LINEAR CONTROL SYSTEMS

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Lecture 7

Time domain design of control systems

Topics to be covered include:

- Introduction.
  - Various controller configurations.
  - Different kind of controllers.
  - Controller realization.
- Controller design in time domain.
  - PID controller design.
  - PD controller design.
  - PI controller design.
  - Lag controller design.
  - Lead controller design.

Introduction

1 Control structure design.

- Feedforward controller.
- Feedback or parallel compensation.
- Series parallel compensation or cascade system.
- Series or cascade compensation.
- Feedforward and series compensation.

2 Controller or compensator type.

- Lead controller or PD.
- Lag controller or PI.
- Lead/lag controller or PID.

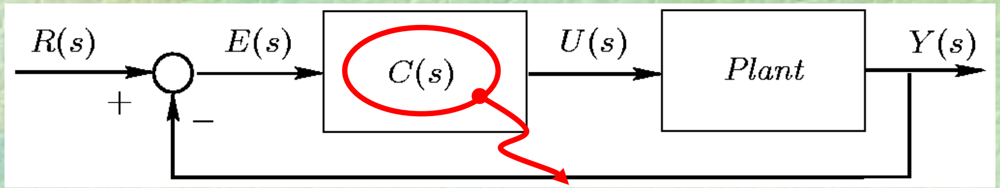
3 Controller or compensator parameter tuning.

- Analytic controller design.
- Graphic controller design.
- Controller design by table.

4 Checking the compensated system.

Series Compensation Structure

In this Course we consider the series or cascade compensation.



Controller

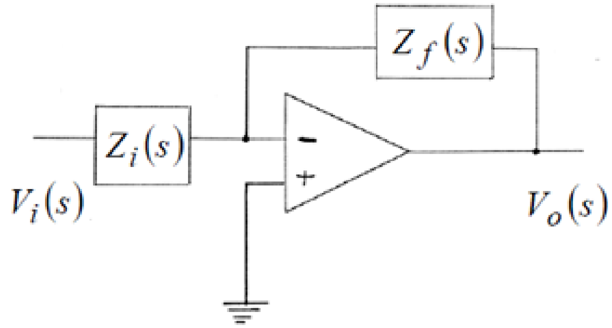
Different kind of controllers

- PID controllers.
- Lead lag controllers
- H_2 Controllers
- H_∞ Controllers
- ……………
- Intelligent Controllers
  - Adaptive Controllers
  - NN Controllers
  - ……………

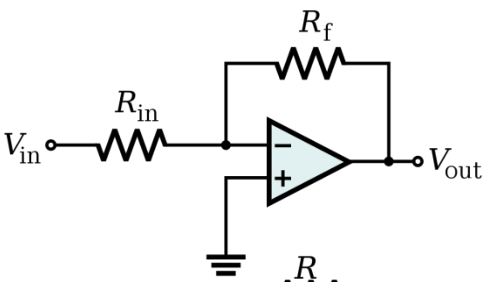
In This course

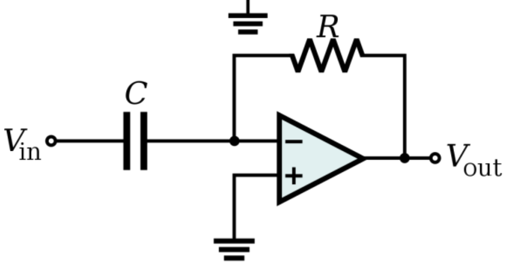
In Graduate courses

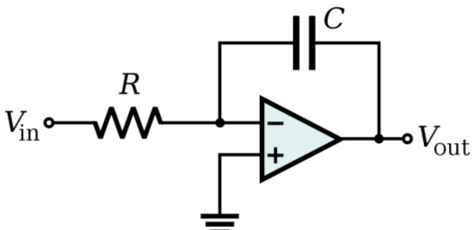
PID and Operational Amplifiers

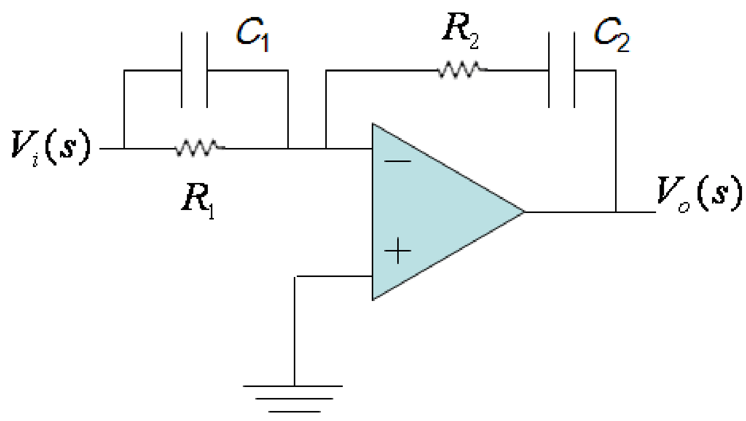


$$G(s)=\frac{V_o(s)}{V_i(s)}=-\frac{Z_f(s)}{Z_i(s)}$$









Time domain design of control systems

- Introduction.
  - Various controller configurations.
  - Different kind of controllers.
  - Controller realization.
- Controller design in time domain.
  - PID controller design.
  - PD controller design.
  - PI controller design.
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Time domain design

Cruise control of a vehicle.

$$m\frac{dv}{dt} = uT(a, v) - mg\sin\theta - mgC_r\, sgn(v) - \frac{1}{2}\rho C_r A v^2$$

now consider v_q=30 m/s and θ_e =0 then we derive u_q

Linear system is:

$$\dot{\tilde{v}} = -0.01\,\tilde{v} + 1.32\tilde{u} - 9.8\tilde{\theta}$$

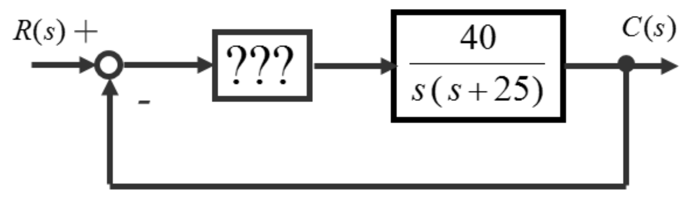
Transfer function ?

Closed loop and its specification?

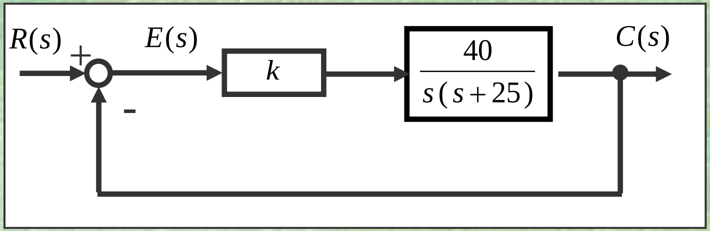
We need zero steady state and settling time less than 1 sec?

Time domain design

Exercise 1: Is it possible to design a P controller such that the damping ratio of complex poles be larger than 0.707 and less than 1?

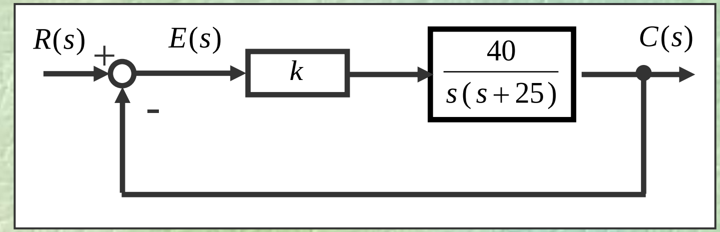


Exercise 2: Is it possible to design a P controller such that the steady state error to unit ramp be less than 0.01?

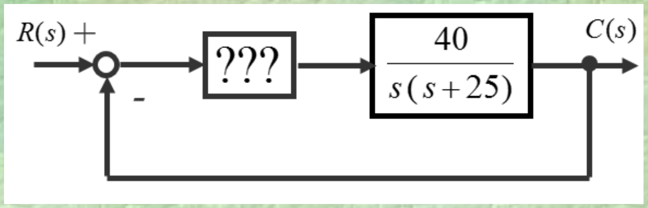


Time domain design

Exercise 3: Is it possible to design a P controller such that the steady state error to unit ramp be less than 0.01 and damping ratio of complex poles be 0.707 ?
What about other controller?



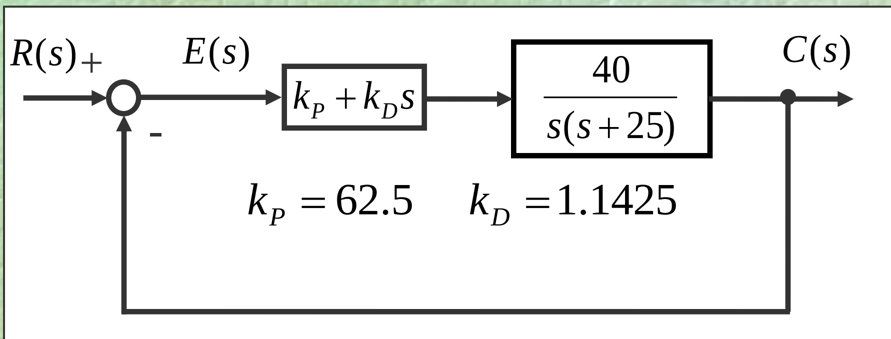
Exercise 4: Design a controller such that the steady state error to unit ramp be less than 0.01 and damping ratio of complex poles be 0.707?

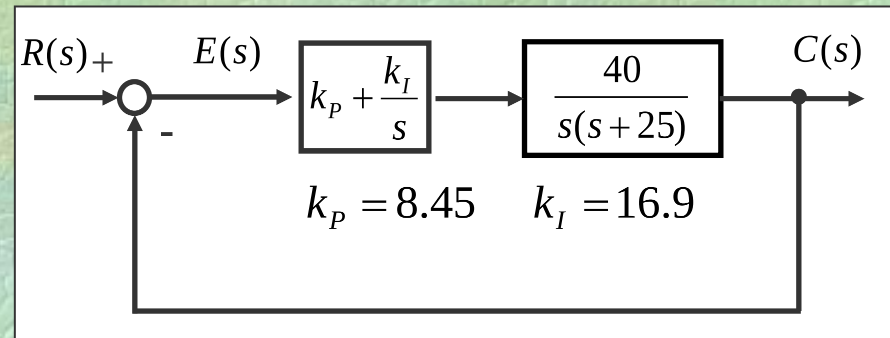


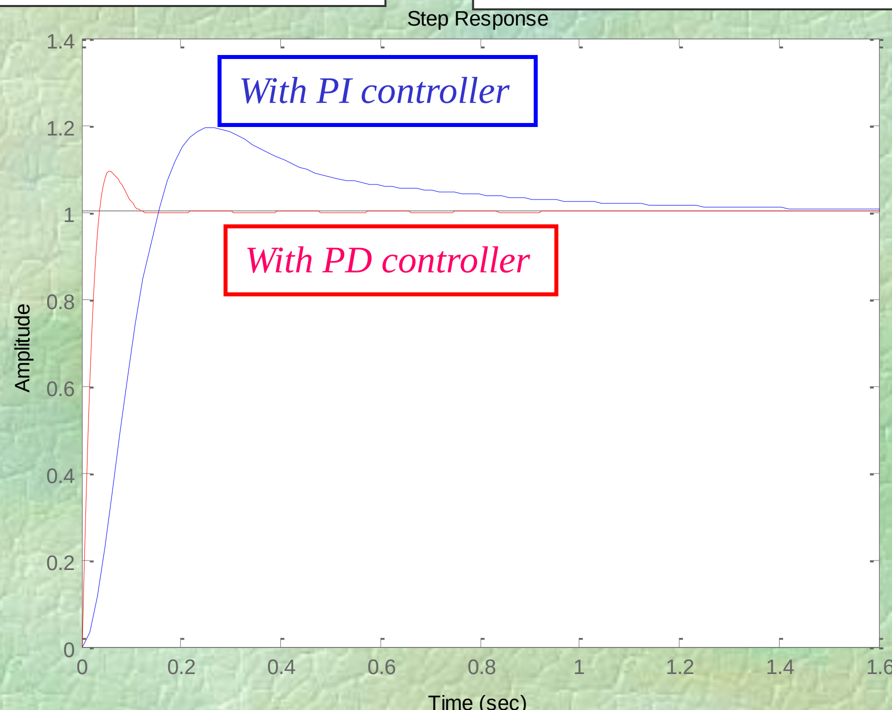
PD controller …..

PI controller …..

Compare PI and PD controllers



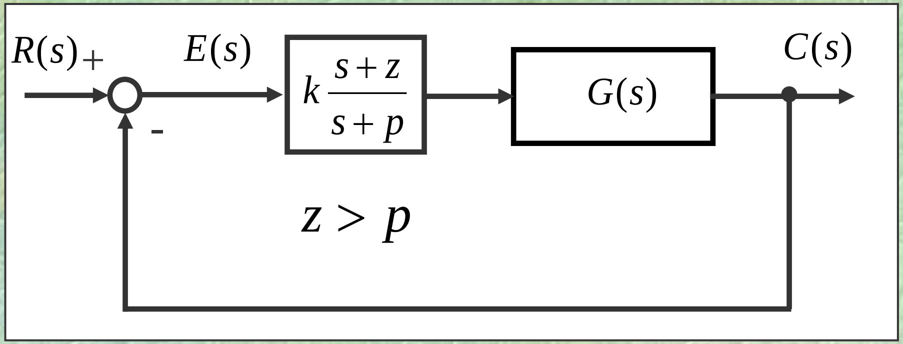




Lag controller design procedure

1- Obtain the root-locus (without controller) and determine the gain k_0 to satisfy the desired damping ratio or …...

2- Find the gain k to satisfy the desired steady state error (without controller). If k is in conflict with k_0 continue.



3- Evaluate the needed controller gain

$$\text{needed gain} = \frac{k}{k_0} = \frac{\text{Gain to satisfy the desired steady-state error}}{\text{Gain to satisfy the desired damping ratio or ...}}$$

Why?

4- Choose pole and zero of controller near origin such that:

$$\frac{z}{p} = \text{needed gain} = \frac{k}{k_0}$$

5 - Choose the controller as:

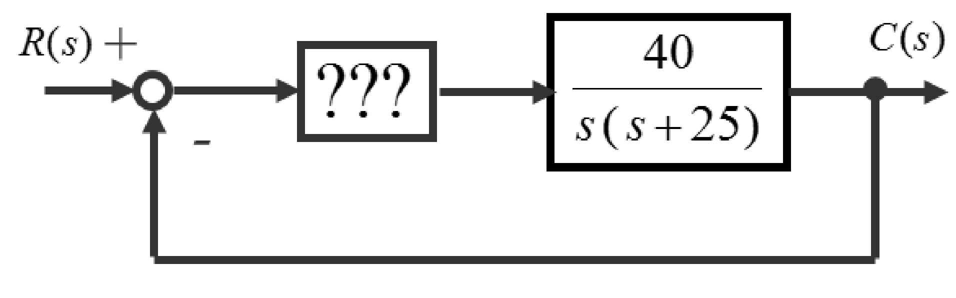
$$G_c(s) = k_0 \frac{s+z}{s+p}$$

What is near?

6 - Check the controller.

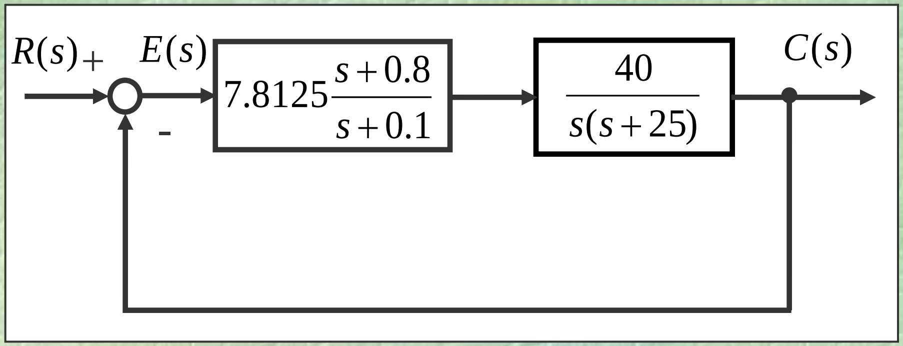
Time domain design

Exercise 5: Design a controller such that the steady state error to unit ramp be less than 0.01 and damping ratio of complex poles be 0.707?



Lag controller …..

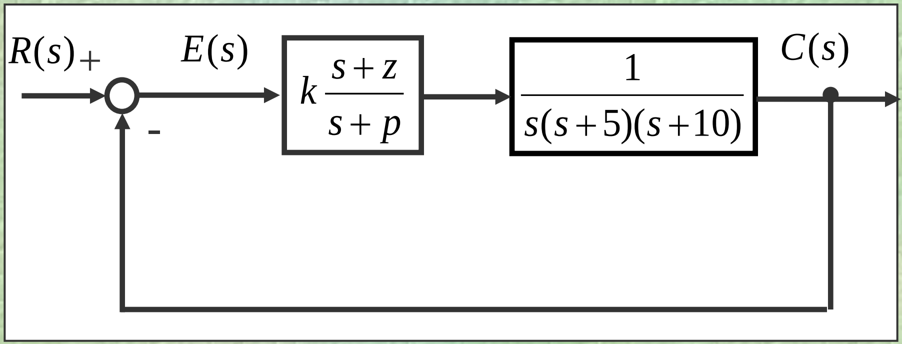
Designing lag controller and its step response



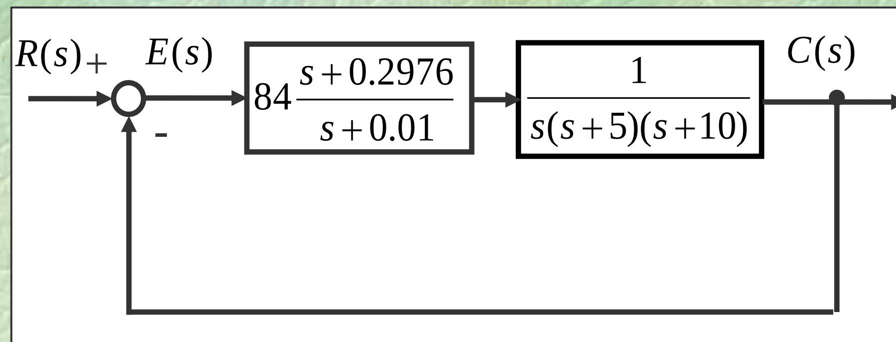
Time domain design

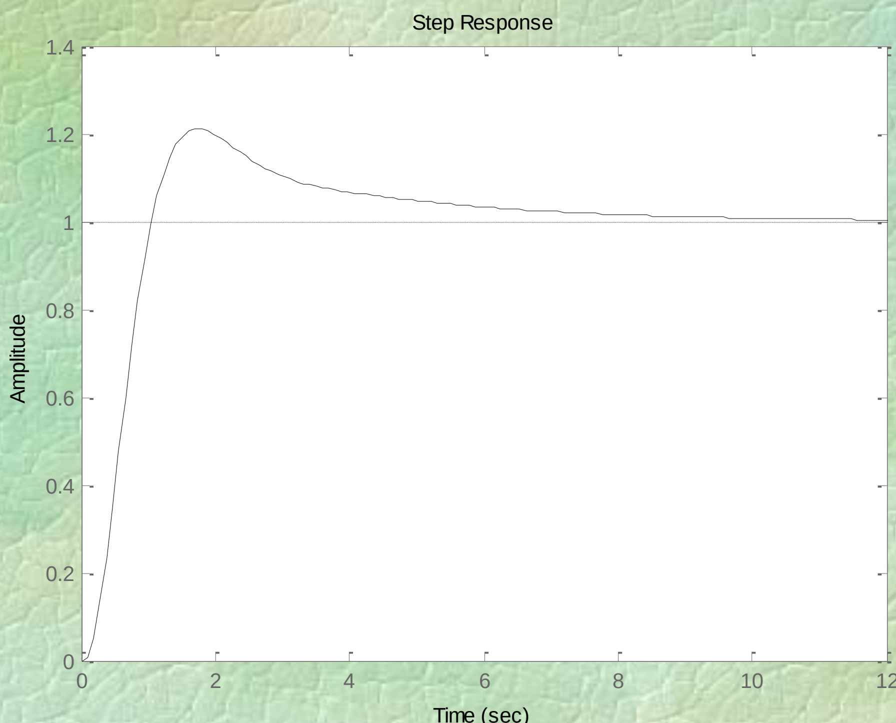
Exercise 6: Design a controller such that the velocity constant be 50 and percent overshoot be less than 16%.

Lag controller …..



Lag controller design





P.O. is not ok

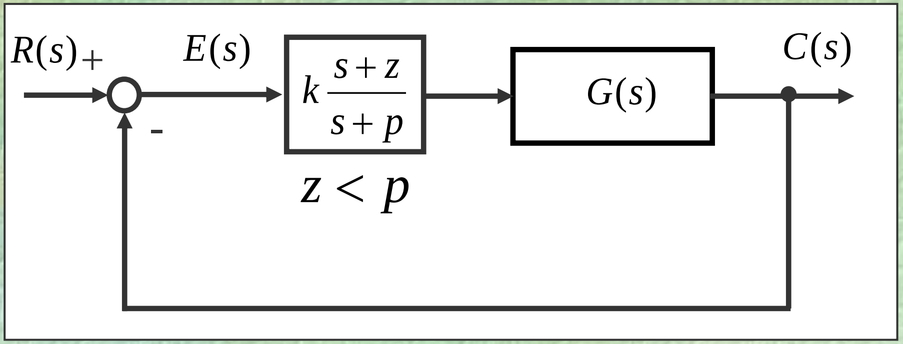
Tune the controller to derive the performance

Lag controller design

When the design of lag controller is not possible?

Lead controller design procedure

1- From the time-domain specifications obtain the desired location of the closed-loop dominant poles.



2- Select the zero of controller. Place the zero on the real value of desired location of the closed-loop dominant poles or on the pole for pole-zero cancellation.

3- Locate the compensator pole so that the angle criterion is satisfied.

4- Determine the compensator gain *k*, such that the magnitude criterion is satisfied.

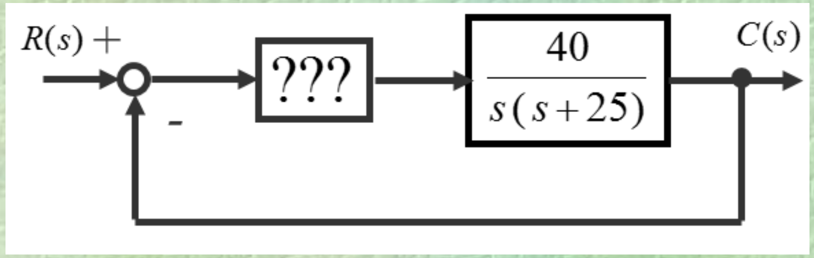
5 - Choose the controller as: $$G_c(s) = k\frac{s+z}{s+p}$$

6 - Check the controller.

7 - If the overall response rise time, overshoot and settling time is not satisfactory, choose another location of the closed-loop dominant poles.

Time domain design

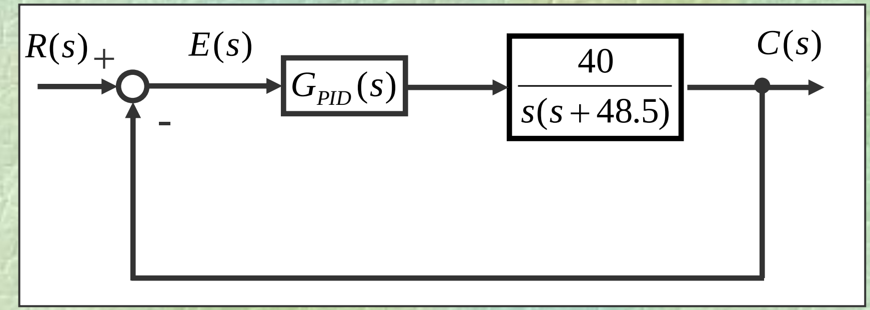
Exercise 7: Design a controller such that the steady state error to unit ramp be less than 0.01 and damping ratio of complex poles be 0.707?



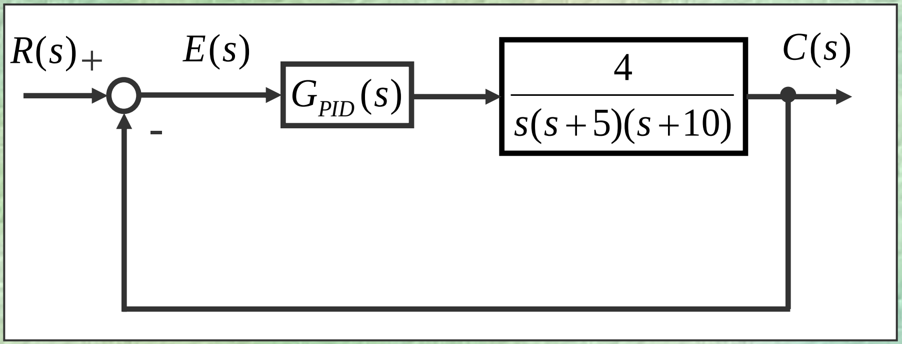
Lead controller …..

Exercises

Exercise 8: In the following system design a PD controller such that that the damping ratio of complex poles be 0.6 and ramp error constant be 80.

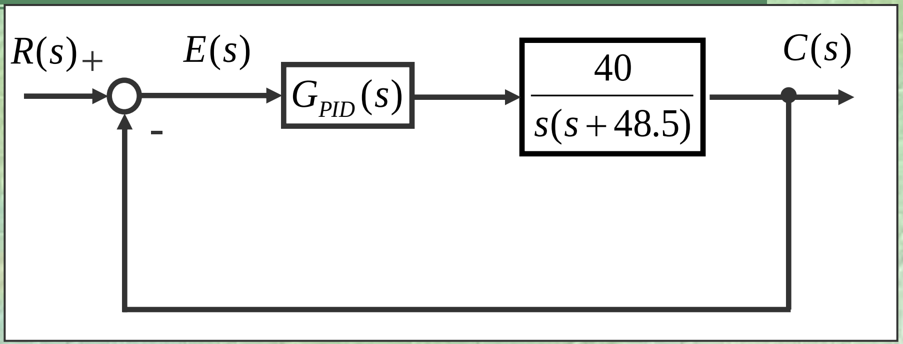


Exercise 9: In the following system design a PD controller such that that the damping ratio of complex poles be 0.6 and ramp error constant be 80.

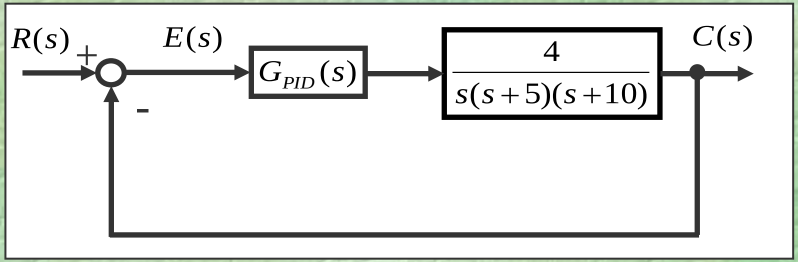


Exercises

Exercise 10: In the following system design a PI controller such that that The damping ratio of complex poles be 0.6 and ramp error constant be 80.

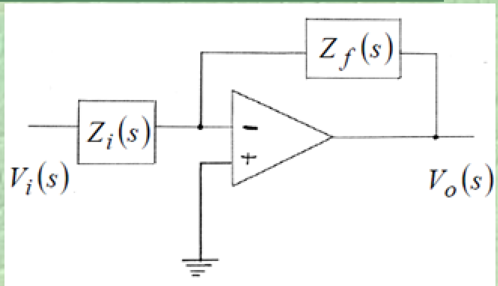


Exercise 11: In the following system design a PI controller such that that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



Exercises

Exercise 12: Consider following structure:



Let the input impedance be generated by a resistor R_2 be in series with a resistor R_1 and a capacitor C_1 that are in parallel, and let the feedback impedance be generated by a resistor R_f in series with a capacitor C_f .

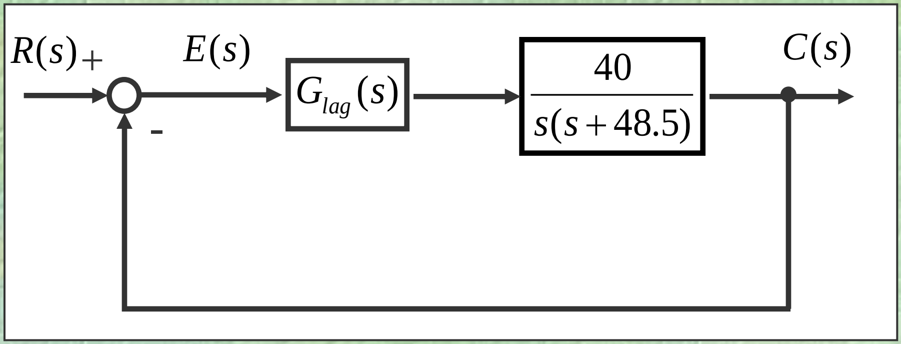
a) Show that this choices lead to form a PID controller with high frequency gain limit as;

$$G(s) = -\left[K_p + \frac{K_i}{s} + K_d s\right] \cdot \frac{1}{\tau s + 1}$$

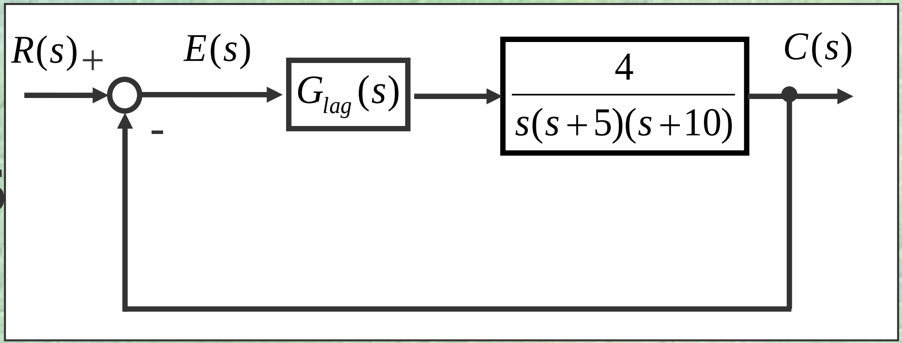
b) Derive the parameters in the controller with respect to resistors and capacitors.

Exercises

Exercise 13: In the following system Design a lag controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



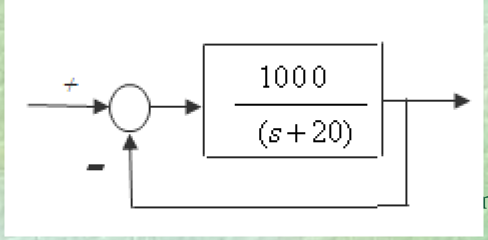
Exercise 14: In the following system design a lag controller such that the damping ratio of complex poles be 0.6 and ramp error constant be 80.



Exercise 15: Design a lead controller for exercise 6.

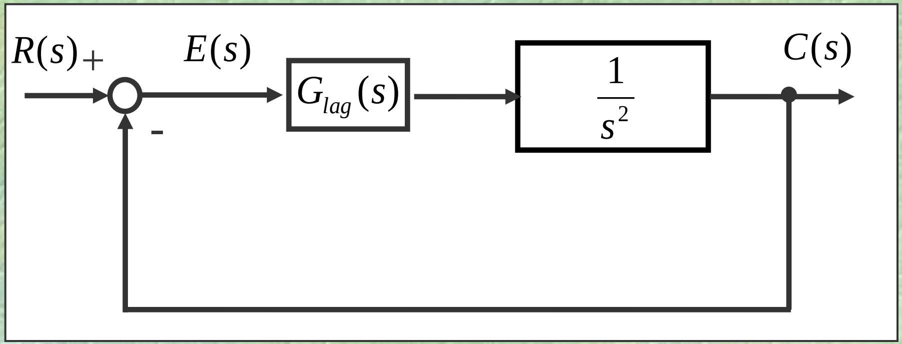
Exercise 16: Design a lead controller for exercise 7.

Exercise 17: In the following system design a controller such that leads to zero steady state error to step and 4.3% P.O. (Final)

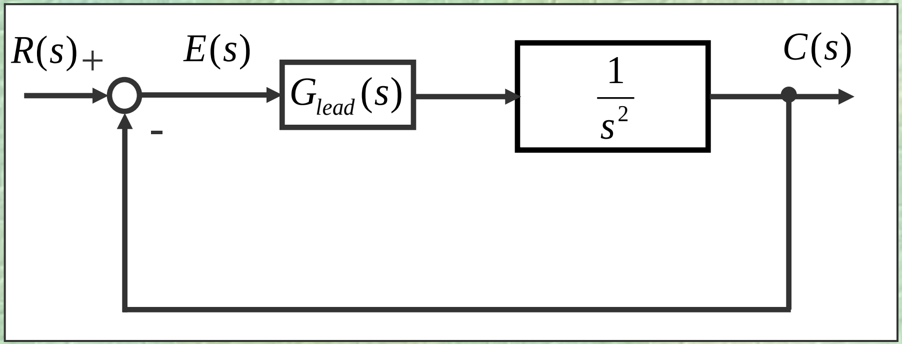


Exercises

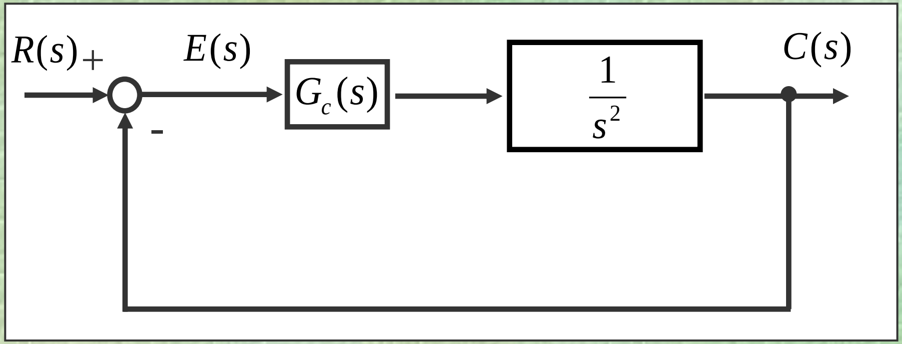
Exercise 18: In the following system design a lag controller such that that the settling time be less than 3 sec.



Exercise 19: In the following system Design a lead controller such that the settling time be less than 3 sec.

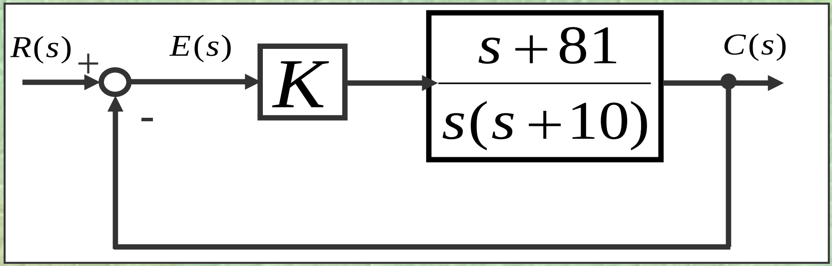


Exercise 20: In the following system design a Lead controller such that the damping ratio of complex poles be 0.4.(Final 1390)



Exercises

Exercise 21: In the following system (Final 1395)



a) Draw root loci for K>0.

b) Derive K for $T_s=0.1^s$ (T_s is Settling time)

c) Derive P.O. and velocity error constant for derived K in part “b”.

d) Derive a controller such that the velocity error constant multiplied by 10 but P.O. and T_s remain the same as part “c”.

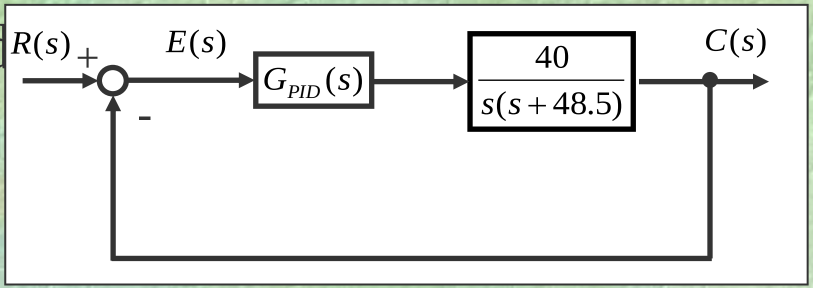
e) Derive a controller such that the velocity error be zero but P.O. and T_s remain the same as part “c”.

f) Derive a controller such that the closed loop dominant poles are on the -15±15j.

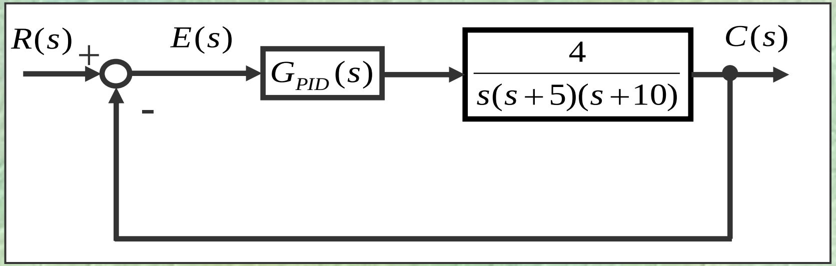
g) Derive step response of system for part “c” , “d” , “e” and “f”.

Exercises

Exercise 22: In the following system d
PID controller with
Ziegler-Nichols Oscillation Method



16- In the following system design a PID controller with Ziegler-Nichols Oscillation Method



Appendix: Ziegler-Nichols Design

This procedure is only valid for open loop stable plants.

- Open-Loop Tuning
- Closed-Loop Tuning

According to Ziegler and Nichols, the open-loop transfer function of a system can be approximated with time delay and single-order system, i.e.

$$G(s) = \frac{K e^{-sT_D}}{1 + sT_1},$$

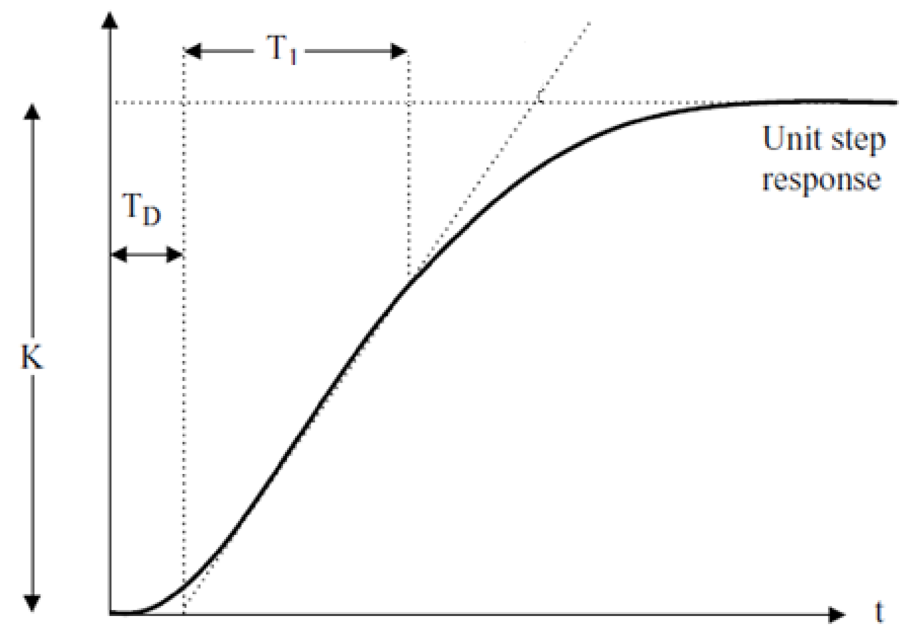
where T_D is the system time delay and T_1 is the time constant.

Appendix: Ziegler-Nichols Reaction Curve Method(Open-Loop Case)

For open-loop tuning, we first find the plant parameters by applying a step input to the open-loop system.

The plant parameters K, TD and T1 are then found from the result of the step test as shown in Figure.

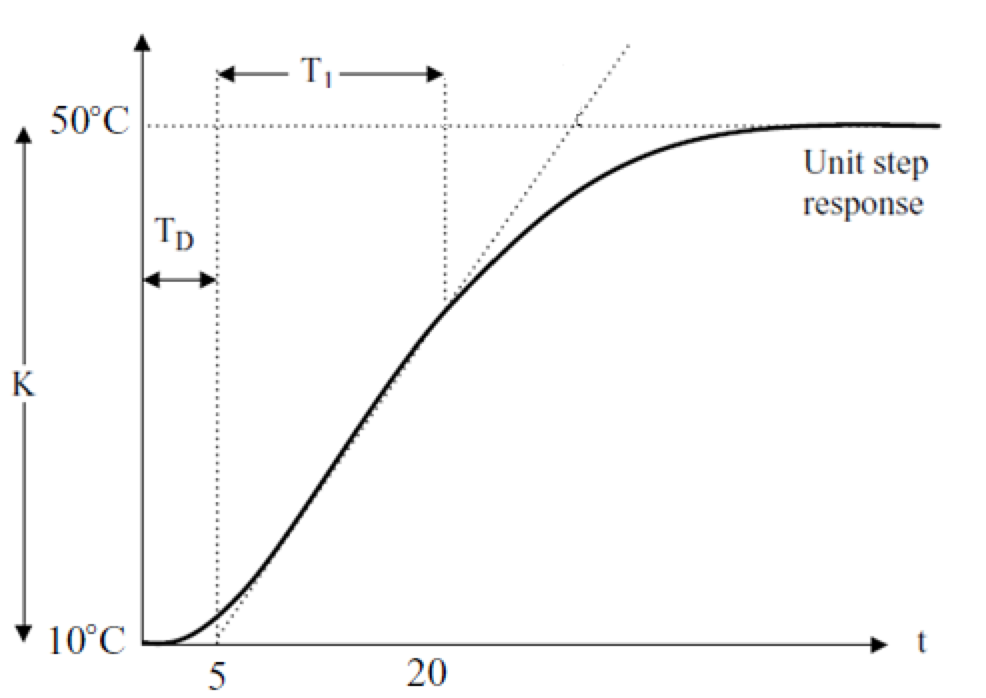
$$G(s) = \frac{K e^{-sT_D}}{1 + sT_1},$$



Appendix: Ziegler-Nichols Reaction Curve Method(Open-Loop Case)

	K_p	K_i	K_d
P	$\frac{T_1}{KT_D}$		
PI	$\frac{0.9T_1}{KT_D}$	$\frac{0.27T_1}{KT_D{}^2}$	
PID	$\frac{1.2T_1}{KT_D}$	$\frac{0.6T_1}{KT_D{}^2}$	$\frac{0.6T_1}{K}$

Example 1: Following figure shows the step response of an open-loop transfer function system. Design a PID controller.



So: $K = 40^{\circ}C, \quad T_D = 5\ \text{sec}, \quad T_1 = 20\,\text{sec} \quad G(s) = \dfrac{40e^{-sT_D}}{1+20s}$

Example 1: Following figure shows the step response of an open-loop transfer function system. Design a PID controller.

$$\text{So}: \quad K = 40^\circ C, \quad T_D = 5\ \text{sec}, \quad T_1 = 20\,\text{sec} \quad G(s) = \frac{40e^{-sT_D}}{1+20s}$$

	K_p	K_i	K_d
P	$\frac{T_1}{KT_D} = 0.1$		
PI	$\frac{0.9T_1}{KT_D} = 0.09$	$\frac{0.27T_1}{KT_D{}^2} = 0.0054$	
PID	$\frac{1.2T_1}{KT_D} = 0.12$	$\frac{0.6T_1}{KT_D{}^2} = 0.012$	$\frac{0.6T_1}{K} = 0.3$

$$K_P(s) = 0.1$$

$$K_{PI}(s) = 0.09 + \frac{0.0054}{s}$$

$$K_{PID}(s) = 0.12 + \frac{0.012}{s} + 0.3s$$

Ziegler-Nichols Oscillation Method(Closed-loop)

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.
- Record the controller critical gain K_c and the oscillation period of the controller output, T.
- Adjust the controller parameters according to Table

Ziegler-Nichols Oscillation Method(Closed-loop)

	K_p	K_i	K_d
P	$0.5K_c$		
PI	$0.45K_c$	$0.54\frac{K_c}{T}$	
PID	$0.6K_c$	$1.2\frac{K_c}{T}$	$0.075K_cT$

Example 2: Design a PID controller for following system.

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s+1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference.

Solution

Applying the procedure we find:

$$K_c = 8 \text{ and } \omega_c = \sqrt{3}. \qquad T\text{=}3.62$$

Hence, from Table, we have

$$K_p = 0.6K_c = 4.8 \qquad K_i = 1.2\frac{K_c}{T} = 2.65 \qquad K_d = 0.075K_cT = 2.17$$

The closed loop response to a unit step in the reference at $t = 0$ is shown in the next figure.

Response to step reference

$$C_{PID}(s) = K_p\left(1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1}\right)$$

$$C_{PID}(s) = 4.8 + \frac{2.65}{s} + 2.17s$$

$$C_{PID}(s) = 4.8 + \frac{2.65}{s} + \frac{2.17s}{0.01 \cdot 2.17s + 1}$$