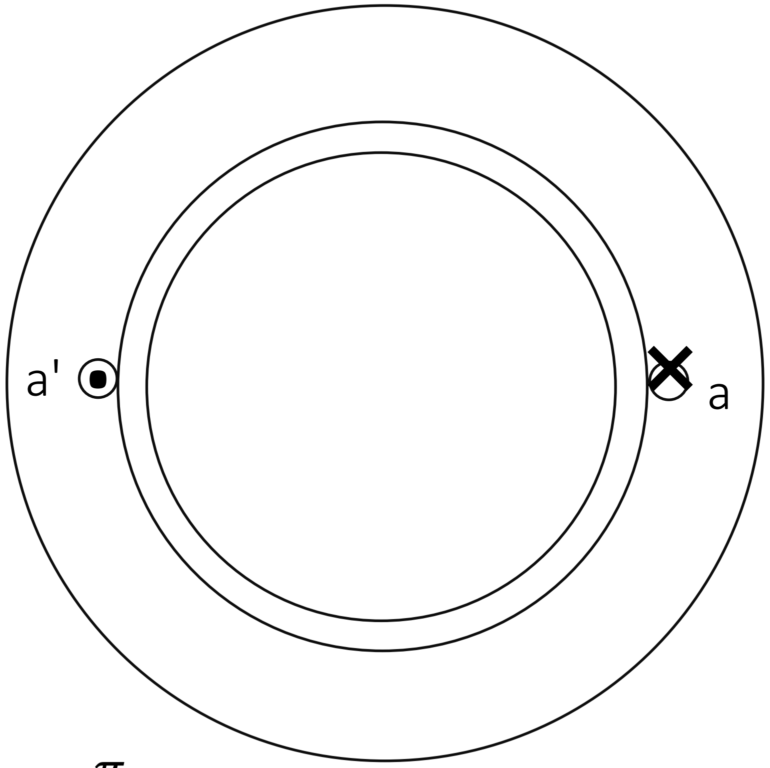
$$L_{ab}i_ai_b + L_{ac}i_ai_c + L_{bc}i_bi_c + L_{ar}i_ai_r + L_{br}i_bi_r + L_{cr}i_ci_r$$

$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta}$$

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{ar} \\ L_{ba} & L_{bb} & L_{bc} & L_{br} \\ L_{ca} & L_{cb} & L_{cc} & L_{cr} \\ L_{ra} & L_{rb} & L_{rc} & L_{rr} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_r \end{bmatrix}$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)

$$\begin{bmatrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_r\end{bmatrix}=\begin{bmatrix}L_{aa} & L_{ab} & L_{ac} & L_{ar}\\ L_{ba} & L_{bb} & L_{bc} & L_{br}\\ L_{ca} & L_{cb} & L_{cc} & L_{cr}\\ L_{ra} & L_{rb} & L_{rc} & L_{rr}\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\\ i_r\end{bmatrix}$$



Stator and rotor self inductances

$$L_{aa}=\frac{\lambda_a}{i_a}\Big|_{i_b\ =\ i_c\ =\ i_r\ =\circ}\qquad \lambda_a=k_\omega N_a\varphi_a$$

$$B_a=\frac{4}{\pi}\,\frac{k_\omega N_a i_a}{2g}\mu_0\cos\theta\qquad \varphi_a=\int B_a dA\qquad \varphi_a=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{4}{\pi}\frac{k_\omega N_a i_a}{2g}\mu_0\cos\theta rld\theta$$

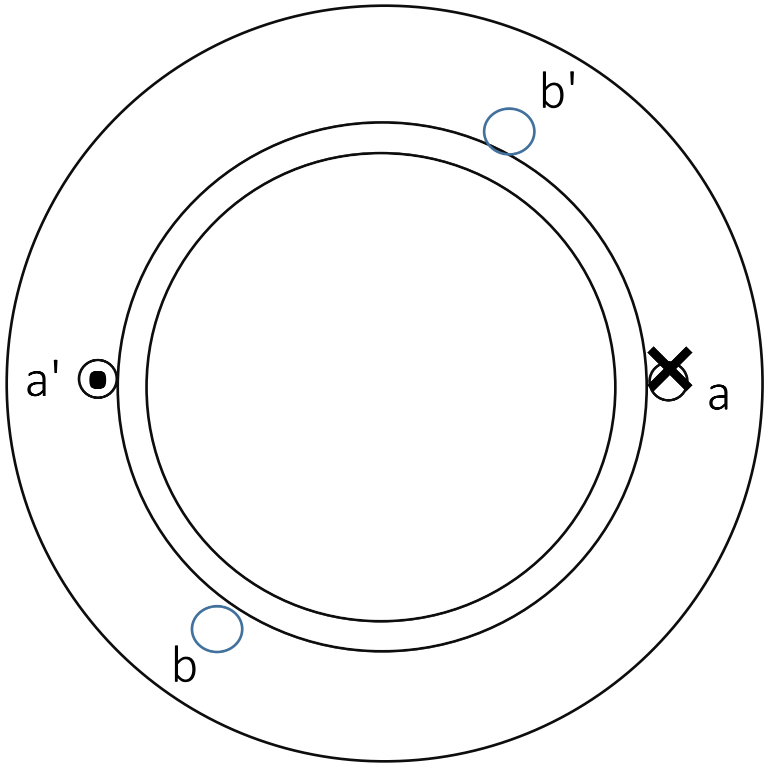
$$L_{aa}=\underbrace{\frac{4k_\omega^2N_a^2}{\pi g}\mu_0 rl}_{L_{aa0}}$$

$$L_{aa}=L_{bb}=L_{cc}=L_{aa0}+l_{al}$$

$$L_{rr}=\frac{4k_{\omega r}^2N_r^2}{\pi g}\mu_0 rl+l_{rl}$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{ar} \\ L_{ba} & L_{bb} & L_{bc} & L_{br} \\ L_{ca} & L_{cb} & L_{cc} & L_{cr} \\ L_{ra} & L_{rb} & L_{rc} & L_{rr} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_r \end{bmatrix}$$



Stator mutual inductances

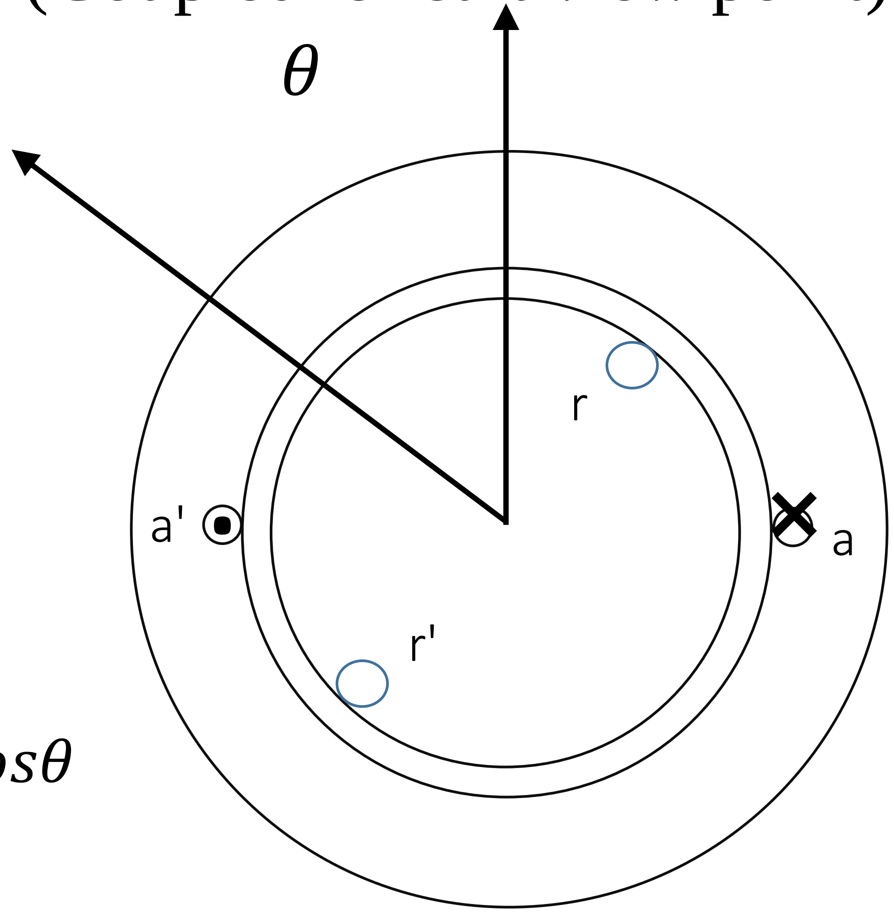
$$L_{ba} = \frac{\lambda_b}{i_a}\Big|_{i_b = i_c = i_r = \circ}$$

$$L_{ba} = \frac{\lambda_a cos120}{i_a}\Big|_{i_b = i_c = i_r = \circ} = -\frac{1}{2}L_{aa0}$$

$$L_{ab} = L_{ba} = L_{ac} = L_{ca} = L_{bc} = L_{cb} = -\frac{1}{2}L_{aa0}$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)

$$\begin{bmatrix}\lambda_a\\ \lambda_b\\ \lambda_c\\ \lambda_r\end{bmatrix}=\begin{bmatrix}L_{aa} & L_{ab} & L_{ac} & L_{ar}\\ L_{ba} & L_{bb} & L_{bc} & L_{br}\\ L_{ca} & L_{cb} & L_{cc} & L_{cr}\\ L_{ra} & L_{rb} & L_{rc} & L_{rr}\end{bmatrix}\begin{bmatrix}i_a\\ i_b\\ i_c\\ i_r\end{bmatrix}$$



Stator and rotor mutual inductances

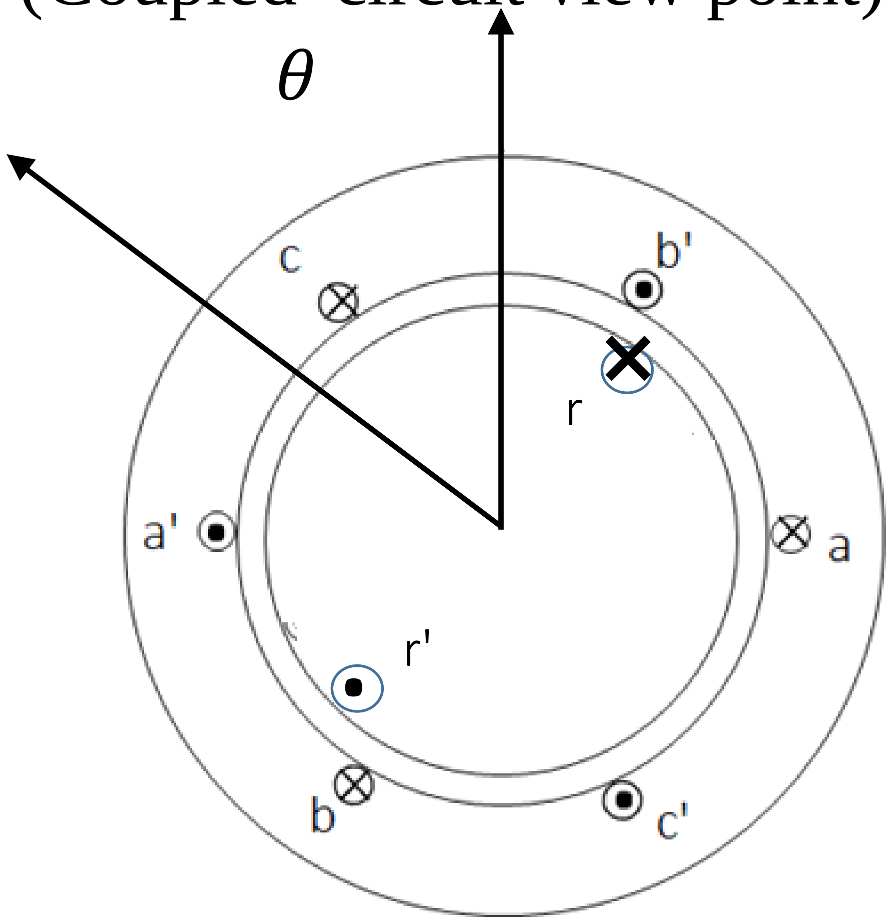
$$L_{ra}=\frac{\lambda_r}{i_a}\Big|_{i_b\,=\,i_c\,=\,i_r\,=\,\circ} \qquad \lambda_r=k_{\omega r}N_r\varphi_r=k_{\omega r}N_r\varphi_a cos\theta$$

$$\varphi_a=\int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}\frac{4}{\pi}\frac{k_\omega N_a i_a}{2g}\mu_0 cos\theta rld\theta \qquad L_{ra}=\underbrace{\frac{4k_{\omega r}N_r k_\omega N_a}{\pi g}}_{M}\mu_0 rlcos\theta$$

$$L_{ra}=L_{ar}=Mcos\theta$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{ar} \\ L_{ba} & L_{bb} & L_{bc} & L_{br} \\ L_{ca} & L_{cb} & L_{cc} & L_{cr} \\ L_{ra} & L_{rb} & L_{rc} & L_{rr} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_r \end{bmatrix}$$



Stator and rotor mutual inductances

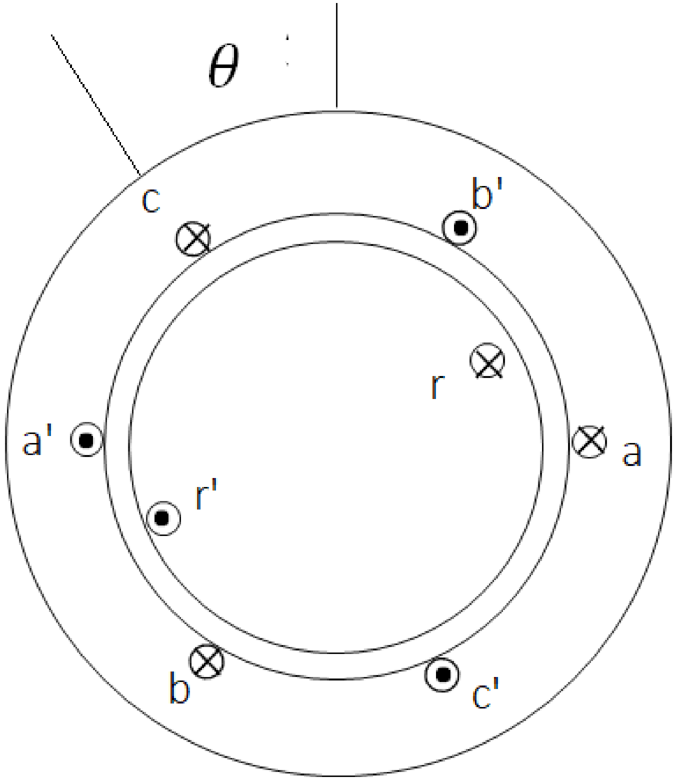
$$L_{ra} = \underbrace{\frac{4k_{\omega r}N_r k_\omega N_a}{\pi g}\mu_0 r l}_{M} cos\theta$$

$$L_{ra} = L_{ar} = Mcos\theta$$

$$L_{rb} = L_{br} = Mcos(\theta + 120)$$

$$L_{rc} = L_{cr} = Mcos(\theta - 120)$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{cc}i_c^2 + \frac{1}{2}L_{rr}i_r^2 +$$

$$L_{ab}i_ai_b + L_{ac}i_ai_c + L_{bc}i_bi_c + L_{ar}i_ai_r + L_{br}i_bi_r + L_{cr}i_ci_r$$

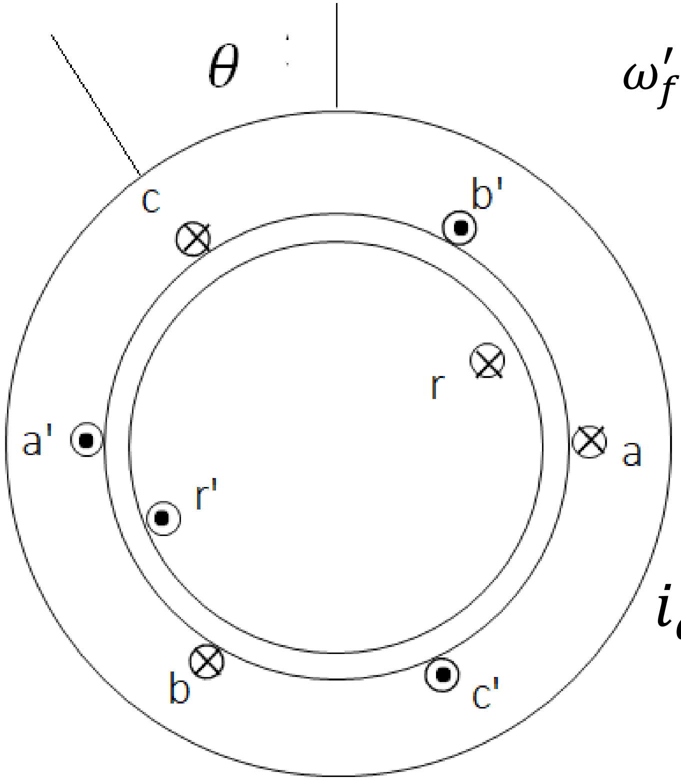
$$L_{aa} = L_{bb} = L_{cc} = L_{aa0} + l_{al}$$

$$L_{ab} = L_{ba} = L_{ac} = L_{ca} = L_{bc} = L_{cb} = -\frac{1}{2}L_{aa0}$$

$$L_{ra} = L_{ar} = Mcos\theta \qquad L_{rb} = L_{br} = Mcos(\theta + 120) \qquad L_{rc} = L_{cr} = Mcos(\theta - 120)$$

$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta} = -Mi_ai_r\sin\theta - Mi_bi_r\sin(\theta + 120) - Mi_ci_r\sin(\theta - 120)$$

Torque determination in cylindrical AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{cc}i_c^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_a i_b + L_{ac}i_a i_c + L_{bc}i_b i_c +$$

$$+L_{ar}i_a i_r + L_{br}i_b i_r + L_{cr}i_c i_r$$

$$\tau = -Mi_a i_r \sin\theta - Mi_b i_r \sin(\theta + 120) - Mi_c i_r \sin(\theta - 120)$$

$$i_a = I_m \cos(\omega t) \qquad i_b = I_m \cos(\omega t + 120) \qquad i_c = I_m \cos(\omega t - 120)$$

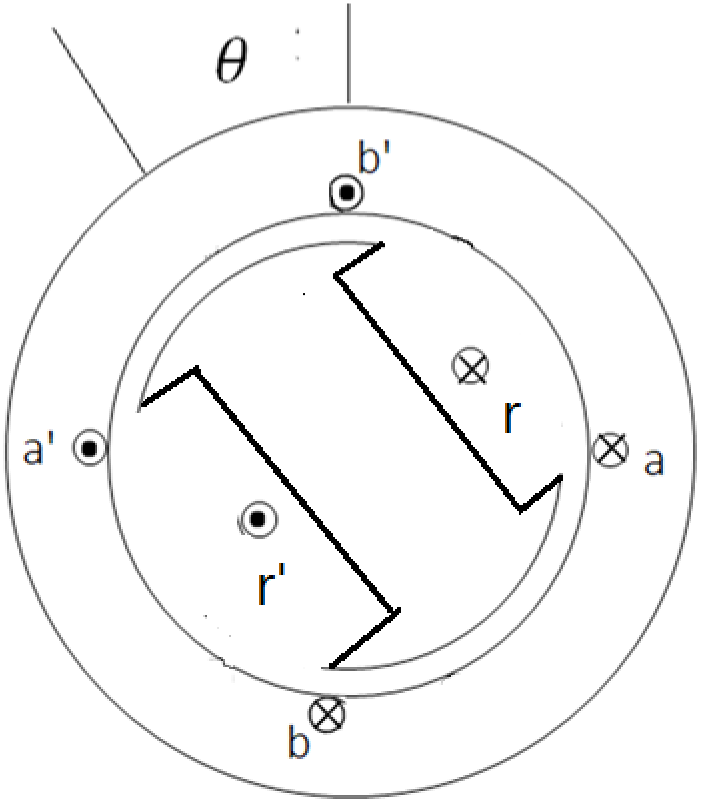
$$i_r = I_r = cte.$$

$$\tau = -MI_m I_r \cos(\omega t)\sin\theta - MI_m I_r \cos(\omega t + 120)\sin(\theta + 120) - MI_m I_r \cos(\omega t - 120)\sin(\theta - 120)$$

$$\tau = -\frac{3}{2}MI_m I_r \sin(\theta - \omega t) = -\frac{3}{2}MI_m I_r \sin(\delta + \omega_m t - \omega t)$$

$$\boxed{\tau_{ss} = -\frac{3}{2}MI_m I_r \sin(\delta) \qquad M = \frac{4k_{\omega r}N_r k_\omega N_a}{\pi g}\mu_0 r l}$$

Torque determination in salient-pole two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_ai_b + L_{ar}i_ai_r + L_{br}i_bi_r$$

$L_{aa} = ??$ $\quad L_{aa} = L_1 + L_2cos2\theta \quad L_{bb} = L_1 - L_2cos2\theta$

$L_{rr} = L_{rr0} + l_{rl} \quad L_{ab} = ?? \quad L_{ab} = -L_2sin2\theta$

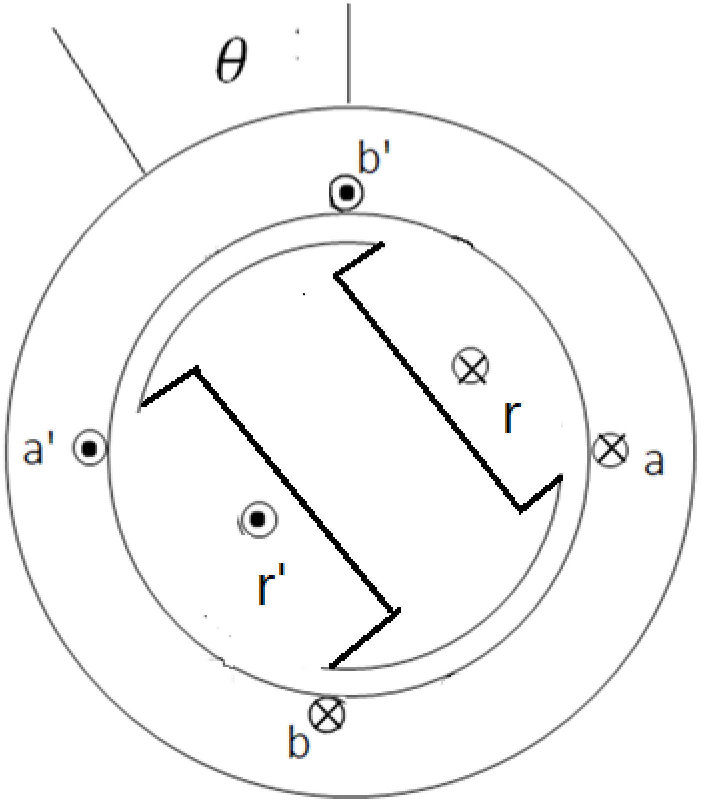
$L_{ar} = ?? \quad L_{ar} = Mcos\theta \quad L_{br} = Mcos(\theta + 90)$

$i_a = I_m\cos(\omega t) \quad i_b = I_m\cos(\omega t + 90) \quad i_r = I_r = cte.$

$$\tau = \frac{\partial \omega'_{fld}}{\partial \theta} \quad = L_2sin2\theta(i_b^2 - i_a^2) - 2L_2cos2\theta i_ai_b - Msin\theta i_ai_r - Msin(\theta + 90)i_bi_r$$

$$\tau = -L_2I_m^2sin2\theta cos2\omega t + L_2I_m^2cos2\theta sin2\omega t - MI_mI_rsin(\theta - \omega t)$$

Torque determination in salient-pole two phases AC Machine (Coupled circuit view point)



$$\omega'_{fld} = \frac{1}{2}L_{aa}i_a^2 + \frac{1}{2}L_{bb}i_b^2 + \frac{1}{2}L_{rr}i_r^2 + L_{ab}i_ai_b + L_{ar}i_ai_r + L_{br}i_bi_r$$

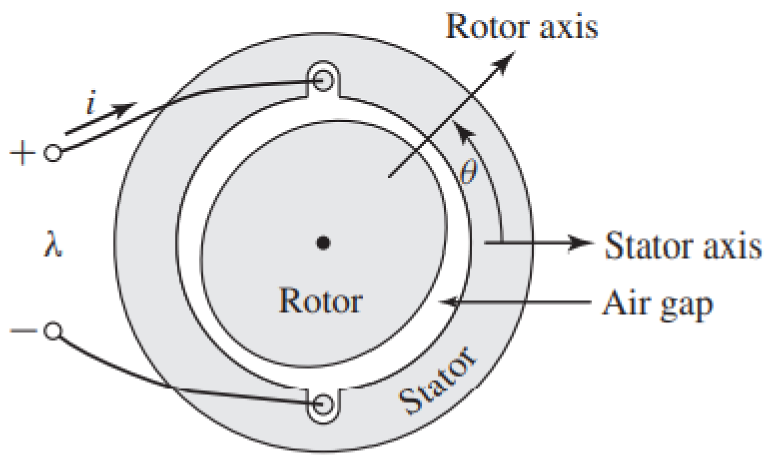
$$\tau = -L_2I_m^2 sin2\theta cos2\omega t + L_2I_m^2 cos2\theta sin2\omega t - MI_mI_r sin(\theta - \omega t)$$

$$\tau = -L_2I_m^2 \sin(2\theta - 2\omega t) - MI_mI_r sin(\theta - \omega t)$$

$$\tau = -L_2I_m^2 \sin(2\delta) - MI_mI_r sin(\delta)$$

Torque determination in salient-pole one phase AC Machine (Coupled circuit view point)

Example 3: The magnetic circuit of following figure consists of a single-coil stator and an oval rotor. Because the air-gap is nonuniform, the coil inductance varies with rotor angular position, measured between the magnetic axis of the stator coil and the major axis of the rotor, as



$$L(\theta) = L_0 + L_2 \cos(2\theta)$$

where L_0 = 10.6 mH and L_2 = 2.7 mH. Find the torque as a function of θ for a coil current of 2 A.

$$T_{\text{fld}}(\theta) = \frac{i^2}{2}\frac{dL(\theta)}{d\theta} = \frac{i^2}{2}\left(-2L_2 \sin(2\theta)\right)$$

$$T_{\text{fld}}(\theta) = -(1.08 \times 10^{-2})\ \sin(2\theta) \quad \text{N} \cdot \text{m}$$