# Electric Machinery II

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Text: A. E. Fitzgerald, Charles Kingsley, Jr And Stephen D. Umans, *Electric Machinery*. Mc GrawHill, 7th Edition

# Electrical Machine II Syllabus

1. Introduction

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2. Introduction to Rotating Machines

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3. DC Machines

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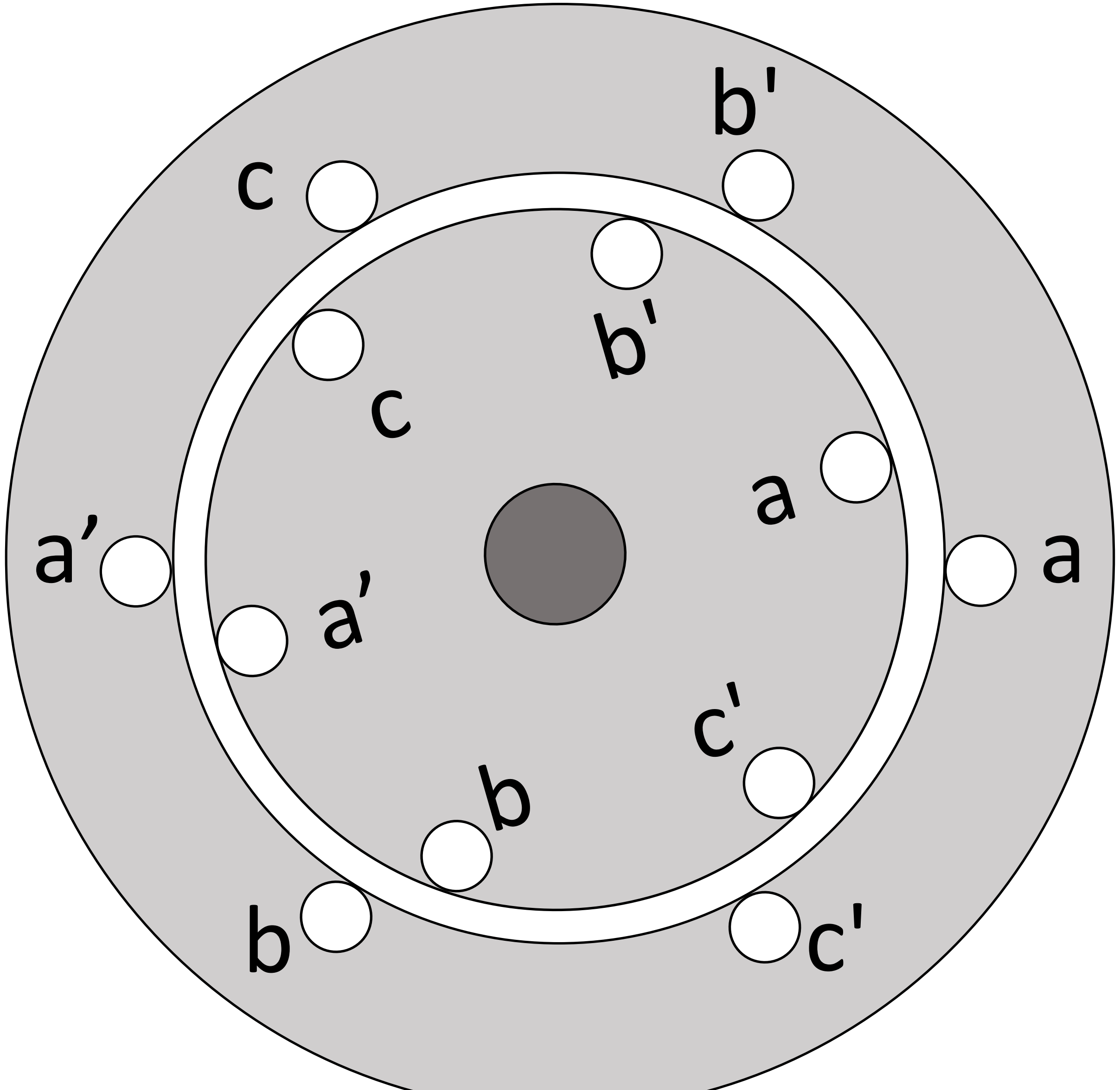
4. Induction Machines
    - Introduction to Polyphase Induction Machines
    - Currents and Fluxes in Polyphase Induction Machines
    - Induction-Motor Equivalent Circuit
    - Analysis of the Equivalent Circuit
    - Torque and Power by Use of Thevenin's Theorem
    - Parameter Determination from No-Load and Blocked-Rotor Tests
    - Effects of Rotor Resistance; Wound and Double-Squirrel-Cage Rotors

# Induction Machines Fundamental

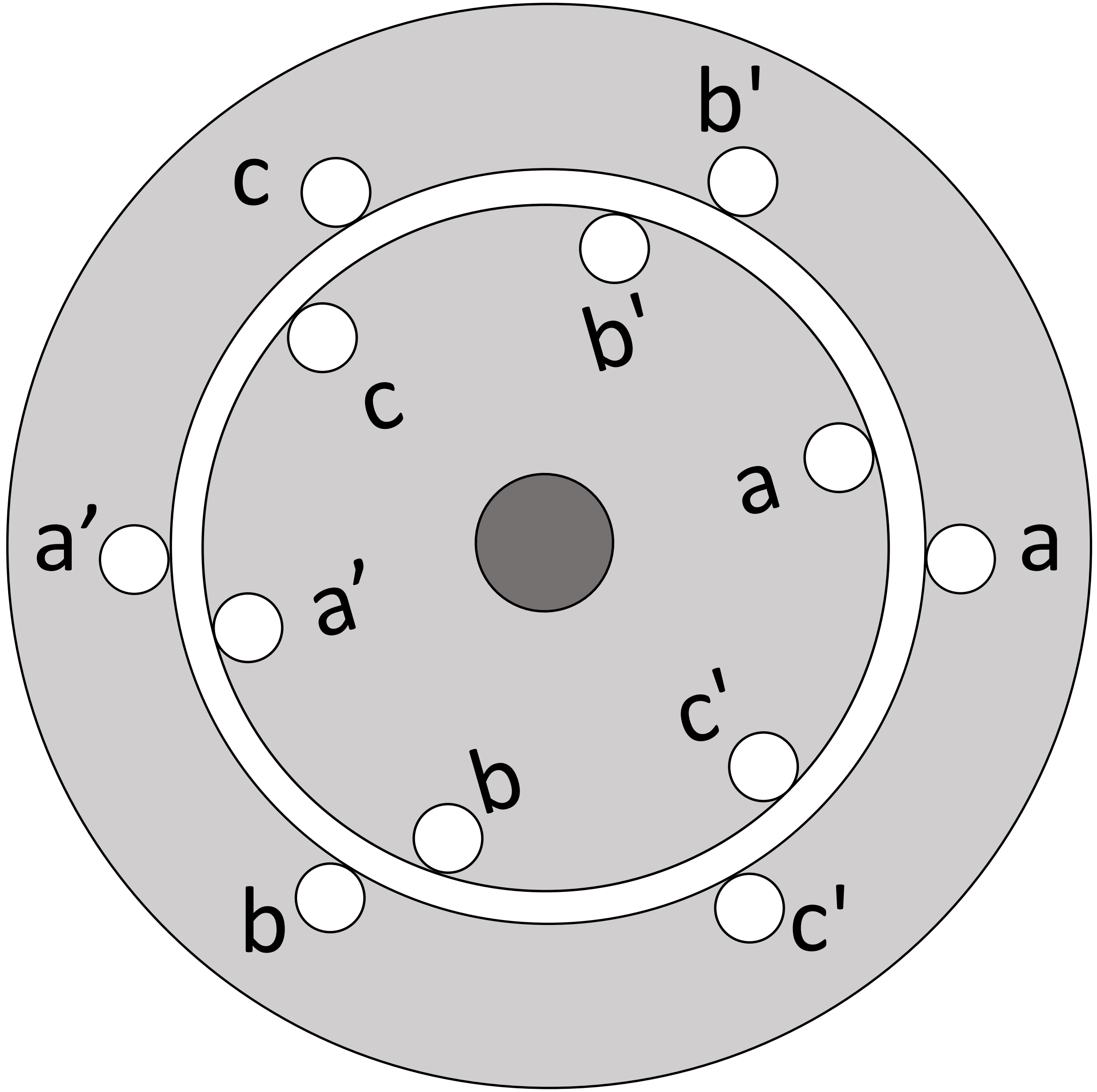
# Structure of an induction machine




## Stator of an AC machine

- Asynchronous machine or Induction machine

# Structure of an induction machine




## Stator of an AC machine

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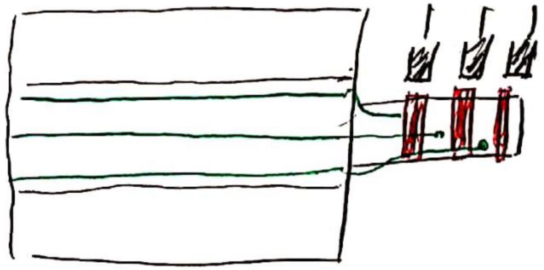
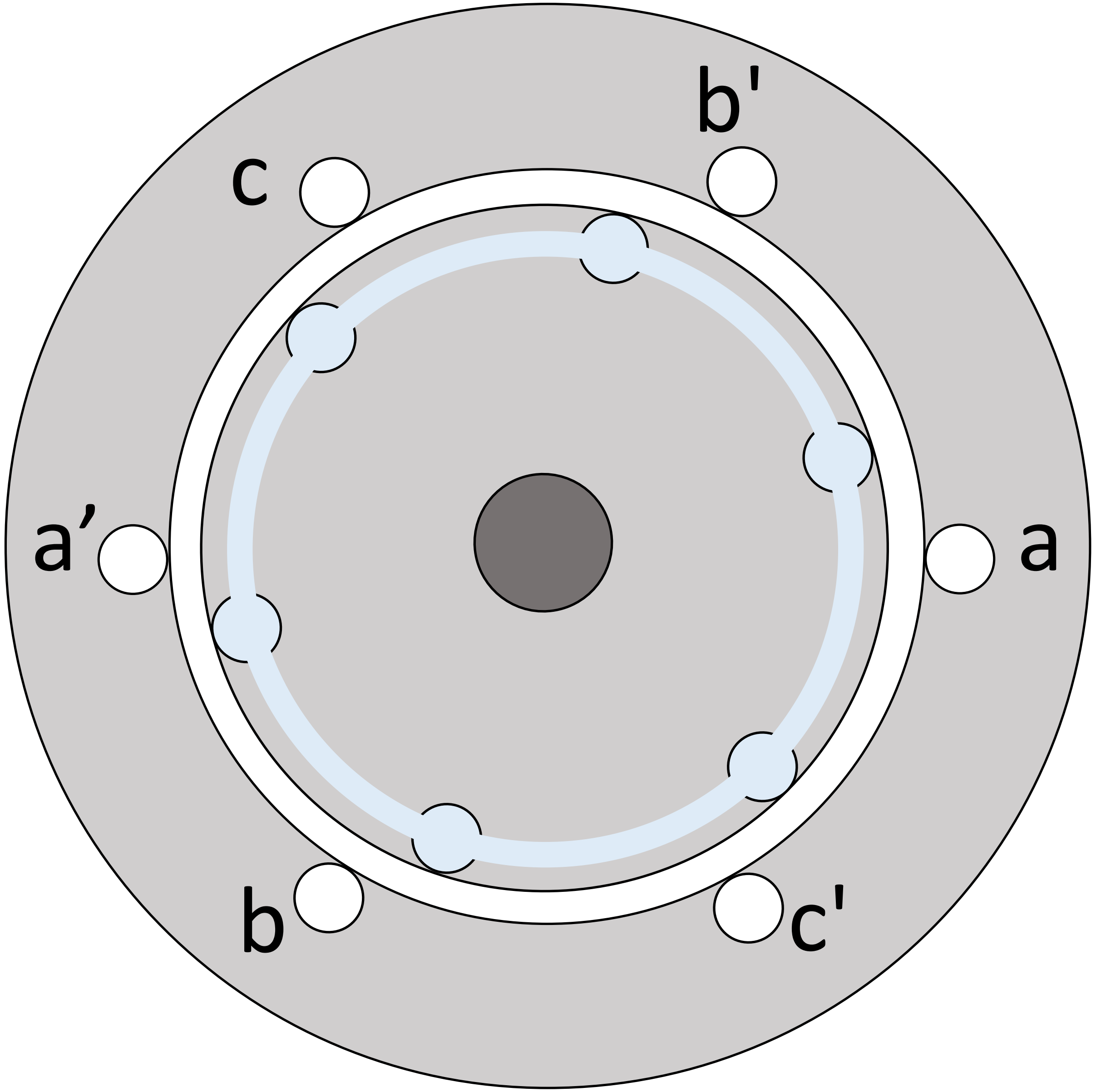


Wound-rotor Induction machine

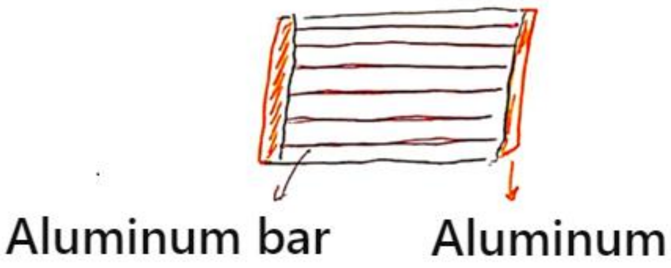
Pros and cons??!!

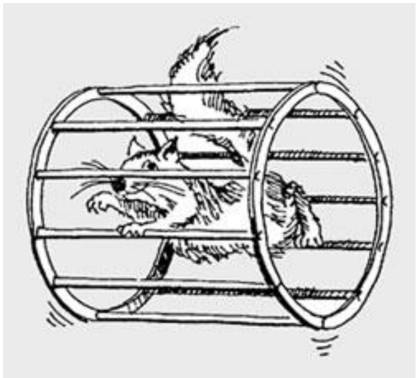
# Structure of an induction machine




## Stator of an AC machine

- Asynchronous machine or Induction machine



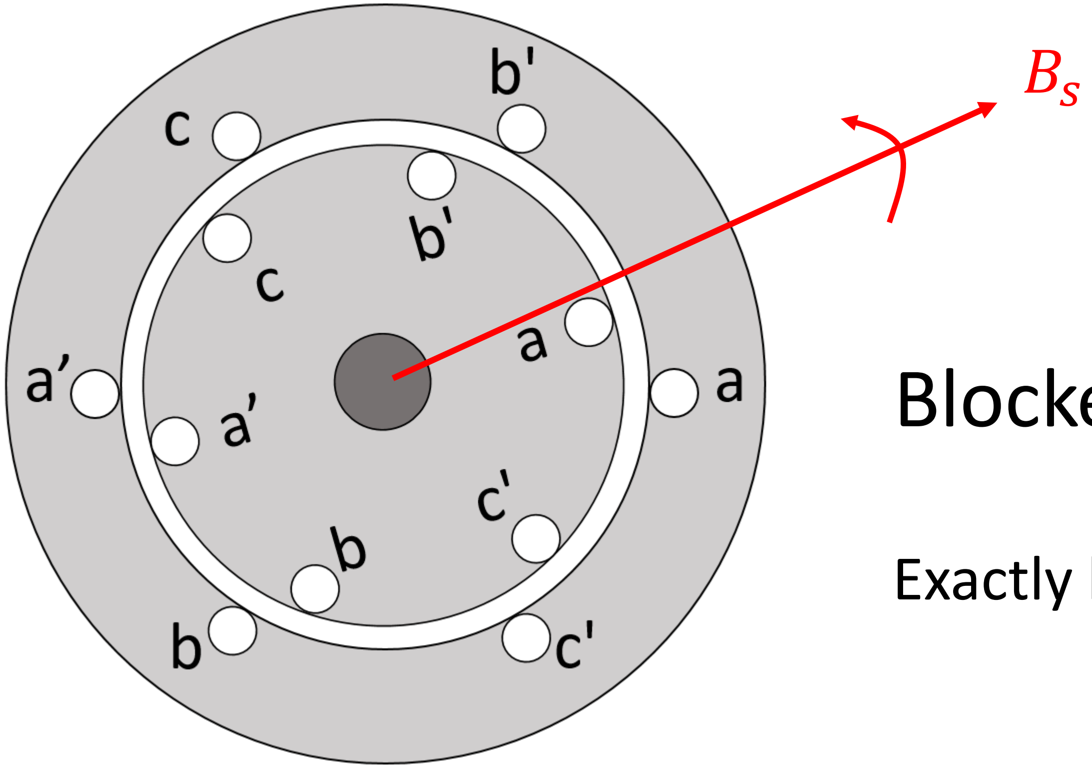



Squirrel-cage Induction machine

Pros and cons??!!

# Structure of an induction machine

Let: $I_a = I_m \sin(2\pi f_s t)\,, I_b = I_m \sin(2\pi f_s t + 120)\,, I_c = I_m \sin(2\pi f_s t - 120)$




$B = B_{max} sin\theta$

$\emptyset_{max} = 2rlB_{max}$

$e_s = N_s \frac{d\emptyset}{dt}$

$E_s = 4.44 N_s f_s \emptyset_{max}$

## Blocked rotor:

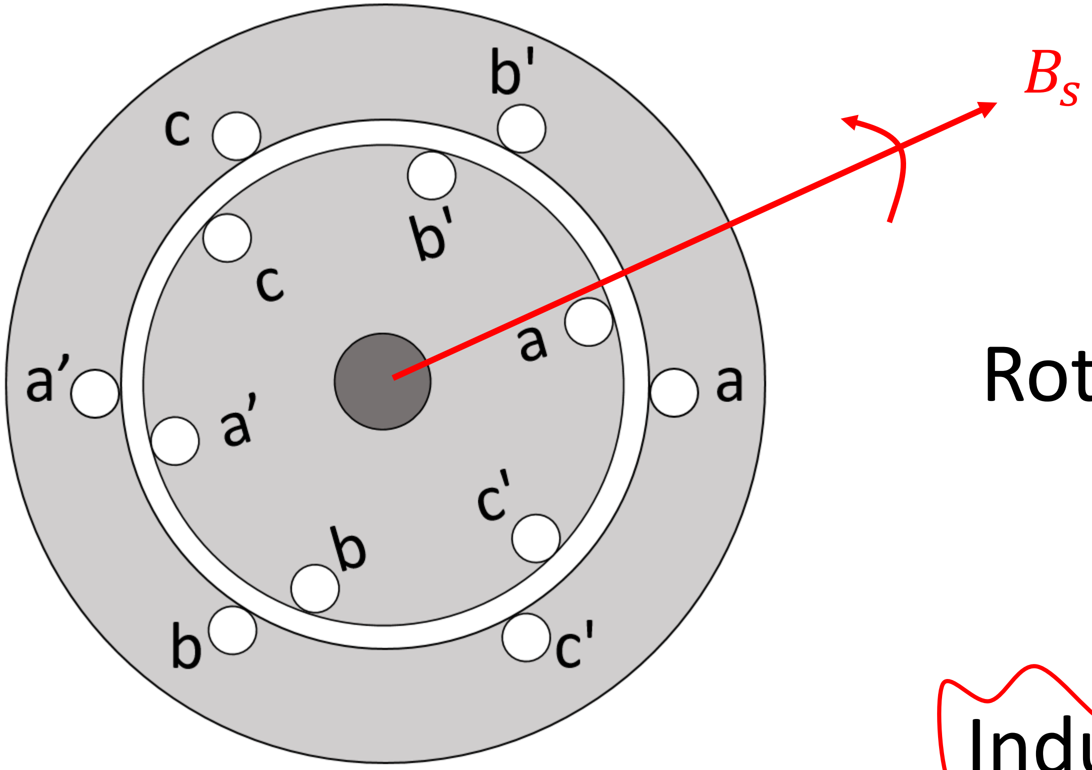
Exactly like a transformer

$E_r = 4.44 N_r f_s \emptyset_{max}$

$$\frac{E_s}{E_r} = \frac{N_s}{N_r}$$

# Structure of an induction machine

Let: $I_a = I_m \sin(2\pi f_s t)\,, I_b = I_m \sin(2\pi f_s t + 120)\,, I_c = I_m \sin(2\pi f_s t - 120)$




$$B = B_{max} sin\theta \qquad \emptyset_{max} = 2rlB_{max}$$

$$e_s = N_s \frac{d\emptyset}{dt} \qquad E_s = 4.44 N_s f_s \emptyset_{max}$$

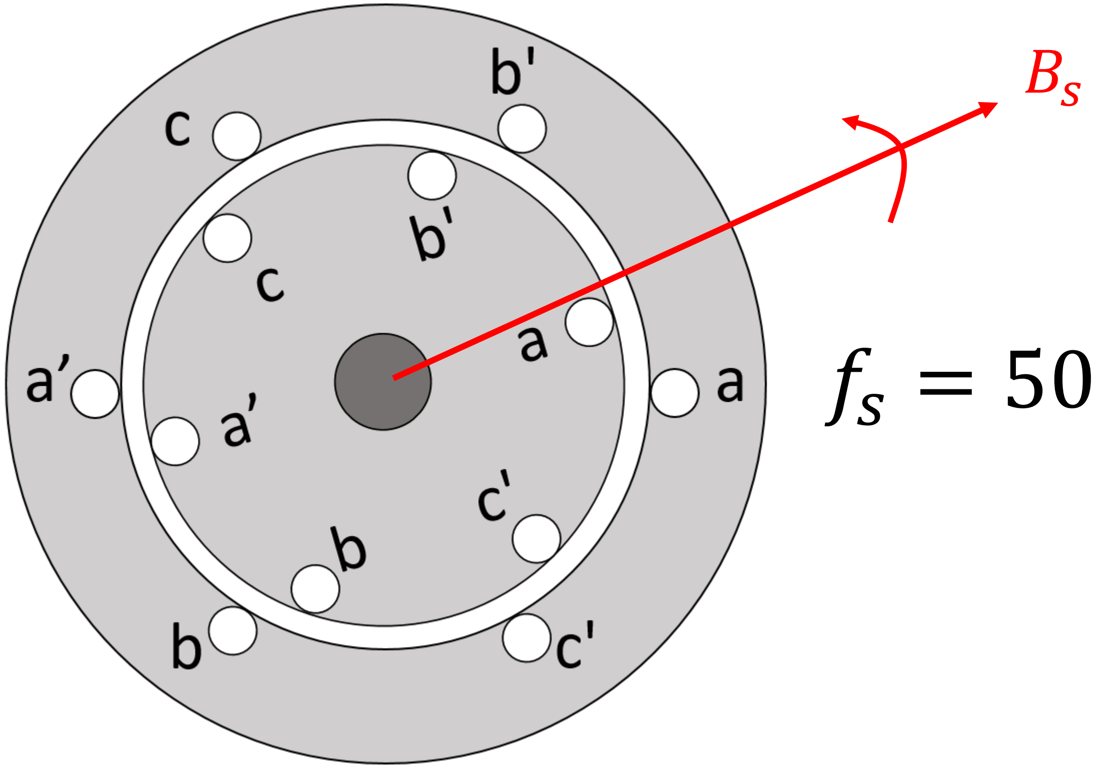
Rotor, rotates at synchronous speed

$$E_r = 0 \qquad \cancel{\frac{E_s}{E_r} = \frac{N_s}{N_r}}$$

Induction machine never rotates at synchronous speed

# Structure of an induction machine

Let: $I_a = I_m \sin(2\pi f_s t)\,, I_b = I_m \sin(2\pi f_s t + 120)\,, I_c = I_m \sin(2\pi f_s t - 120)$




$$B = B_{max} sin\theta \qquad \emptyset_{max} = 2rlB_{max}$$

$$e_s = N_s \frac{d\emptyset}{dt} \qquad E_s = 4.44 N_s f_s \emptyset_{max}$$

$f_s = 50$ and rotor rotates at 40 rps(2400 rpm)

$$E_r = 4.44 N_r f_r \emptyset_{max} \qquad \frac{E_s}{E_r} = \frac{N_s f_s}{N_r f_r}$$

# Structure of an induction machine

Let: $I_a = I_m \sin(2\pi f_s t)\,, I_b = I_m \sin(2\pi f_s t + 120)\,, I_c = I_m \sin(2\pi f_s t - 120)$

$$n_s = \frac{120 f_s}{p} \ \text{(rpm)} \qquad n_s = \frac{2 f_s}{p} \ \text{(rps)}$$

$n_s$ =Synchronous speed

$$\omega_s = \frac{4\pi f_s}{p} \ \text{(rad/s)}$$

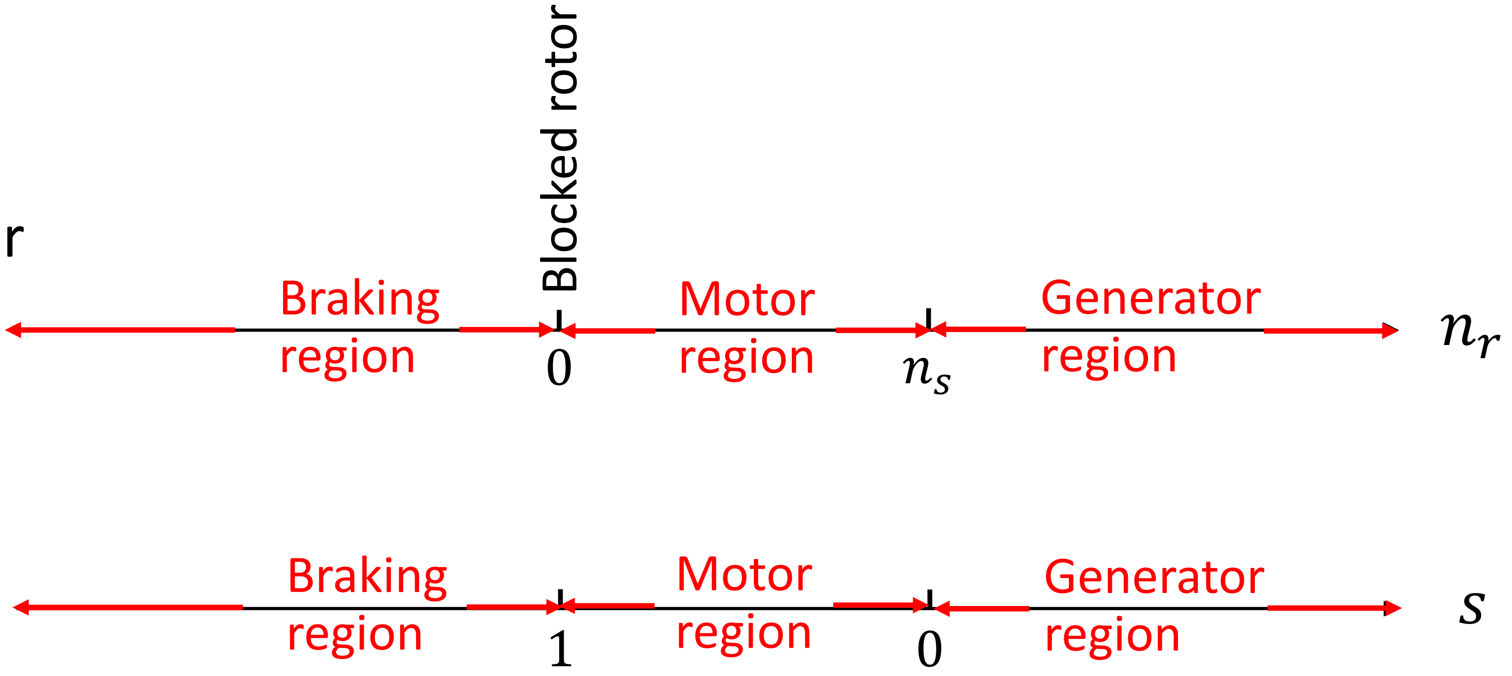
$s$ =Slip

$$s = \frac{n_s - n_r}{n_s}$$

# Different region according to s

$$s = \frac{n_s - n_r}{n_s}$$

$n_r = 0 \qquad s = 1$ Blocked rotor




$n_r = n_s \qquad s = 0$

$0 < n_r < n_s \qquad 0 < s < 1$ Motor region

$n_r < 0 \qquad s > 1$ Braking region

$n_r > n_s \qquad s < 0$ Generator region

# Structure of an induction machine

$$s = \frac{n_s - n_r}{n_s}$$

$$n_r = (1 - s)n_s$$

Case 1: Consider a 2-pole 50 Hz induction machine. Discuss the rotor voltage frequency.

$$n_s = \frac{2f_s}{p} = 50 \ \ (\text{rps})$$

Let $n_r = 0$ $\quad s = 1$ $\quad f_r = 50\ Hz$

Let $n_r = 10\ rps$ $\quad s = 0.8$ $\quad f_r = 40\ Hz$

Let $n_r = 40\ rps$ $\quad s = 0.2$ $\quad f_r = 10\ Hz$

Let $n_r = 49\ rps$ $\quad s = 0.02$ $\quad f_r = 1\ Hz$

$$f_r = sf_s$$

# Structure of an induction machine

Case 2: Consider an induction machine. Discuss the rotor voltage(rotor is open circuit).

$$E_s = 4.44 N_s f_s \emptyset_{max}$$

$$E_r = 4.44 N_r f_r \emptyset_{max}$$

$$\frac{E_s}{E_r} = \frac{N_s f_s}{N_r f_r}$$

$$\frac{E_s}{E_r} = \frac{N_s f_s}{N_r s f_s}$$

$E_{Br}$

$$E_r = s \frac{N_r}{N_s} E_s$$

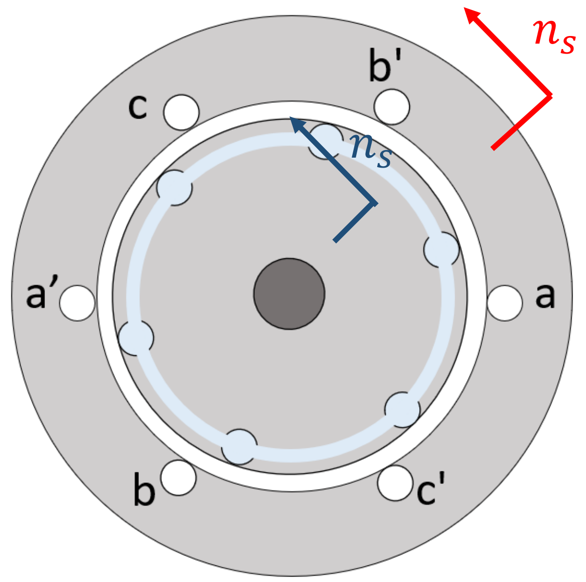
As the $n_r \rightarrow n_s, s \rightarrow 0$ and thus rotor voltage and rotor voltage frequency converges to 0.

# Magnetic fields in induction machines

Case 3: Consider an induction machine. Discuss the rotor voltage and rotor circuit when rotor winding is close.

$n_r = 0 \quad s = 1$ Blocked rotor $\quad f_r = sf_s \; = f_s$

$$Z_r = R_r + iL_r\omega_r \; = R_r + isL_r\omega_s \; = R_r + iL_r\omega_s \; = R_r + iX_{Br}$$



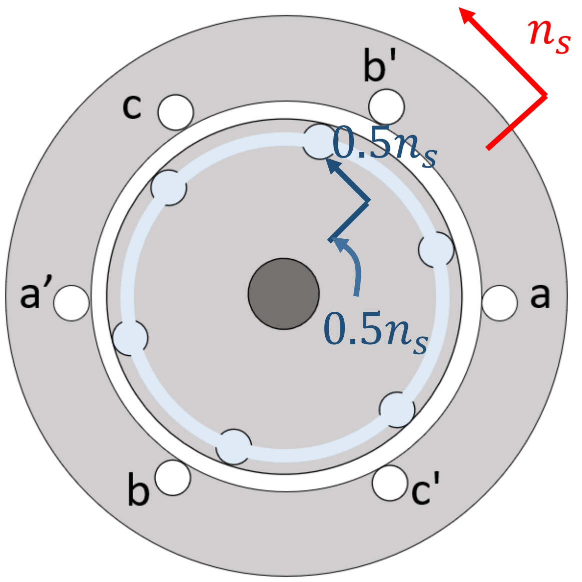

Stator magnetic field is in synchronous with rotor magnetic field but rotor speed is different from magnetic field.

# Magnetic fields in induction machines

Case 3: Consider an induction machine. Discuss the rotor voltage and rotor circuit when rotor winding is close.

$n_r = 0.5n_s$ $\quad s = 0.5$ $\quad f_r = sf_s = 0.5f_s$

$$Z_r = R_r + iL_r\omega_r = R_r + isL_r\omega_s = R_r + i0.5L_r\omega_s = R_r + i0.5X_{Br}$$



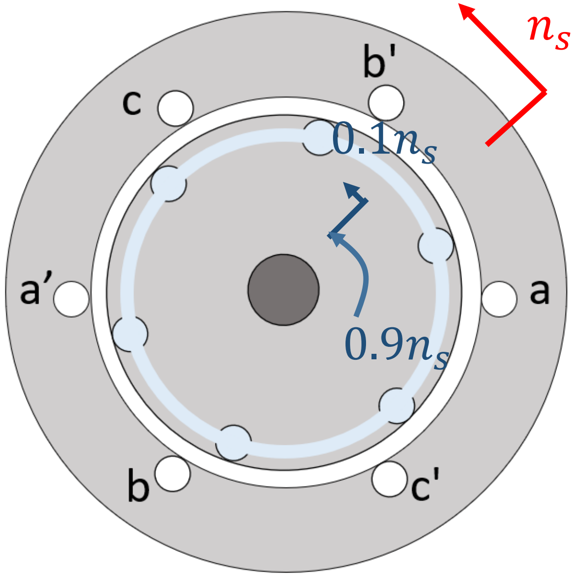

Stator magnetic field is in synchronous with rotor magnetic field but rotor speed is different from magnetic field.

# Magnetic fields in induction machines

Case 3: Consider an induction machine. Discuss the rotor voltage and rotor circuit when rotor winding is close.

$n_r = 0.9n_s$ $\quad s = 0.1$ $\quad f_r = sf_s = 0.1f_s$

$$Z_r = R_r + iL_r\omega_r = R_r + isL_r\omega_s = R_r + i0.1L_r\omega_s = R_r + i0.1X_{Br}$$




Stator magnetic field is in synchronous with rotor magnetic field but rotor speed is different from magnetic field.

This happens in any situation so we call it Asynchronous machine.

# Magnetic fields in induction machines

**Exercise 1:** The nameplate on a 400-V, 35-kW, 50-Hz, four-pole induction motor indicates that its speed at rated load is 1458 r/min. Assume the motor to be operating at rated load.

a. What is the slip of the rotor?

b. What is the frequency of the rotor currents in Hz?

c. What is the angular velocity of the stator-produced air-gap flux wave with respect to the stator in rad/sec? With respect to the rotor?

d. What is the angular velocity of the rotor-produced air-gap flux wave with respect to the stator in rad/sec? With respect to the rotor?

# Magnetic fields in induction machines

Exercise 2: A 60-Hz, two-pole, 208-V wound-rotor induction motor has a three-phase stator winding of 42 series turns/phase and a rotor winding of 38 seriesturns/phase. When operating at rated terminal voltage, the motor is observed to be operating at a speed of 3517 r/min. Calculations indicate that under this operating condition, the air-gap flux wave induces a voltage of 193 V, line-line in the stator winding. Calculate the corresponding voltage induced in the rotor winding.

# Magnetic fields in induction machines

Exercise 3: A three-phase induction motor runs at 1198 r/min at no load and 1119 r/min at full load when supplied from a 60-Hz, three-phase source.

a. How many poles does this motor have?

b. What is the slip in percent at full load?

c. What is the corresponding frequency of the rotor currents?

d. What is the speed in r/min of the rotor field with respect to the rotor? With respect to the stator?

# Magnetic fields in induction machines

**Exercise 4: (Final 2022) A three-phase, 4-pole, 50 Hz induction motor is available. This motor operates at its rated voltage and frequency with a steady-state speed of 1470 RPM.**

a) What is the synchronous speed of the stator's rotating field relative to an external observer in RPM?

b) What is the frequency of the rotor current in Hertz?

c) What is the speed of the rotor's rotating field relative to the rotor in RPM?

d) What is the speed of the rotor's rotating field relative to an external observer in RPM?

e) If both the rotor and stator are laminated, are the iron losses greater in the rotor or the stator? Why?

f) If the load on the motor increases slightly, the motor speed will slightly ___________, and if the load on the motor decreases slightly, the motor speed will slightly ___________.

# Magnetic fields in induction machines

Exercise 5: Linear induction motors have been proposed for a variety of applications including high-speed ground transportation. A linear motor based on the induction-motor principle consists of a car riding on a track. The track is a developed squirrel-cage winding, and the car, which is 6.7 m long and 1.75 m wide, has a developed three-phase, 10-pole-pair armature winding. Power at 40 Hz is fed to the car from arms extending through slots to rails below ground level.

a. What is the synchronous speed in km/hr?

b. Will the car reach this speed? Explain your answer.

c. What is the slip if the car is traveling 89 km/hr? What is the frequency of the track currents under this condition?

d. If the control system controls the magnitude and frequency of the car currents to maintain constant slip, what is the frequency of the armature-winding currents when the car is traveling 75 km/hr? What is the frequency of the track currents under this condition?

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# Equivalent Circuit of Induction Machines

# Structure of an induction machine

$$E_s = 4.44 N_s f_s \emptyset_{max}$$

$$E_r = 4.44 N_r f_r \emptyset_{max}$$

$$\frac{E_s}{E_r} = \frac{N_s f_s}{N_r f_r}$$

$$\frac{E_s}{E_r} = \frac{N_s f_s}{N_r s f_s}$$

$$E_r = s \frac{N_r}{N_s} E_s$$

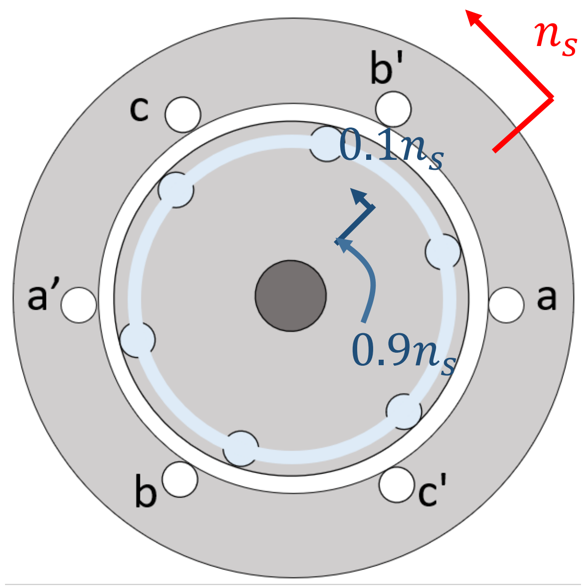
$E_{Br}$

As the $n_r \to n_s, s \to 0$ and thus rotor voltage and rotor voltage frequency converges to 0.

# Magnetic fields in induction machines

$n_r = 0.9n_s$ $s = 0.1$ $f_r = sf_s = 0.1f_s$

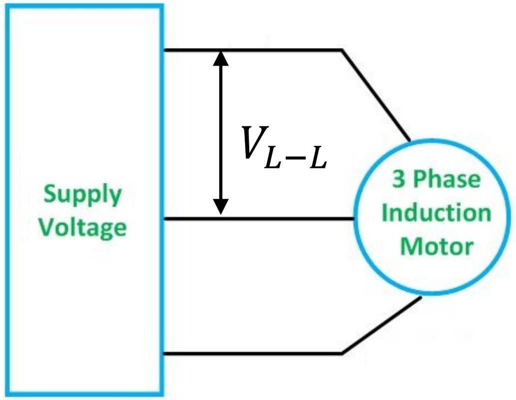
$$Z_r = R_r + jL_r\omega_r = R_r + jsL_r\omega_s = R_r + j0.1L_r\omega_s = R_r + j0.1X_{Br}$$

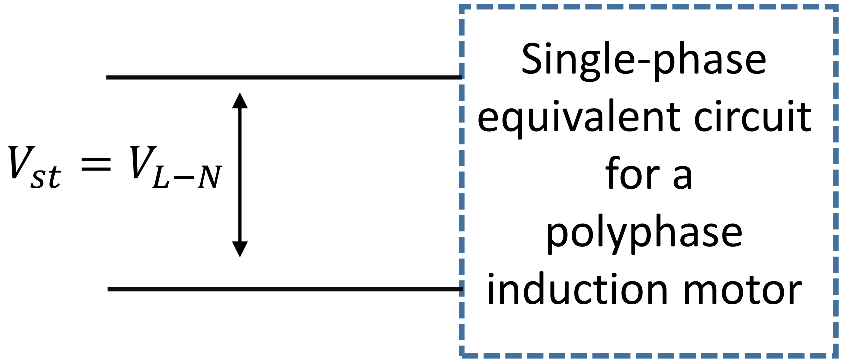



Stator magnetic field is in synchronous with rotor magnetic field but rotor speed is different from magnetic field.

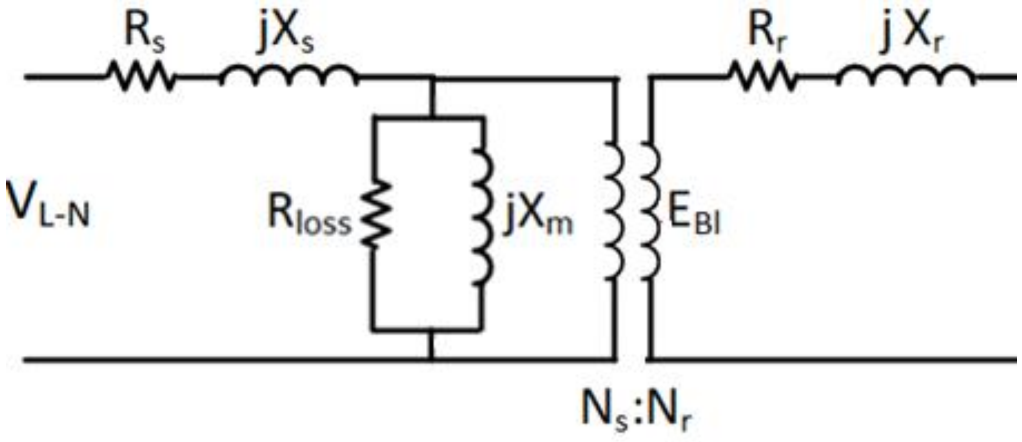
This happens in any situation so we call it asynchronous machine.

# Equivalent circuit of induction machine



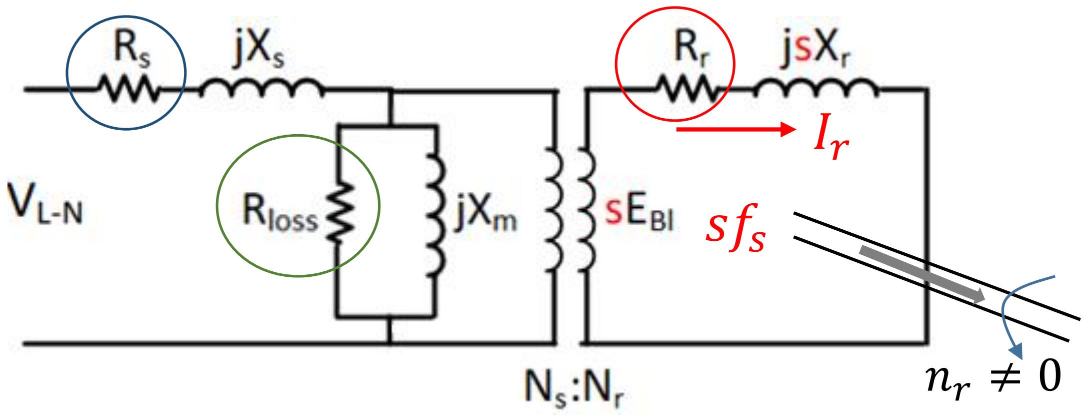

Equivalent circuit for a polyphase induction motor(rotor is open loop and blocked)

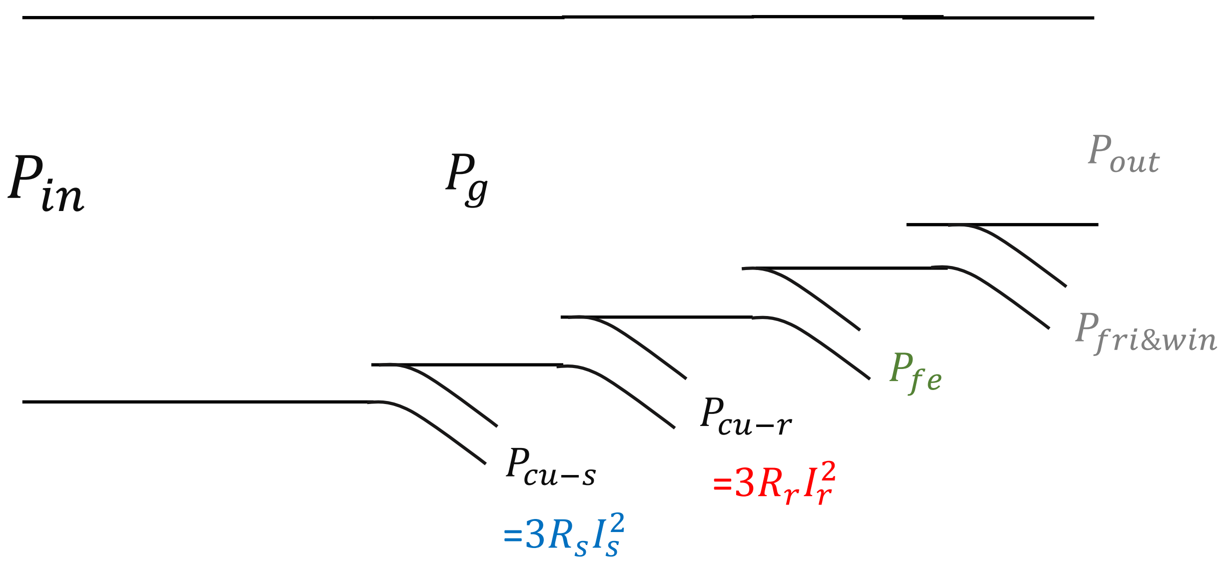



Similar to transformer

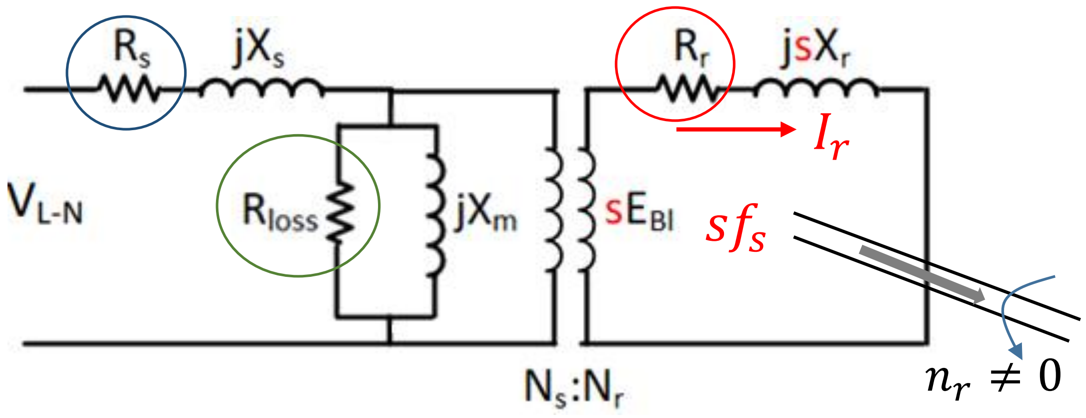
Shunt current is about a few percent in transformer and 30-50% of rated current in induction machine!

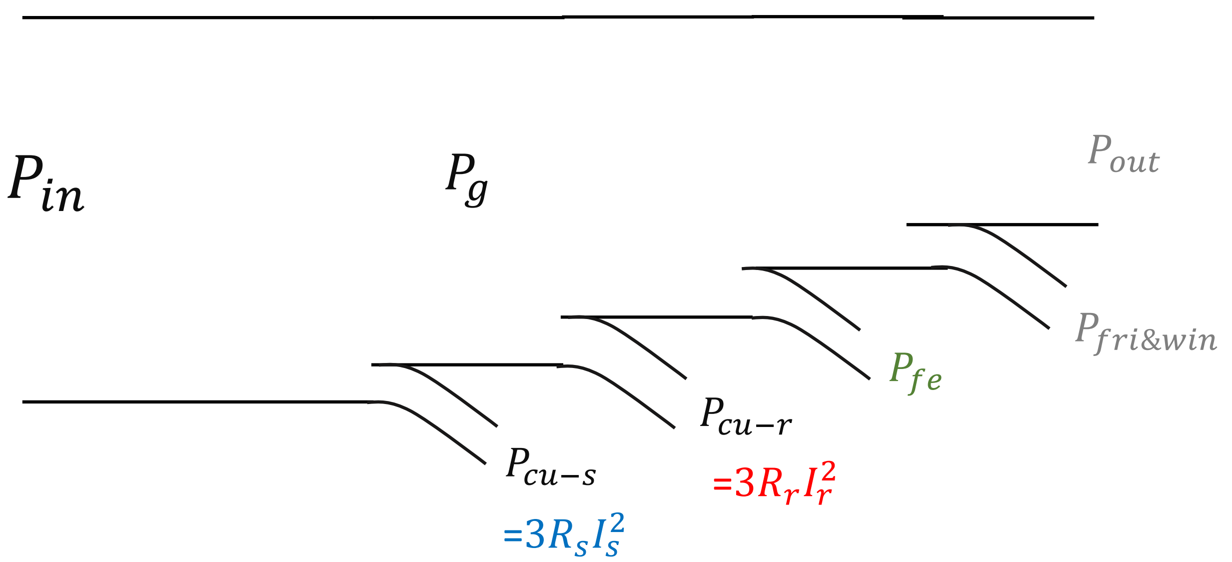
# Equivalent circuit of induction machine

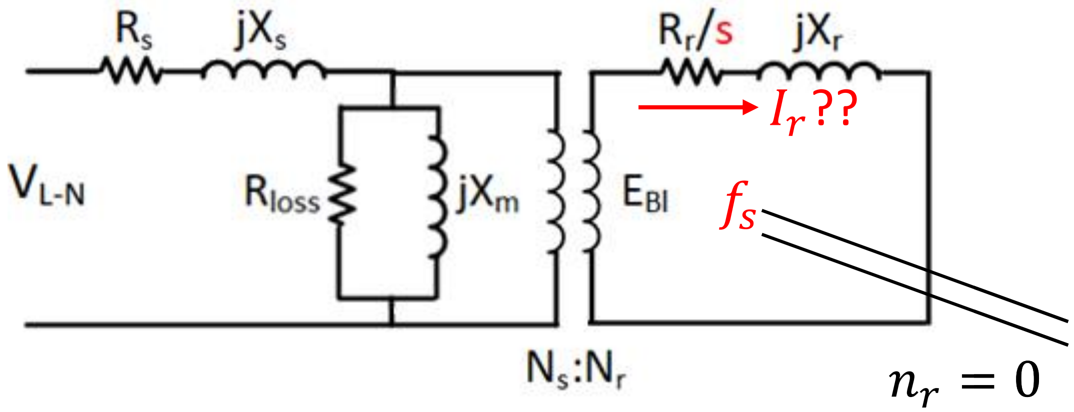


# Equivalent circuit of induction machine

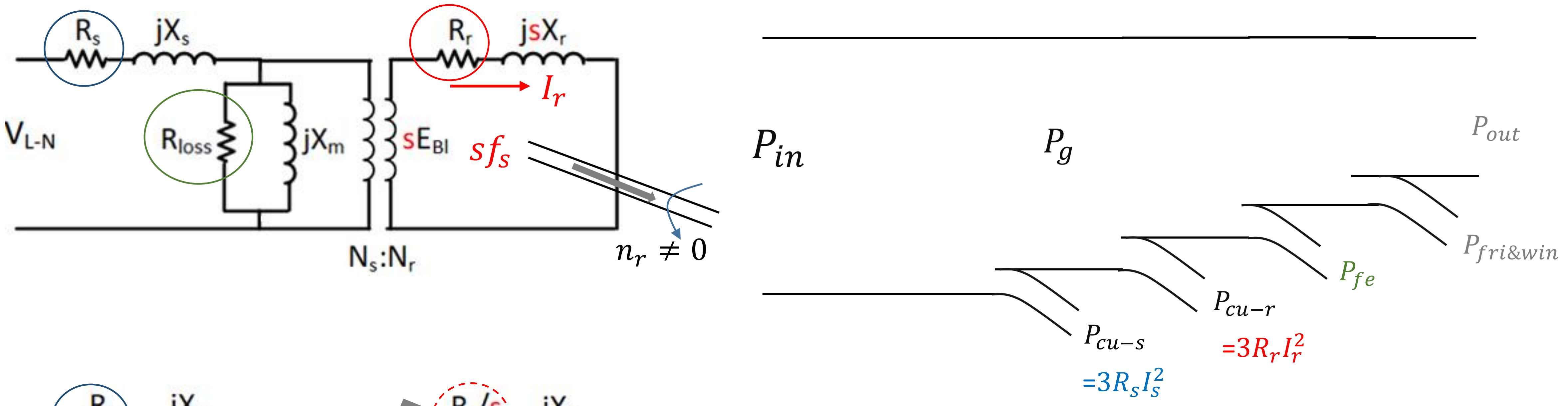


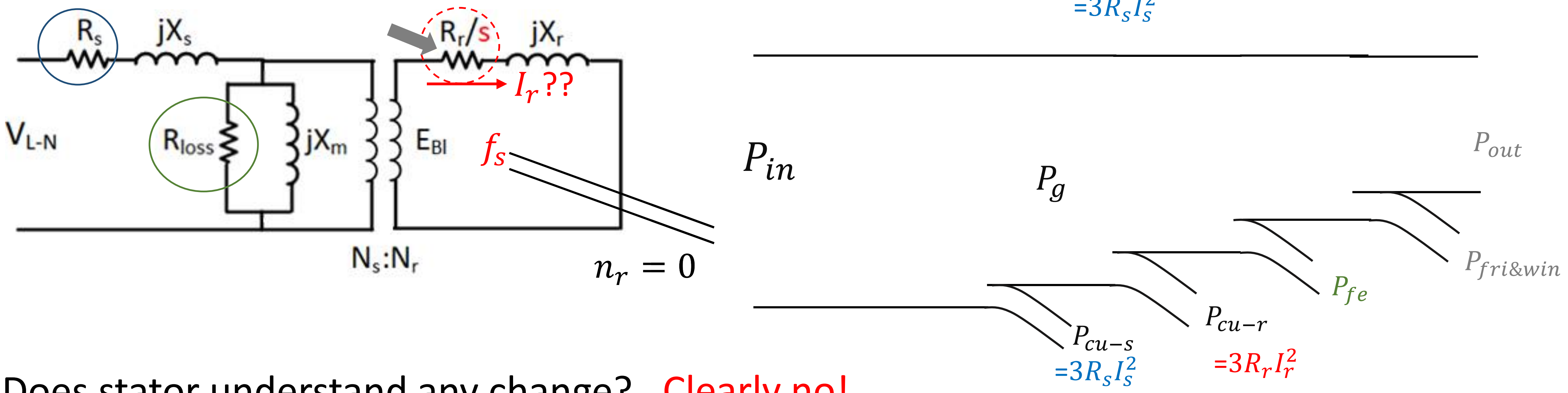




Does stator understand any change? Clearly no!

# Equivalent circuit of induction machine

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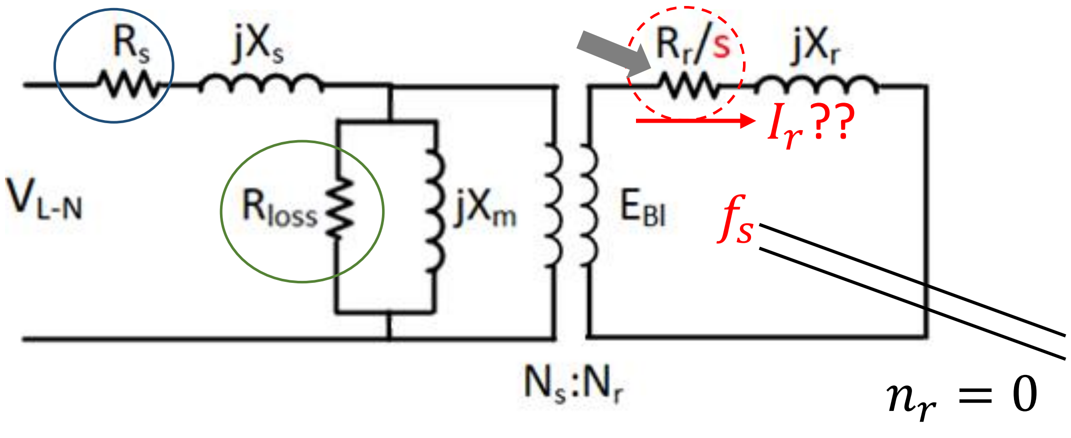
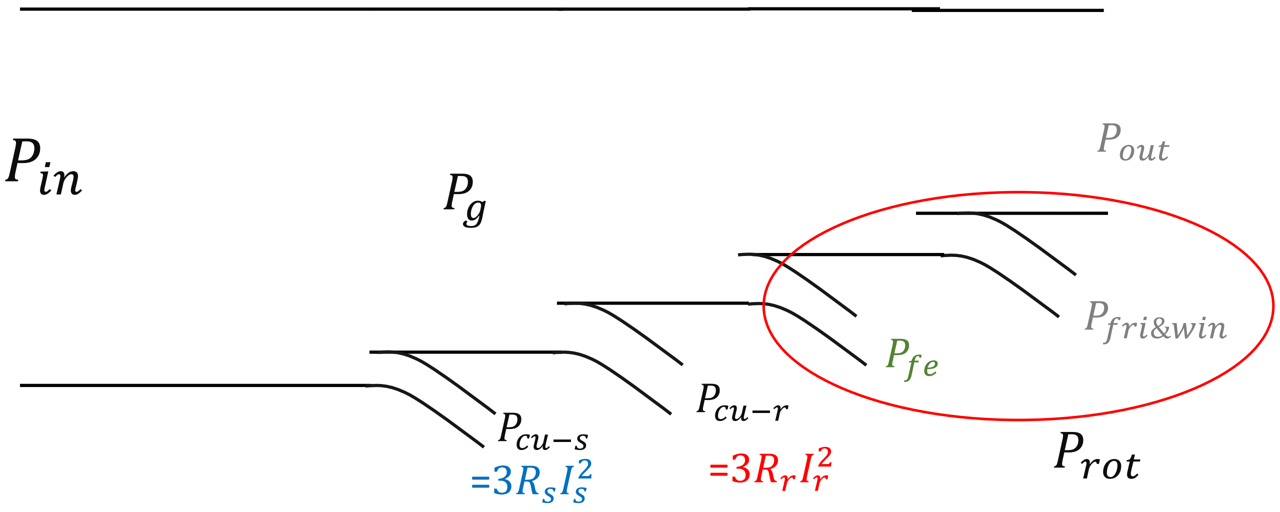
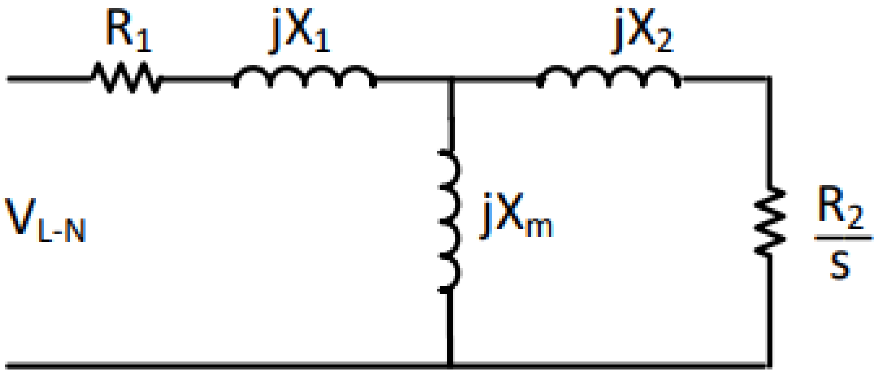



$R_1$: Stator resistance

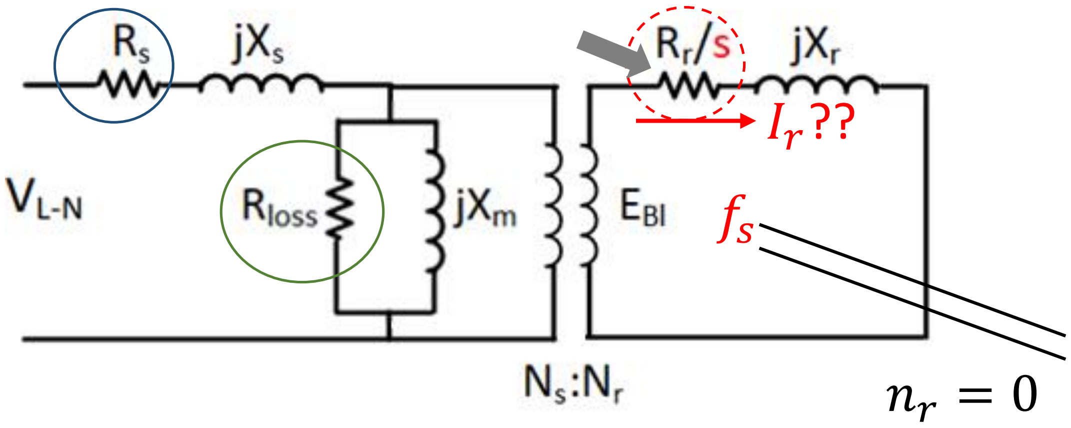
$X_1$: Stator reactance

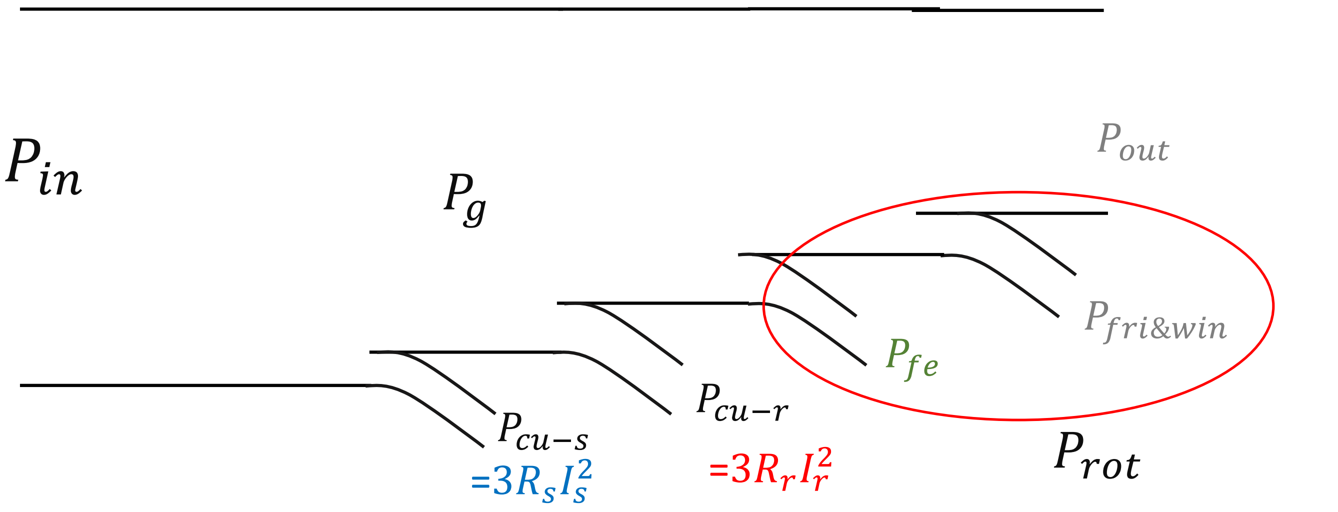
$X_m$: Magnetizing reactance

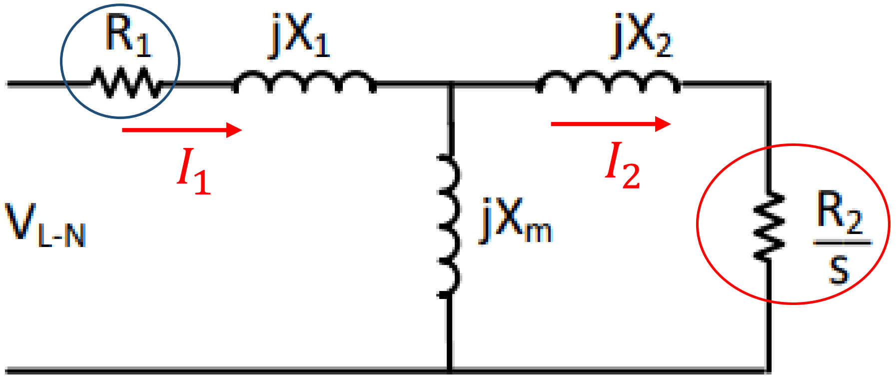
$R_2$: Rotor resistance in stator side

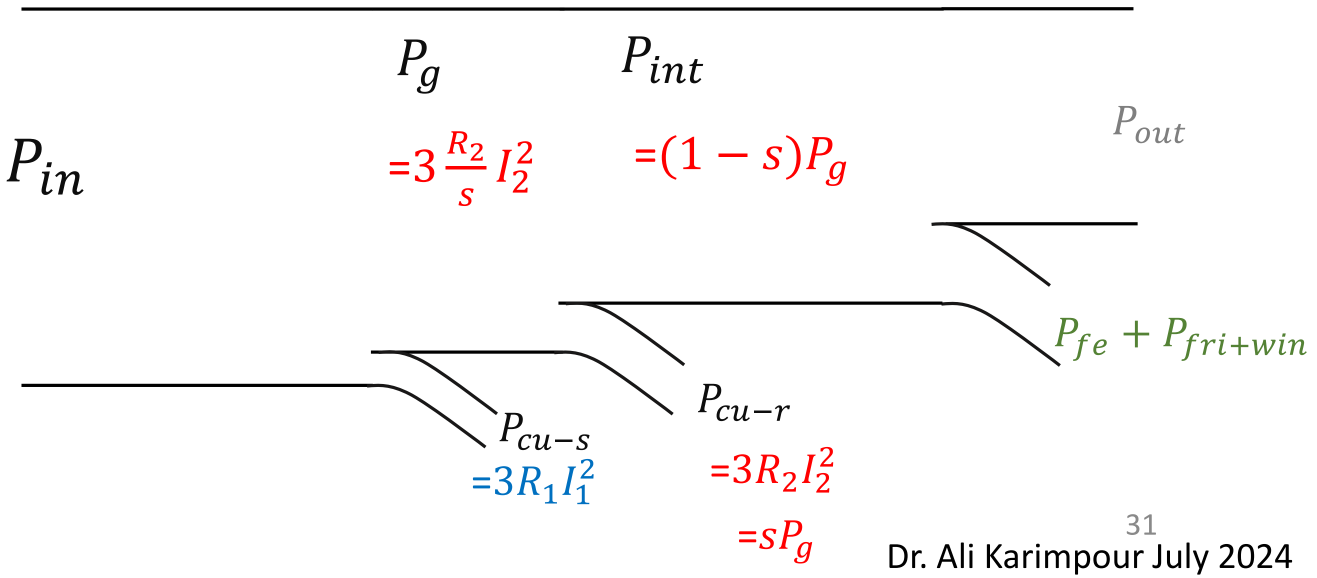
$X_2$: Rotor reactance in stator side

# Equivalent circuit of induction machine

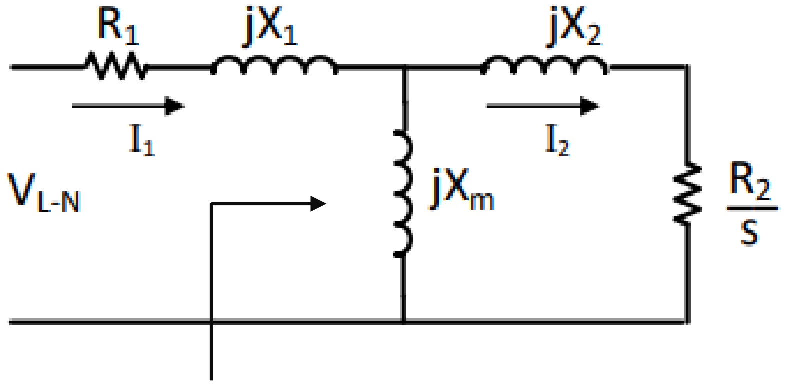


# Equivalent circuit of induction machine




$P_{in}$

$= 3V_{L-N}\, I_1 cos\varphi$

$P_g$
$= P_{in} - 3R_1 I_1^2$
$= 3R_f I_1^2$
$= 3\frac{R_2}{s} I_2^2$

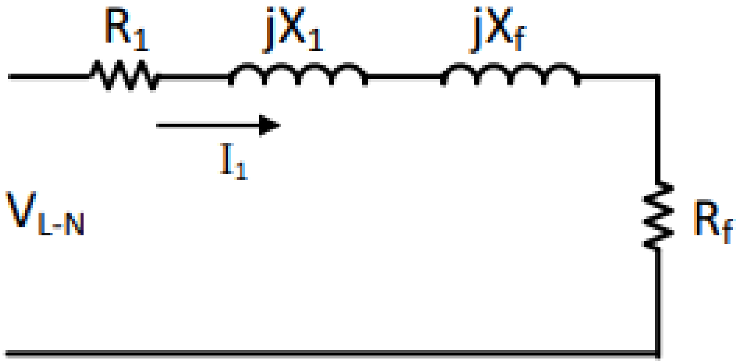
$P_{cu-s}$
$=3R_1 I_1^2$

$P_{int}$
$= (1-s)P_g$

$P_{cu-r}$
$=3R_2 I_2^2$
$=sP_g$

$P_{out}$
$= P_{int} - P_{rot}$

$P_{rot} =$
$P_{fe} + P_{windage}$

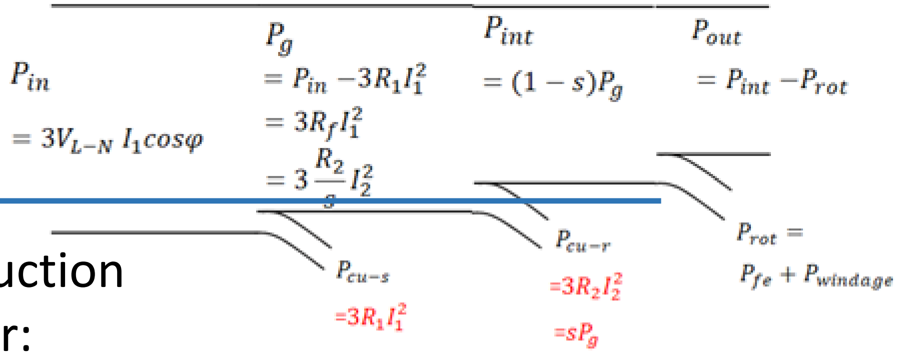



$$\tau_{int} = \frac{P_{int}}{\omega_r} = \frac{(1-s)P_g}{(1-s)\omega_s} = \frac{P_g}{\omega_s}$$

$$\boxed{\tau_{int} = \frac{P_g}{\omega_s}}$$

$$\tau_{out} = \frac{P_{out}}{\omega_r}$$

$$\eta = \frac{P_{out}}{P_{in}}$$



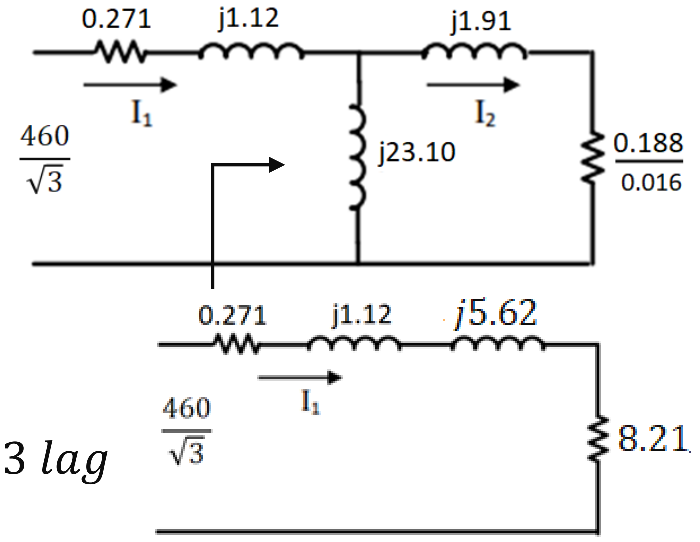

# Example 1

A three-phase Y-connected 460-V (line-to-line) 20-kW 60-Hz six-pole induction motor has the following parameter values in /phase referred to the stator:

R1 = 0.271 R2 = 0.188 X1 = 1.12 X2 = 1.91 Xm = 23.10

The total friction, windage, and core losses may be assumed to be constant at 320 W, independent of load.

a) For a slip of 1.6 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.




$$n_s = \frac{120 * 60}{6} = 1200 \qquad n_r = (1 - 0.016) * 1200 = 1181\, rpm$$

$$R_f + jX_f = (R_2/s + jX_2)||\mathrm{j}X_m = 8.21 + j5.62$$

$$\eta = 93.2\%$$

$$I_1 = \frac{460/\sqrt{3}}{(0.271 + 8.21) + j(1.12 + 5.62)} = 24.5 < -38.5° \qquad pf = \cos(-38.5°) = 0.783\, lag$$

$$P_{int} = (1 - 0.016) 3R_f I_1^2 = 14548\, w \quad P_{out} = 14548 - 320 = 14228\, w \quad \tau_{out} = \frac{14228}{2\pi * 1181/60} = 115\, N.m$$

# Example 1

A three-phase Y-connected 460-V (line-to-line) 20-kW 60-Hz six-pole induction motor has the following parameter values in /phase referred to the stator:

R1 = 0.271     R2 = 0.188     X1 = 1.12     X2 = 1.91     Xm = 23.10

The total friction, windage, and core losses may be assumed to be constant at 320 W, independent of load.

b) For a slip of 1 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.

c) For a rated output power, compute the speed, output torque, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.

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3. DC Machines

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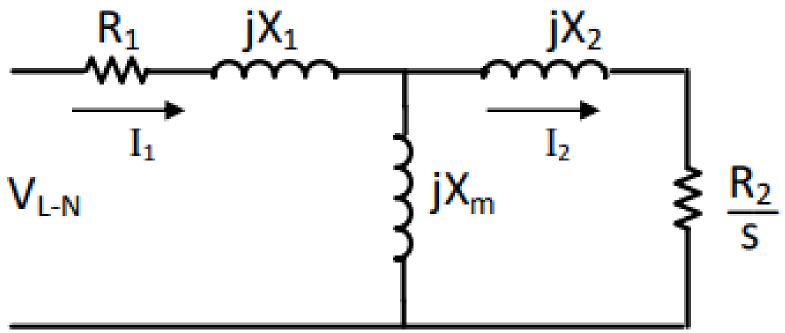
4. Induction Machines
   - Introduction to Polyphase Induction Machines
   - Currents and Fluxes in Polyphase Induction Machines
   - Induction-Motor Equivalent Circuit
   - Analysis of the Equivalent Circuit
   - Torque and Power by Use of Thevenin’s Theorem
   - Parameter Determination from No-Load and Blocked-Rotor Tests
   - Effects of Rotor Resistance; Wound and Double-Squirrel-Cage Rotors

# Torque – Speed Characteristic in Induction Machines

# Torque-Speed Characteristic in induction machine




$P_{in}$

$= 3V_{L-N}\, I_1 cos\varphi$

$P_g$
$= P_{in} - 3R_1 I_1^2$
$= 3R_f I_1^2$
$= 3\frac{R_2}{s} I_2^2$

$P_{cu-s}$

$P_{int} = (1-s)P_g$

$\tau_{int} = \frac{P_g}{\omega_s}$

$P_{cu-r} = sP_g$

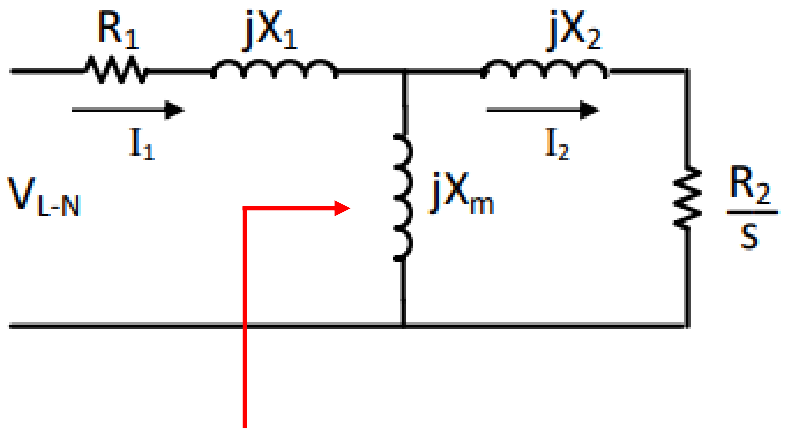
$P_{out} = P_{int} - P_{rot}$

$\tau_{out} = \frac{P_{out}}{\omega_r}$

$P_{rot} = P_{fe} + P_{windage}$

$\tau_{int} = f(s) or\ g(n_r)??$

# Torque-Speed Characteristic in induction machine




$P_{in}$

$= 3V_{L-N}\, I_1 cos\varphi$

$P_g$

$= P_{in} - 3R_1 I_1^2$

$= 3R_f I_1^2$

$= 3\frac{R_2}{s} I_2^2$

$P_{int} = (1-s)P_g$

$P_{out} = P_{int} - P_{rot}$

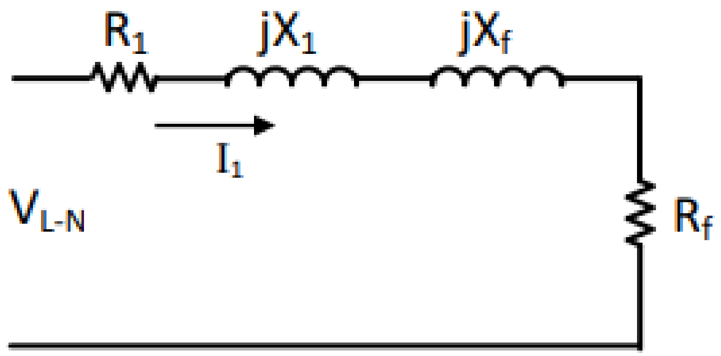
$\tau_{int} = \frac{P_g}{\omega_s}$

$\tau_{out} = \frac{P_{out}}{\omega_r}$

$P_{cu-s}$

$P_{cu-r} = sP_g$

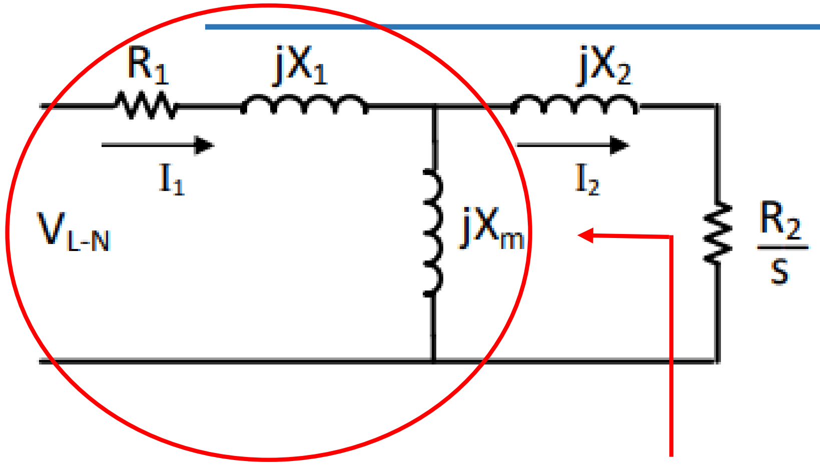
$P_{rot} = P_{fe} + P_{windage}$




$\tau_{int} = \frac{P_g}{\omega_s}$ ??

$n_r or\ s$ ??

# Torque-Speed Characteristic in induction machine




$$P_{in} = 3V_{L-N}\, I_1 cos\varphi$$

$$P_g = P_{in} - 3R_1 I_1^2 = 3R_f I_1^2 = 3\frac{R_2}{s} I_2^2$$

$$P_{cu-s}$$

$$P_{int} = (1-s)P_g$$

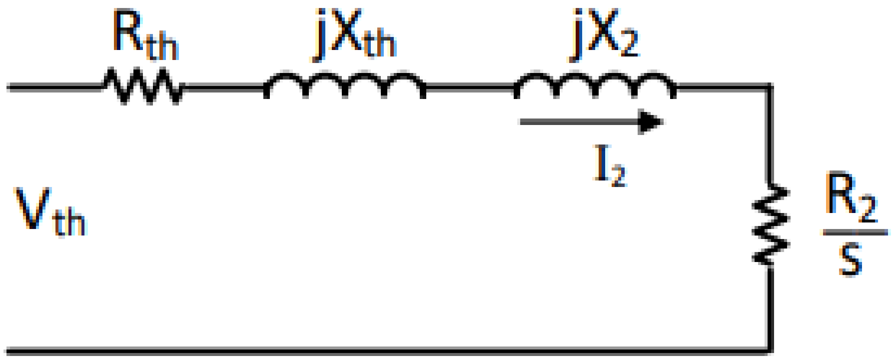
$$\tau_{int} = \frac{P_g}{\omega_s}$$

$$P_{cu-r} = sP_g$$

$$P_{out} = P_{int} - P_{rot}$$

$$\tau_{out} = \frac{P_{out}}{\omega_r}$$

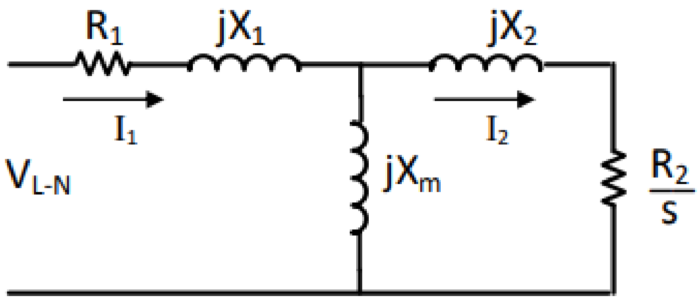
$$P_{rot} = P_{fe} + P_{windage}$$

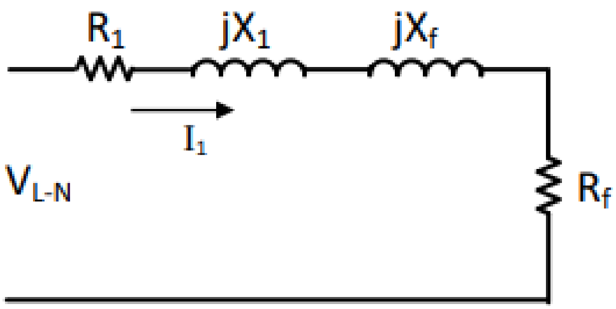


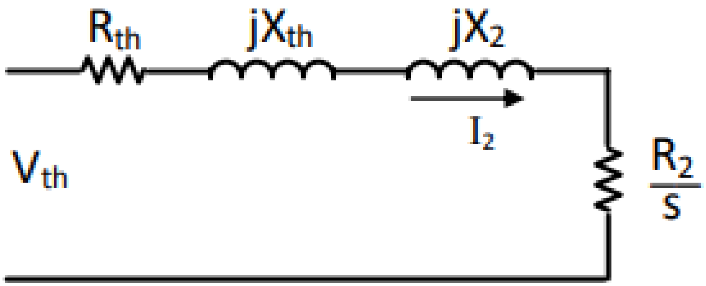

$$V_{th} = \frac{V_{L-N}}{R_1 + j(X_1 + X_m)} jX_m$$

$$R_{th} + jX_{th} = jX_m || (R_1 + jX_1) = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

# Torque-Speed Characteristic in induction machine




$P_{in} =??$

$pf =??$

$I_1 =??$

$P_{cu-st} =??$

$\eta =??$

$I_2 =??$

$P_g =?? \quad = 3\frac{R_2}{s}I_2^2$

$P_{in} =??$

$pf =??$

$I_1 =??$

$P_{cu-st} =??$

$\eta =??$

$I_2 =??$

$P_g =?? \quad = 3R_f I_1^2$

$P_{in} =??$

$pf =??$

$I_1 =??$

$P_{cu-st} =??$

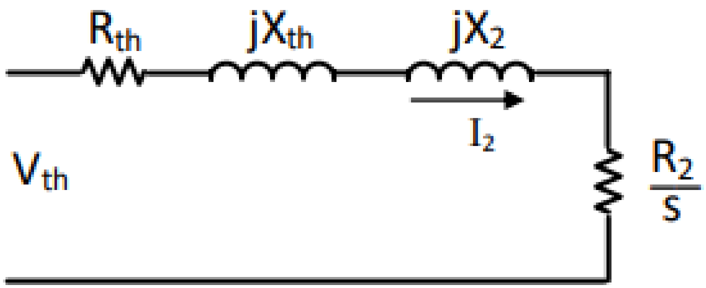
$\eta =??$

$I_2 =??$

$P_g =?? \quad = 3\frac{R_2}{s}I_2^2$

$$\tau_{int} = f(s) or\ g(n_r)??$$

# Torque-Speed Characteristic in induction machine



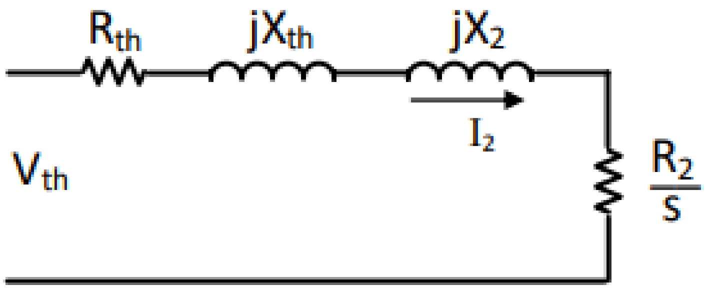

$$\tau_{int} = f(s) or\ g(n_r)??$$

$$P_g = 3\frac{R_2}{s}I_2^2 \quad = 3\frac{V_{th}^2\frac{R_2}{s}}{(R_{th}+\frac{R_2}{s})^2+(X_{th}+X_2)^2}$$

$$\tau_{int} = \frac{P_g}{\omega_s}$$

$$\tau_{int} = \frac{3}{\omega_s}\frac{V_{th}^2\frac{R_2}{s}}{(R_{th}+\frac{R_2}{s})^2+(X_{th}+X_2)^2}$$

# Torque-Speed Characteristic in induction machine

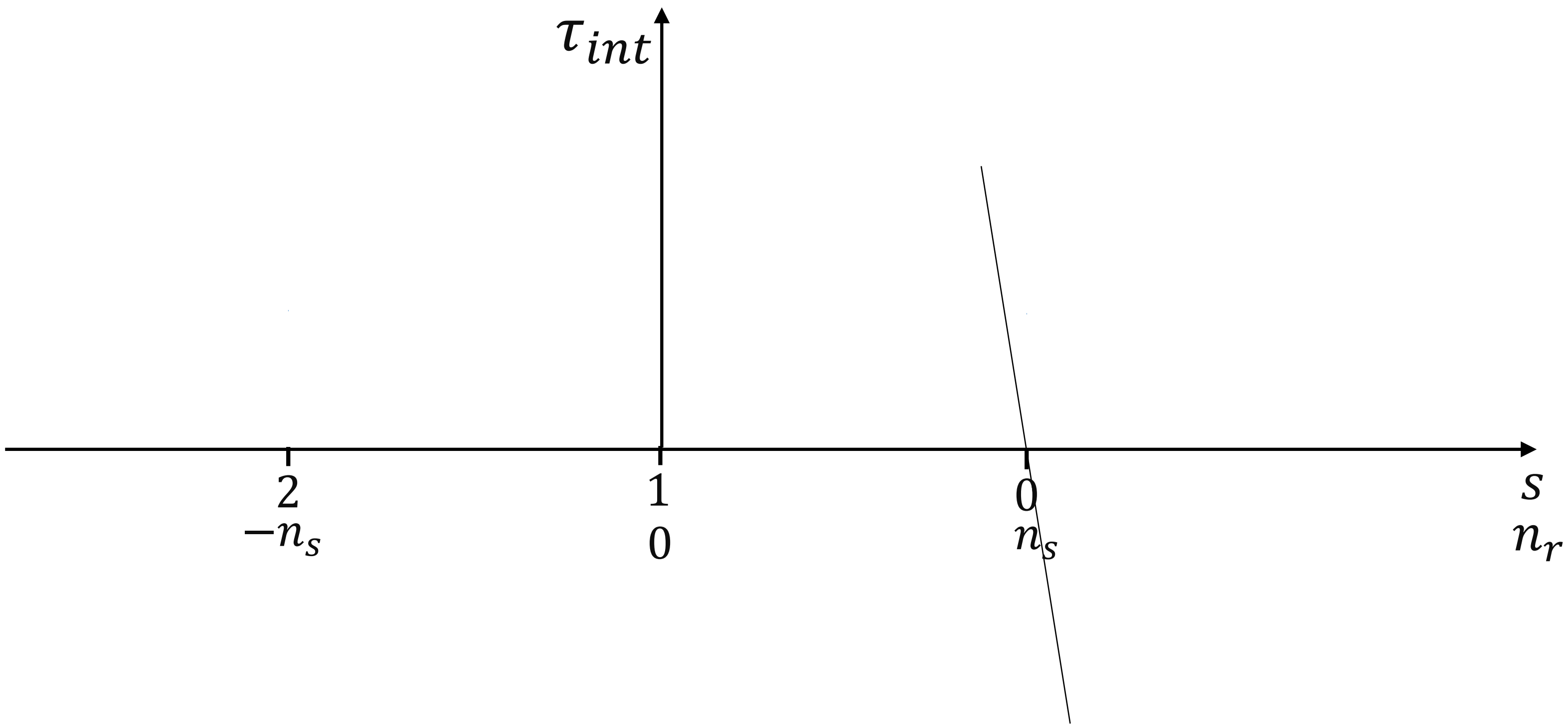



$$\tau_{int} = \frac{3}{\omega_s} \frac{V_{th}^2 \frac{R_2}{s}}{(R_{th} + \frac{R_2}{s})^2 + (X_{th} + X_2)^2}$$

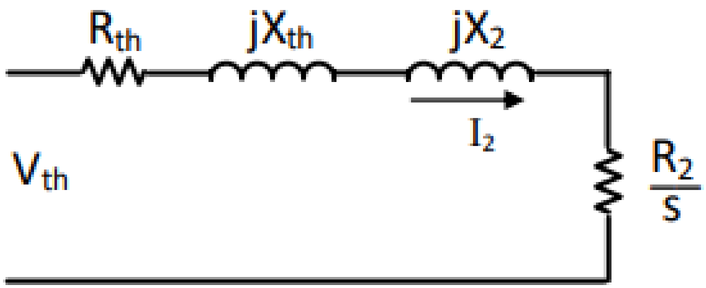
$$|s| \longrightarrow 0$$

$$\tau_{int} \cong \frac{3}{\omega_s} \frac{V_{th}^2 \frac{R_2}{s}}{(\frac{R_2}{s})^2}$$

$$\tau_{int} \cong \frac{3}{\omega_s} \frac{V_{th}^2}{R_2} s$$



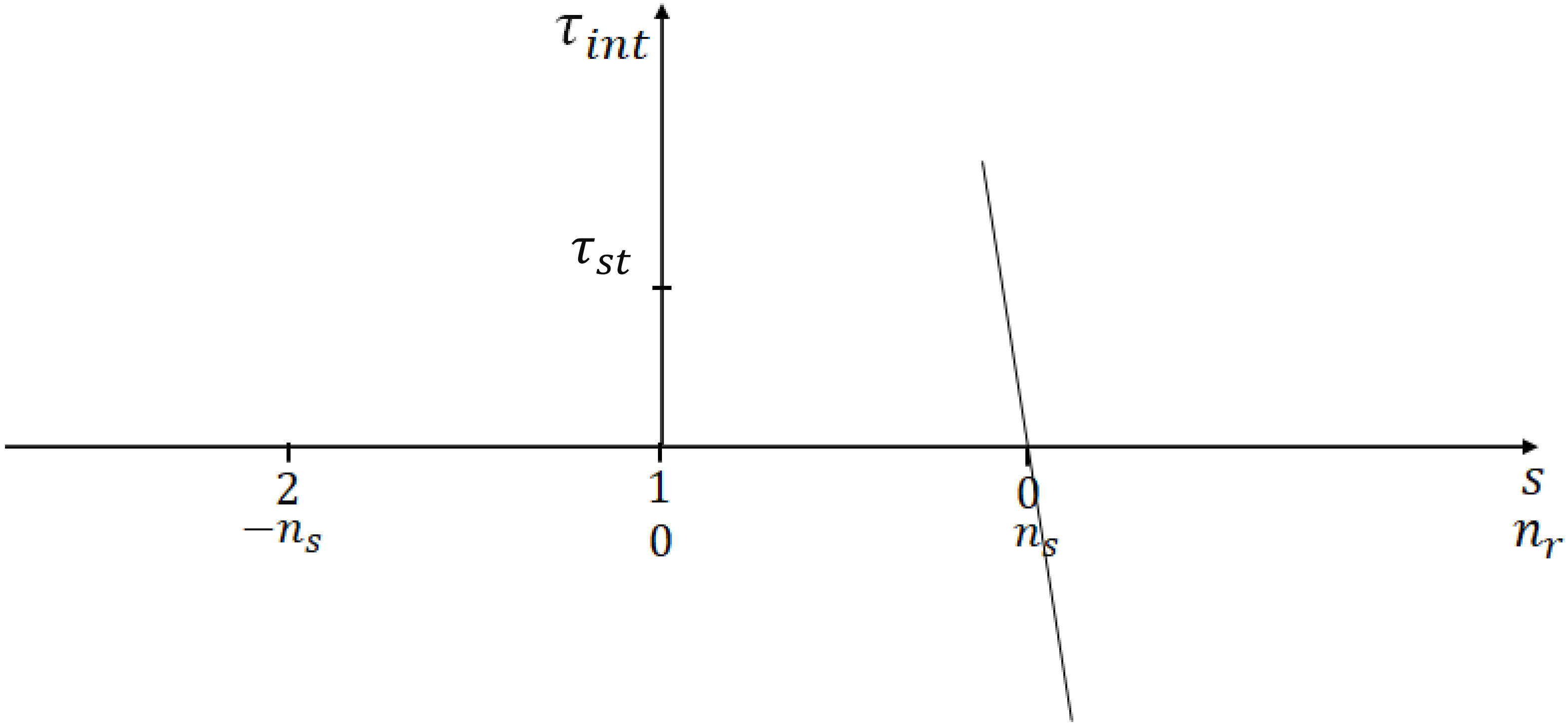
# Torque-Speed Characteristic in induction machine




$$\tau_{int} = \frac{3}{\omega_s}\frac{V_{th}^2\frac{R_2}{s}}{(R_{th}+\frac{R_2}{s})^2+(X_{th}+X_2)^2}$$

$$s = 1$$

$$\tau_{st} = \frac{3}{\omega_s}\frac{V_{th}^2 R_2}{(R_{th}+R_2)^2+(X_{th}+X_2)^2}$$



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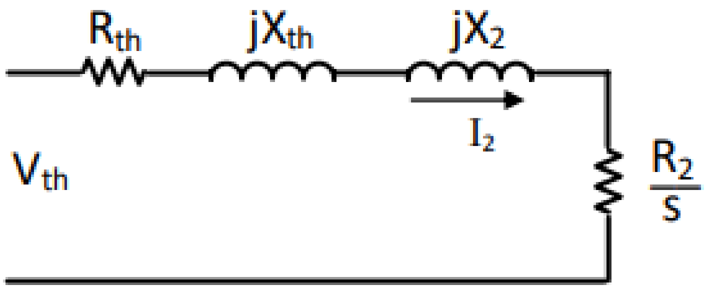
# Torque-Speed Characteristic in induction machine

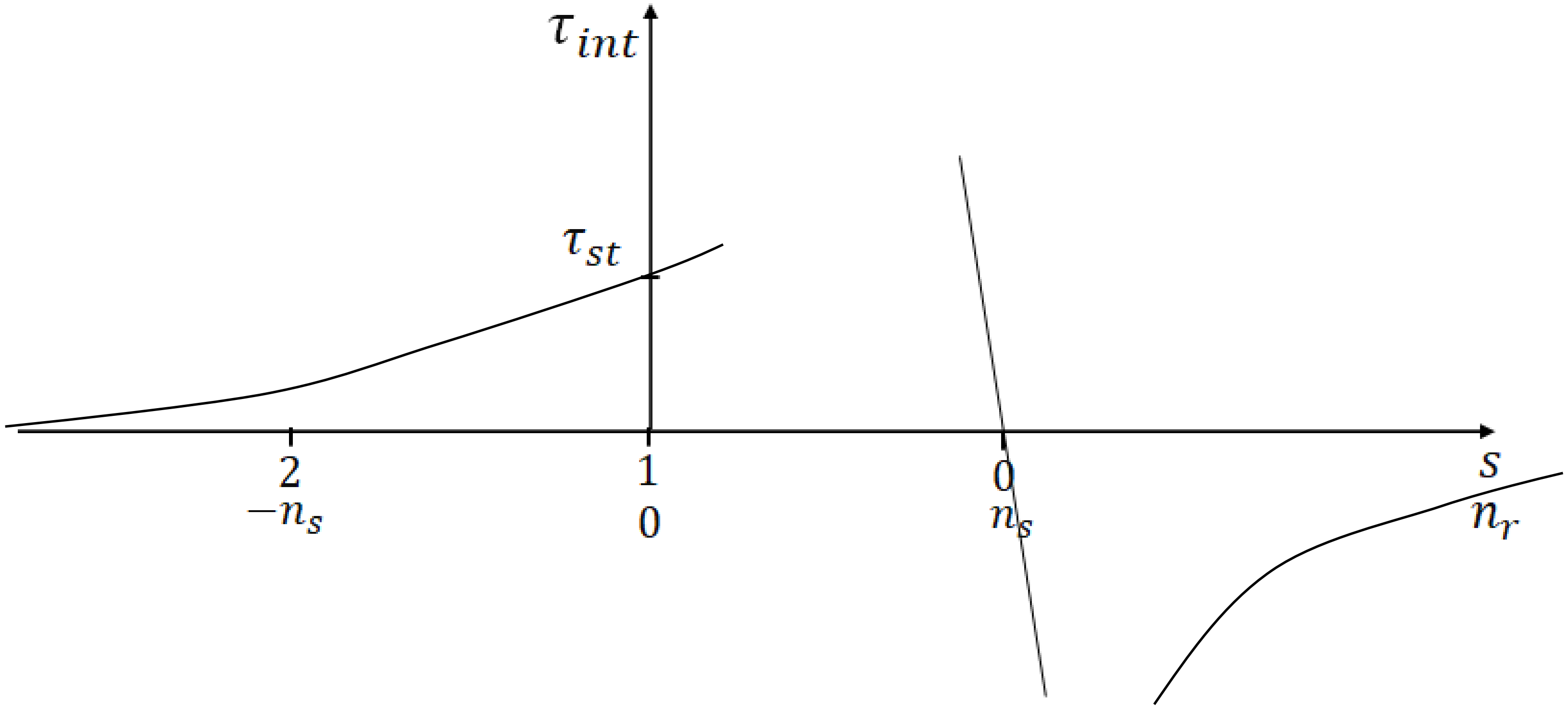



$$\tau_{int} = \frac{3}{\omega_s}\frac{V_{th}^2\frac{R_2}{s}}{(R_{th}+\frac{R_2}{s})^2+(X_{th}+X_2)^2}$$

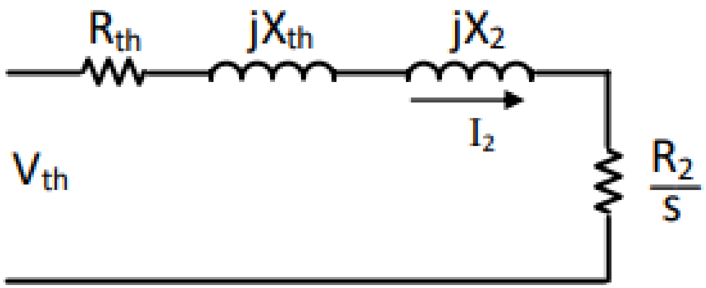
$$|s| \gg 0$$

$$\tau_{int} \cong \frac{3}{\omega_s}\frac{V_{th}^2\frac{R_2}{s}}{(R_{th})^2+(X_{th}+X_2)^2}$$

$$\tau_{int} \propto \frac{k}{s}$$



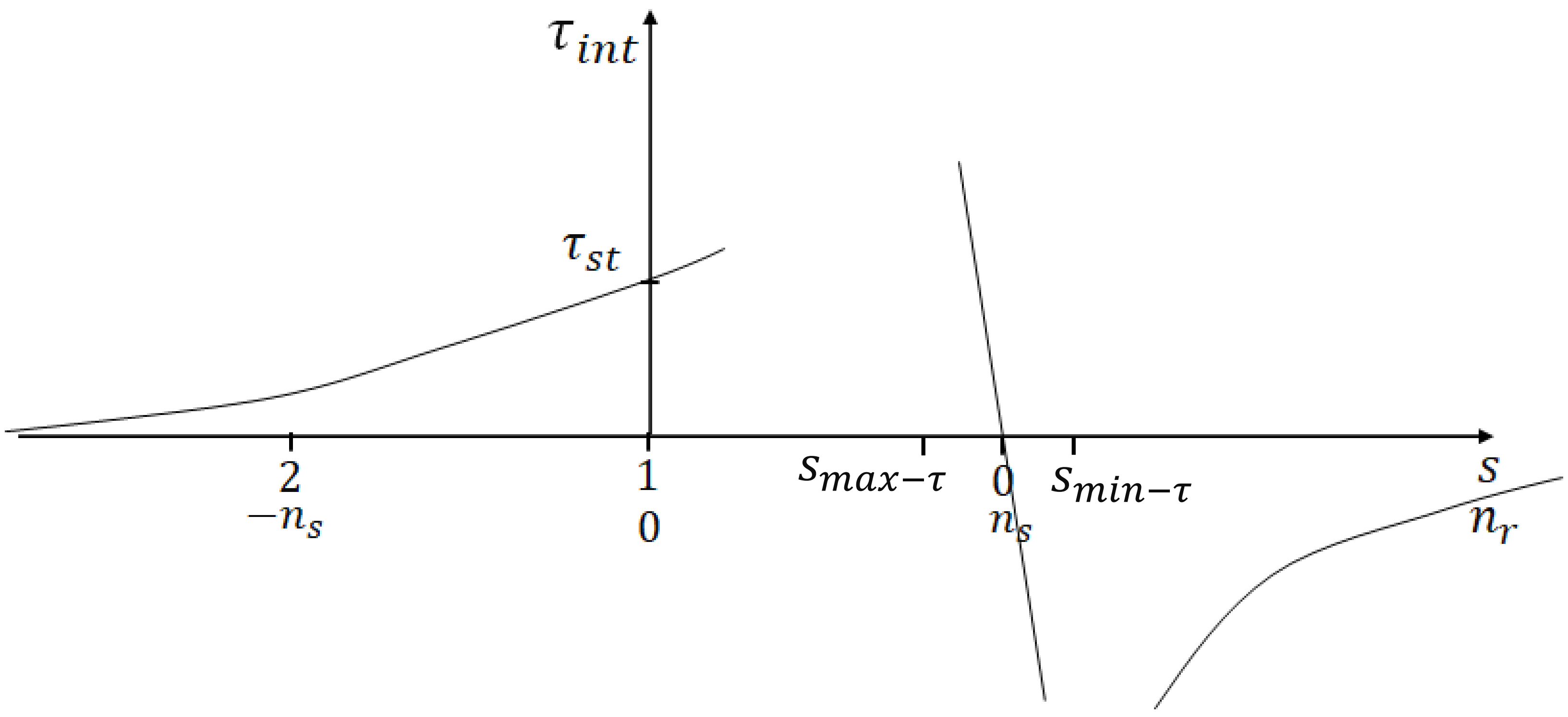
# Torque-Speed Characteristic in induction machine




$$\tau_{int} = \frac{3}{\omega_s} \frac{V_{th}^2 \frac{R_2}{s}}{(R_{th} + \frac{R_2}{s})^2 + (X_{th} + X_2)^2}$$

$$\frac{\partial \tau_{int}}{\partial s} = ??$$

$$s_{max-\tau}, s_{min-\tau} = ??$$





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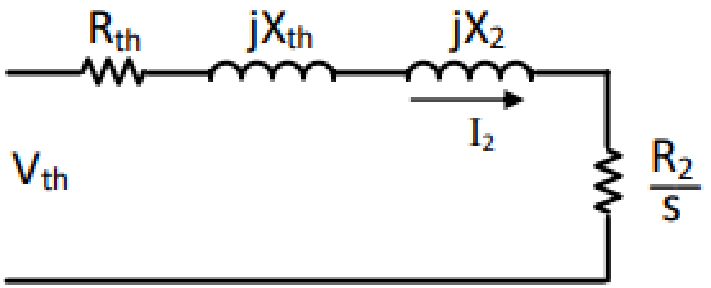
# Torque-Speed Characteristic in induction machine

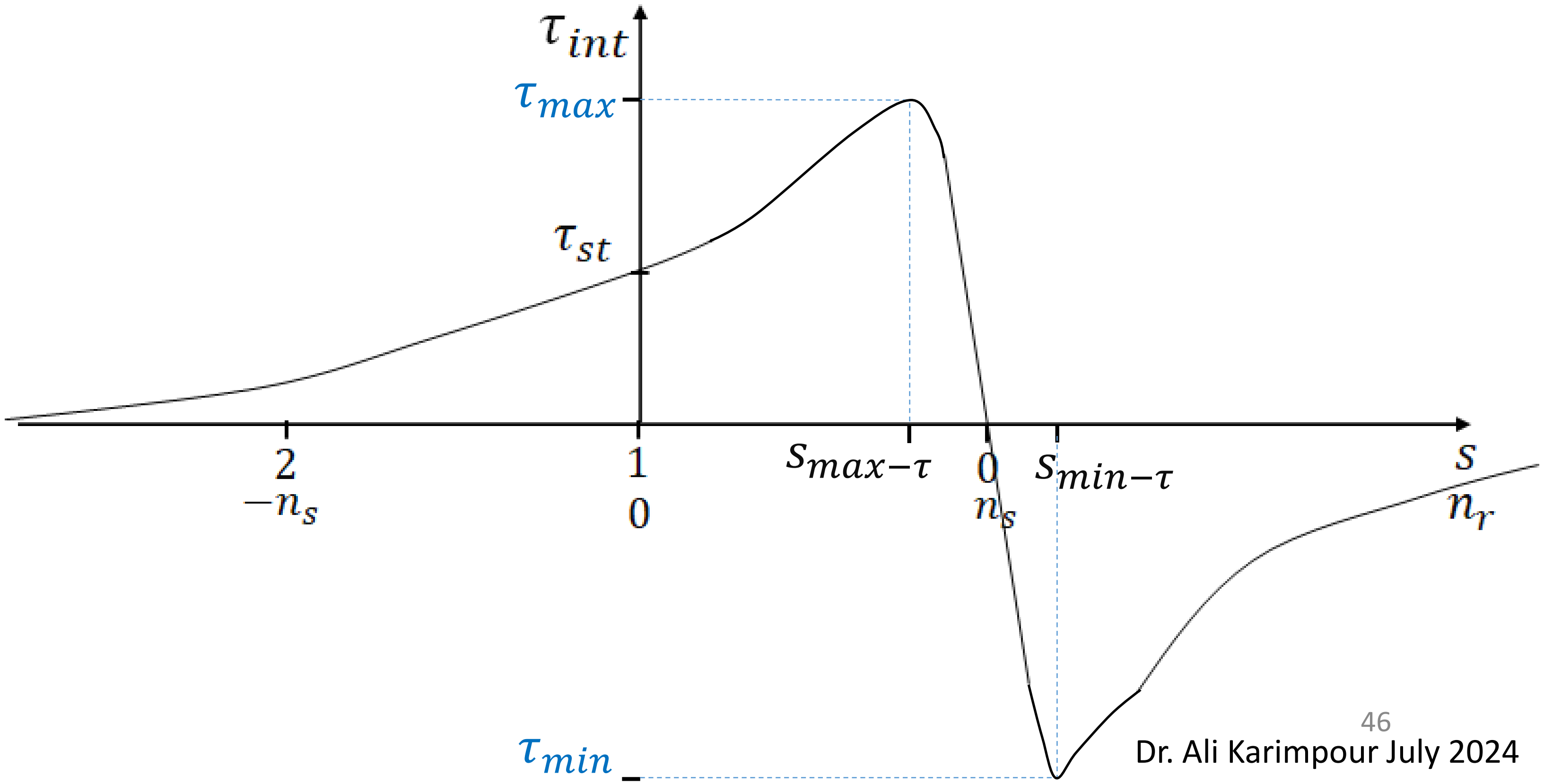



$$\tau_{int} = \frac{3}{\omega_s}\frac{V_{th}^2 \frac{R_2}{s}}{(R_{th} + \frac{R_2}{s})^2 + (X_{th} + X_2)^2}$$

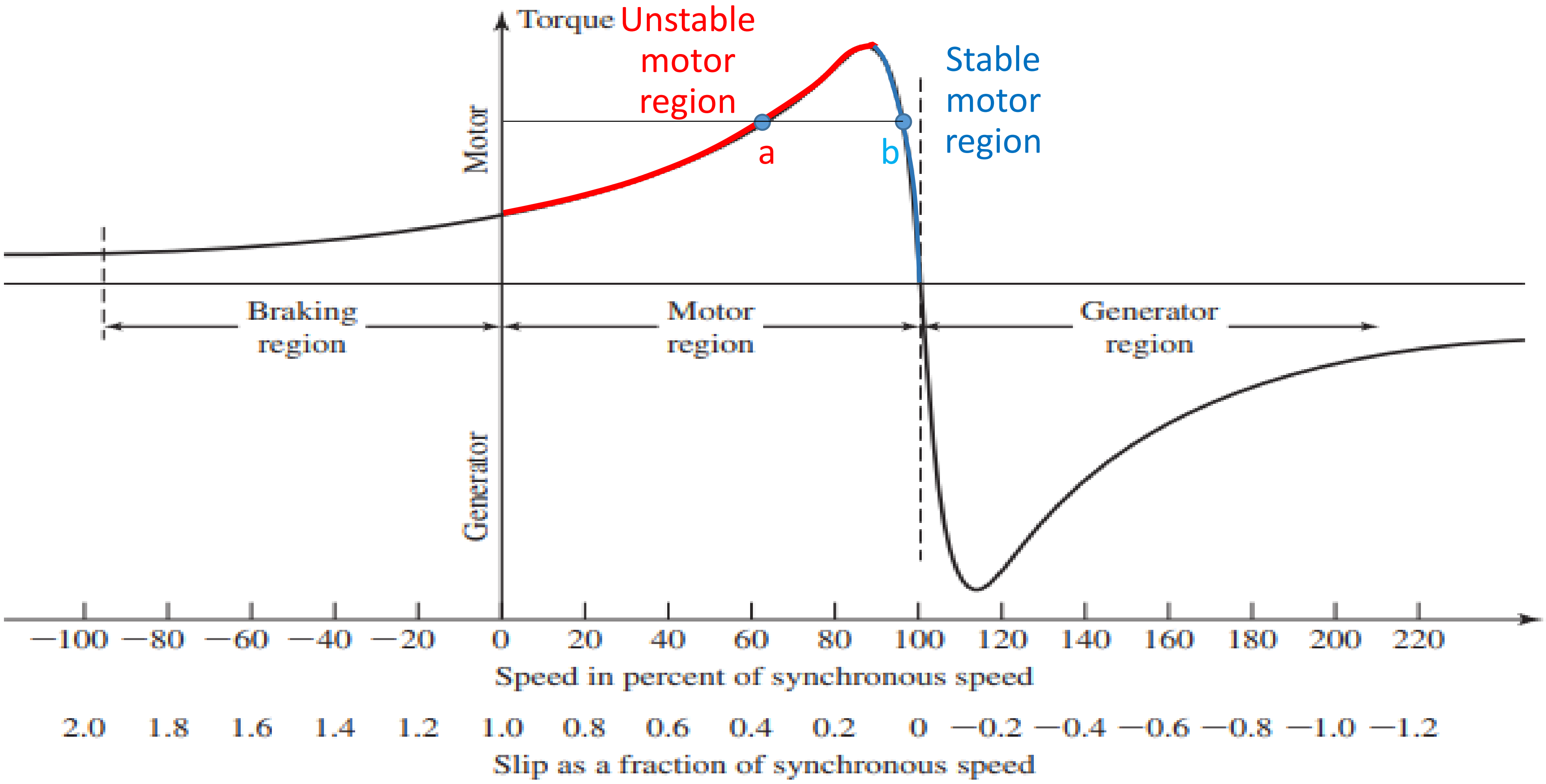
$$\frac{\partial \tau_{int}}{\partial s} = ??$$

$$s_{max-\tau}, s_{min-\tau} = ??$$

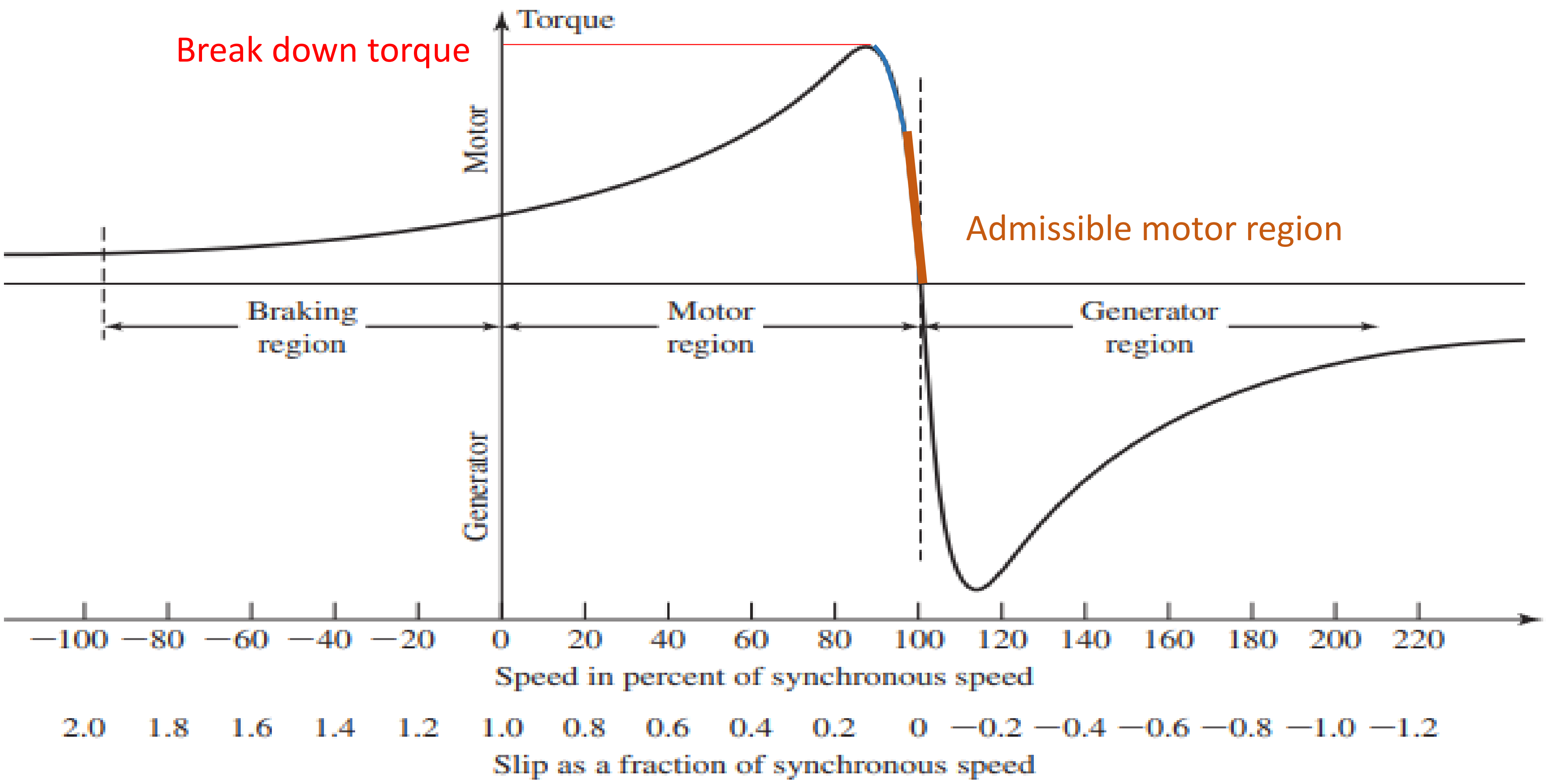
$$\tau_{max}, \tau_{min} = ??$$



# Torque-Speed Characteristic in induction machine



# Torque-Speed Characteristic in induction machine



# Example 2: Final 2022

The parameters for the Thevenin equivalent circuit of a three-phase, 4-pole, 50 Hz induction motor are given as follows:

$$V_{th} = 250\,V, \qquad R_{th} = 0.25\,\Omega, \qquad X_{th} = 1\,\Omega$$

The rotor circuit parameters from the stator perspective are given as follows:

$$R_2 = 0.2\,\Omega, \qquad X_2 = 2\,\Omega,$$

The total losses due to windage, friction, and core losses are constant and equal to 100 watts. Determine the following items. For any items that cannot be calculated with the given information, state the reason why. Assume the motor is operating at its rated voltage and frequency.

**a)** If the rotor is operating at a speed of 1455 RPM, determine the output torque and input power to the motor.

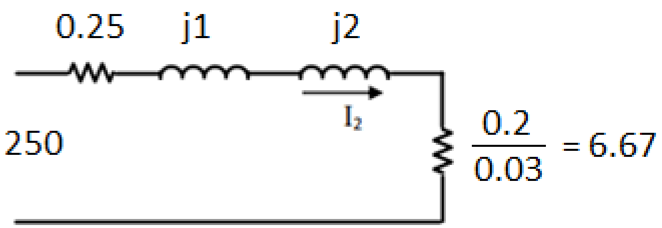
**b)** Determine the maximum internal torque that the motor can produce, along with the corresponding speed and stator current for that torque.

**c)** Determine the internal starting torque of the motor and the corresponding stator starting current.

# Example 2: Final 2022

**a)** At a speed of 1455 RPM, the slip is: $s = \frac{n_s - n_r}{n_s} = \frac{120*50/4 - 1455}{120*50/4} = 0.03$

The equivalent circuit is as follows:




$$I_2 = \frac{250}{(6.67+0.25)+j(2+1)} = 33.15 < \cdots \qquad P_{out} = (1-s)P_g - P_{rot} = (1-0.03)\times 3\times 6.67\times 33.15^2 - 100 = 21230\ W$$

$$I_2 = \frac{250}{(0.2+0.25)+j(2+1)} = 82.4 < \cdots. \qquad P_{int\_st} = P_g = 3\times 0.2\times 82.4^2 = 4075\ W$$

$$\tau_{out} = \frac{P_{out}}{\omega_r} = \frac{21230}{2\pi\frac{1455}{60}} = 139.3\ N.m$$

since the stator current cannot be calculated from the Thevenin equivalent, the input power cannot be determined.

$$s_{max\tau} = \frac{R_2}{\sqrt{R_{th}^2 + (X_{th}+X_2)^2}} = 0.0664 \qquad n_r = (1-0.0664)*\frac{120*50}{4} = 1404\ rpm \qquad \tau_{max} = \cdots = 183\ N.m$$ (ب

Clearly the stator current cannot be calculated from the Thevenin equivalent.

**c)** At startup:

$$\tau_{out} = \frac{P_g}{\omega_s} = \frac{4075}{2\pi\frac{1500}{60}} = 25.94\ N.m$$

Since the stator current cannot be calculated from the Thevenin equivalent, the stator current corresponding to the starting torque cannot be determined.

# Exercise 5 & 6: Final 2022

Exercise 6: A three-phase, 50 Hz, 6-pole induction motor delivers its maximum torque at a slip of 20%. Write down the maximum and minimum stable operating speeds of this motor.

Does the motor allow operation at the minimum speed? Why?

Exercise 7: A three-phase, 50 Hz, 8-pole induction motor delivers its rated torque at a slip of 10%. Write down the permissible speed range for this machine.

The permissible speed range refers to the speeds corresponding to a torque that is less than or equal to the rated torque.

# Exercise 8

A three-phase Y-connected 460-V (line-to-line) 20-kW 60-Hz six-pole induction motor has the following parameter values in /phase referred to the stator:

$R1 = 0.271$ $R2 = 0.188$ $X1 = 1.12$ $X2 = 1.91$ $Xm = 23.10$

Derive torque-speed(slip) characteristic for this machine.

**Review 1**: Examine the variations in the torque-speed (slip) curve as the rotor resistance changes from its current value to 2 ohms.

- Effect of Rotor Resistance on Speed Control
- Effect of Rotor Resistance on Starting Torque
- Effect of Rotor Resistance on Starting Current
- How to Change Rotor Resistance
- Appropriate Rotor Resistance

# Exercise 9

A three-phase Y-connected 460-V (line-to-line) 20-kW 60-Hz six-pole induction motor has the following parameter values in /phase referred to the stator:

$R1 = 0.271$ $R2 = 0.188$ $X1 = 1.12$ $X2 = 1.91$ $Xm = 23.10$

Derive torque-speed(slip) characteristic for this machine.

**Review 2**: Examine the variations in the torque-speed (slip) curve as the terminal voltage changes.

- Effect of Terminal Voltage Variation on Speed Control
- Effect of Terminal Voltage Variation on Starting Torque
- Effect of Terminal Voltage Variation on Starting Current
- How to Change Terminal Voltage
- Appropriate Terminal Voltage

# Exercise 10

A three-phase Y-connected 460-V (line-to-line) 20-kW 60-Hz six-pole induction motor has the following parameter values in /phase referred to the stator:

$R1 = 0.271$ $R2 = 0.188$ $X1 = 1.12$ $X2 = 1.91$ $Xm = 23.10$

Derive torque-speed(slip) characteristic for this machine.

**Review 3**: Examine the variations in the torque-speed (slip) curve as the frequency changes.

- Effect of Frequency Variation on Speed Control
- Effect of Frequency Variation on Starting Torque
- Effect of Frequency Variation on Starting Current
- How to Change Frequency
- Why Control of Ratio of Voltage to Frequency

# Electrical Machine II Syllabus

1. Introduction

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2. Introduction to Rotating Machines

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3. DC Machines

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4. Induction Machines
   - Introduction to Polyphase Induction Machines
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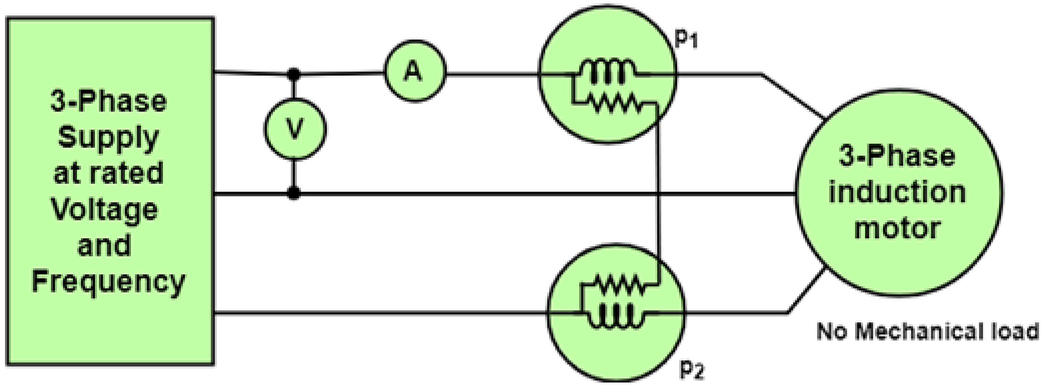
# Parameter Determination in Induction Machines

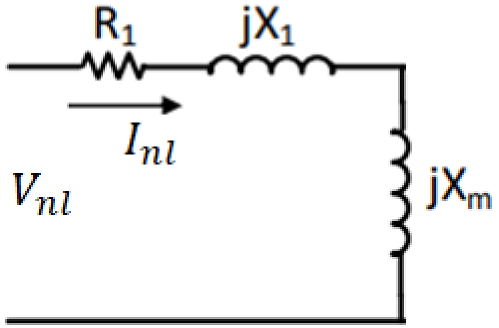
There are two tests plus one measurement for induction machine for finding all parameters and all losses except stray load losses. For more information see IEEE Std. 112-2004.

- No load test.
- Blocked rotor test.
- measurements of the dc resistances of the stator windings.

# No load test




$$P_{nl}\ , V_{nl}\ , I_{nl}$$



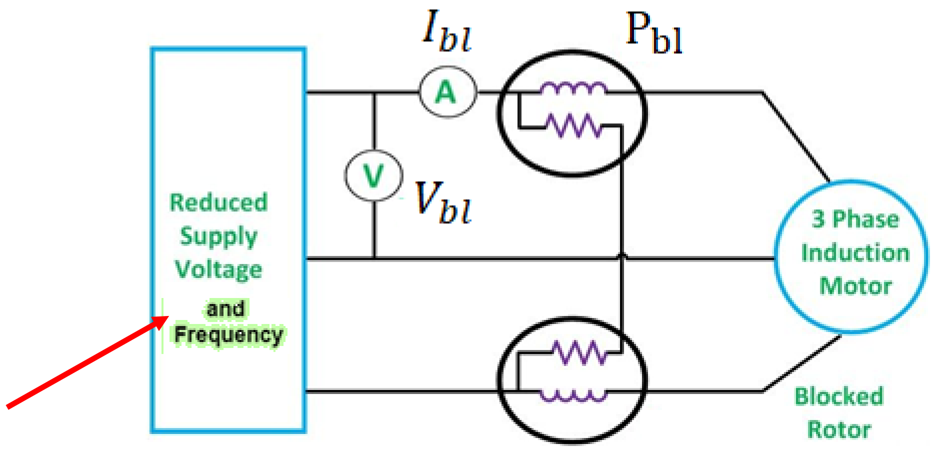

$$P_{rot} = P_{nl} - 3R_1 I_{nl}^2$$

$$S_{nl} = \sqrt{3} V_{nl} I_{nl}$$

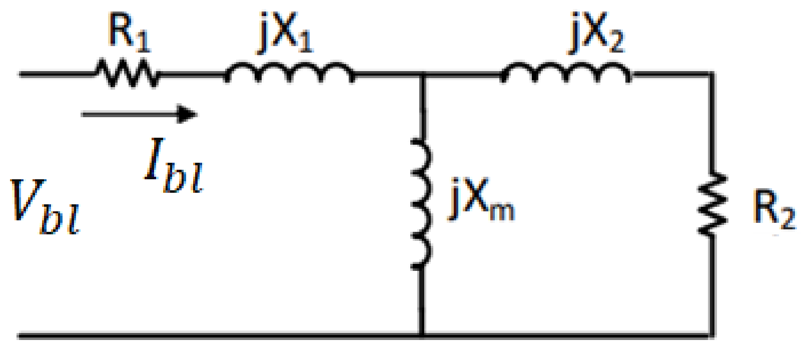
$$Q_{nl} = \sqrt{S_{nl}^2 - P_{nl}^2}$$

$$X_{nl} = X_1 + X_m = \frac{Q_{nl}/3}{I_{nl}^2}$$

# Blocked rotor test




$$P_{bl}, V_{bl}, I_{bl}$$




$$S_{bl} = \sqrt{3} V_{bl} I_{bl} \qquad Q_{bl} = \sqrt{S_{bl}^2 - P_{bl}^2}$$

$$R_{bl} = \frac{P_{bl}/3}{I_{bl}^2} \qquad X_{bl} = \frac{Q_{bl}/3}{I_{bl}^2} \frac{f_n}{f_{test}}$$

$$R_{bl} = R_1 + RE\{jX_m || (R_2 + jX_2)\} \cong R_1 + R_2 \left(\frac{X_m}{X_m + X_2}\right)^2$$

$$X_{bl} = X_1 + IM\{jX_m || (R_2 + jX_2)\} \cong X_1 + X_2 \left(\frac{X_m}{X_m + X_2}\right)$$

$$R_2 = (R_{bl} - R_1) \left(\frac{X_m + X_2}{X_m}\right)^2$$

$$X_2 = (X_{bl} - X_1) \left(\frac{X_{nl} - X_1}{X_{nl} - X_{bl}}\right)$$

# Blocked rotor test

(1)

$$X_{nl} = X_1 + X_m$$

(1) (2)

(2) (3)

$$R_2 = (R_{bl} - R_1)\left(\frac{X_m + X_2}{X_m}\right)^2$$

(4)

(3)

$$X_2 = (X_{bl} - X_1)\left(\frac{X_{nl} - X_1}{X_{nl} - X_{bl}}\right)$$

(4)

| Motor class | Description | Fraction of $(X_1 + X_2)$ $X_1$ | $X_2$ |
|---|---|---|---|
| A | Normal starting torque, normal starting current | 0.5 | 0.5 |
| B | Normal starting torque, low starting current | 0.4 | 0.6 |
| C | High starting torque, low starting current | 0.3 | 0.7 |
| D | High starting torque, high slip | 0.5 | 0.5 |
| Wound rotor | Performance varies with rotor resistance | 0.5 | 0.5 |

Source: IEEE Standard 112.

$X_1$ $X_2$ $X_m$ $R_2$

# Example 3

The following test data apply to a 135 kW (180-hp), three-phase, 460-V, 60-Hz, four-pole induction motor with a double-squirrel-cage rotor of design class B (normal-starting-torque, low-starting-current type):

Test 1: No-load test at 60 Hz

- Applied voltage $V_{nl}$ = 459 V line-to-line
- Average phase current $I_{nl}$ = 34.1 A
- Power $P_{nl}$ = 1.25 kW

Test 2: Blocked-rotor test at 15 Hz

- Applied voltage $V_{bl}$ = 42.3 V line-to-line
- Average phase current $I_{bl}$ = 169
- A Power $P_{bl}$ = 4.44 kW

Test 3: Average dc resistance per stator phase (measured immediately after test 2) $R_1$ = 30.3 $m\Omega$

Test 4: Blocked-rotor test at 60 Hz

- Applied voltage $V_{bl}$ = 455 V line-to-line
- Average phase current $I_{1,bl}$ = 725
- A Power $P_{bl}$ = 163 kW Measured starting torque $\tau_{st}$ = 603 N·m

# Example 3

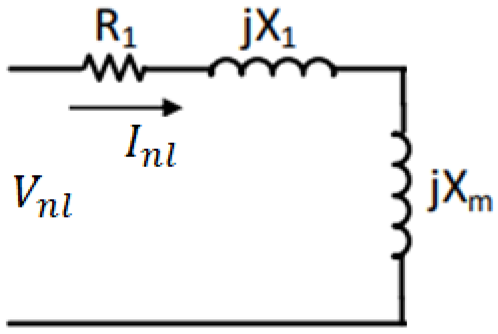
a) Compute the no-load rotational loss and the equivalent-circuit parameters applying to normal running conditions. Assume the same temperature as in test 3. Neglect any effects of core loss, assuming that core loss can be lumped in with the rotational losses.

b) Compute the electromechanical starting torque from the input measurements of test 4. Assume the same temperature as in test 3.

# Example 3

a) Compute the no-load rotational loss and the equivalent-circuit parameters applying to normal running conditions. Assume the same temperature as in test 3. Neglect any effects of core loss, assuming that core loss can be lumped in with the rotational losses.

$$P_{rot} = P_{nl} - 3R_1 I_{nl}^2 = 1250 - 3 * 0.0303 * 34.1^2 = 1144\,W$$




$$S_{nl} = \sqrt{3} V_{nl} I_{nl} = 27110\,VA \qquad Q_{nl} = \sqrt{S_{nl}^2 - P_{nl}^2} = 27085\,VAR$$

$$X_{nl} = X_1 + X_m = \frac{Q_{nl}/3}{I_{nl}^2} = 7.76\,\Omega$$

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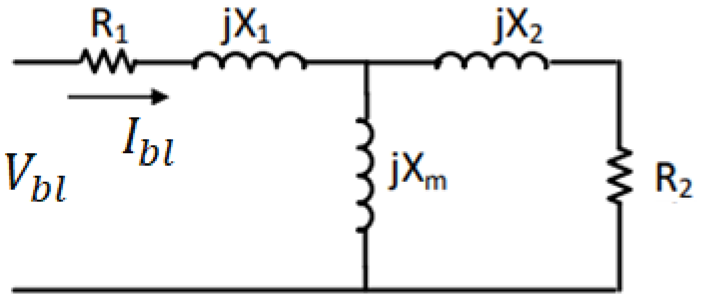



$$S_{bl} = \sqrt{3} V_{bl} I_{bl} = 12382\,VA \qquad Q_{bl} = \sqrt{S_{bl}^2 - P_{bl}^2} = 11558\,VAR$$

$$X_{bl}' = \frac{Q_{bl}/3}{I_{bl}^2} = 0.135\,\Omega \qquad X_{bl} = 0.54\,\Omega$$

$$R_{bl} = \frac{P_{bl}/3}{I_{bl}^2} = 0.052\,\Omega$$

# Example 3

(1)

$$7.76 = X_1 + X_m$$

(2)

$$R_2 = (0.052 - 0.0303)\left(\frac{X_m + X_2}{X_m}\right)^2$$

(3)

$$X_2 = (0.54 - X_1)\left(\frac{7.76 - X_1}{7.76 - 0.54}\right)$$

(4)

| | | Fraction of $(X_1 + X_2)$ | |
|---|---|---|---|
| Motor class | Description | $X_1$ | $X_2$ |
| B | Normal starting torque, low starting current | 0.4 | 0.6 |

$$X_2 = 0.6(X_2 + X_1)$$

$$X_m = 7.54$$

$$R_2 = 0.0237$$

$$X_1 = 0.221$$

$$X_2 = 0.332$$

b) Compute the electromechanical starting torque from the input measurements of test 4. Assume the same temperature as in test 3.

$$P_g = P_{in} - 3R_1 I_{bl}^2 = 163000 - 3 * 0.0303 * 725^2 = 115694\ W$$

$$\tau_{int-st} = \frac{P_g}{\omega_s} = \frac{115694}{60\pi} = 614\ N.m$$

The difference with 603 N.m is??

# Example 4: Final 2022

The following tests were conducted on a 100 horsepower, three-phase, 4-pole, 60 Hz, 460 V induction motor with a standard squirrel-cage rotor (Class A):

No-load test at 60 Hz: $V_{nl} = 460\ V,\ i_{nl} = 34\ A, P_{nl} = 1250\ W$

Locked-rotor test at 15 Hz: $V_{bl} = 42\ V,\ i_{bl} = 170\ A, P_{bl} = 4440\ W$

**The resistance of each stator phase is also measured to be 30 milliohms.**

**a) Determine the rotational losses of the motor.**

b) **Determine the equivalent circuit parameters of the motor.**

# Example 4: Final 2022

a) $P_{rot} = P_{nl} - 3R_sI_s{}^2 = 1250 - 3 \times 0.03 \times 34^2 = 1146\ W$

b)

$S_{nl} = \sqrt{3}I_{nl}V_{nl} = 27089\ \ VA,$ $Q_{nl} = \sqrt{S_{nl}{}^2 - P_{nl}{}^2} = 27060\ VAR$ $X_{nl} = X_1 + X_m = \dfrac{Q_{nl}/3}{I_{nl}{}^2} = 7.8$

$S_{bl} = \sqrt{3}I_{bl}V_{bl} = 12367\ \ VA,$ $Q_{bl} = \sqrt{S_{bl}{}^2 - P_{bl}{}^2} = 11542\ VAR$ $X_{bl} = \dfrac{60}{15}\dfrac{Q_{bl}/3}{I_{bl}{}^2} = 0.5325 = X_1 + X_2\dfrac{X_m}{X_2 + X_m}$

$Since\ it\ is\ a\ class\ A\ motor\ so$: $X_1 = X_2$

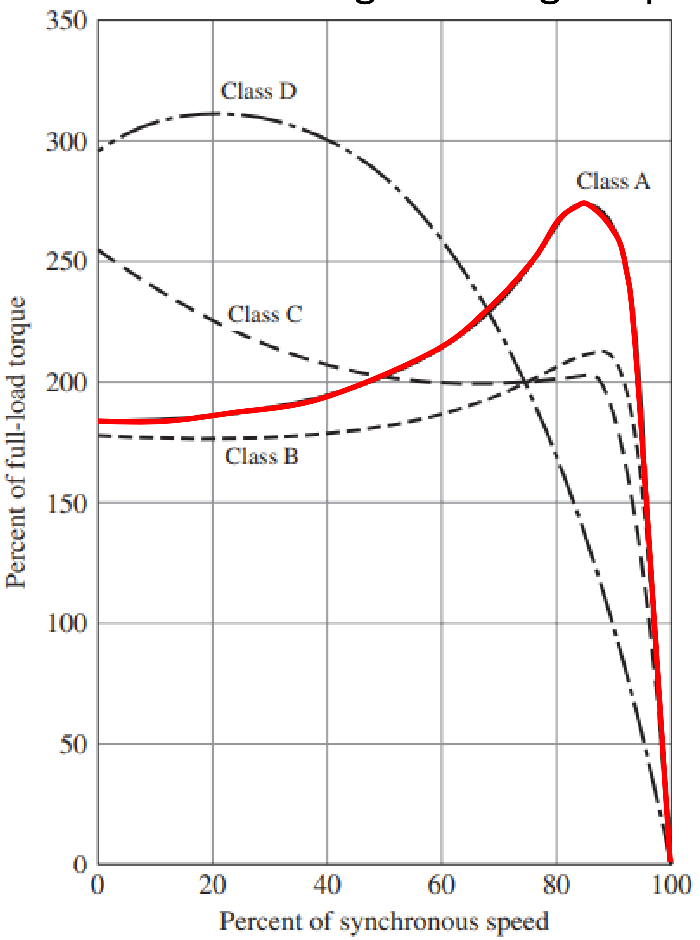
$$0.5325 = X_1 + X_1\frac{X_m}{X_1 + X_m} = X_1 + X_1\frac{X_{nl} - X_1}{X_{nl}} = X_1 + X_1\frac{7.8 - X_1}{7.8}$$ $X_1{}^2 - 15.6X_1 + 4.1535 = 0,$

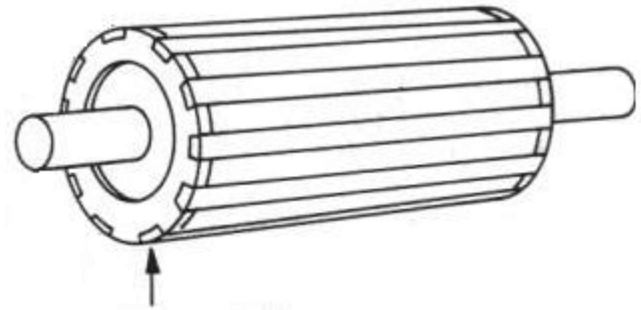
$X_1 = 0.271\ \Omega,$ $X_1 = 15.33\ \Omega$ غ ق ق $X_1 = X_2 = 0.271\ \ \Omega,$ $X_m = 7.8 - 0.271 = 7.53\ \ \Omega$

$$R_{bl} = \frac{P_{bl}/3}{I_{bl}{}^2} = 0.0512\ \Omega = R_1 + R_2\left(\frac{X_m}{X_m + X_2}\right)^2$$ $$R_2 = (R_{bl} - R_1)\left(\frac{X_m + X_2}{X_m}\right)^2 = 0.0227\ \Omega$$

# Class A: Normal Starting Torque, Normal Starting Current, Low Slip

| Motor class | Description |
|---|---|
| A | Normal starting torque, normal starting current |
| B | Normal starting torque, low starting current |
| C | High starting torque, low starting current |
| d | High starting torque, high slip |



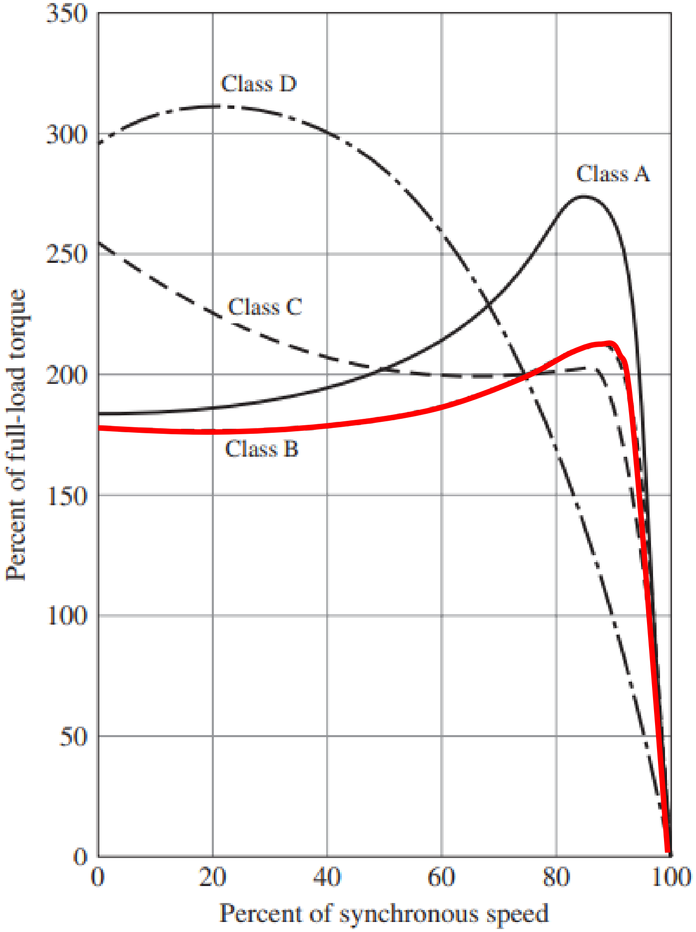



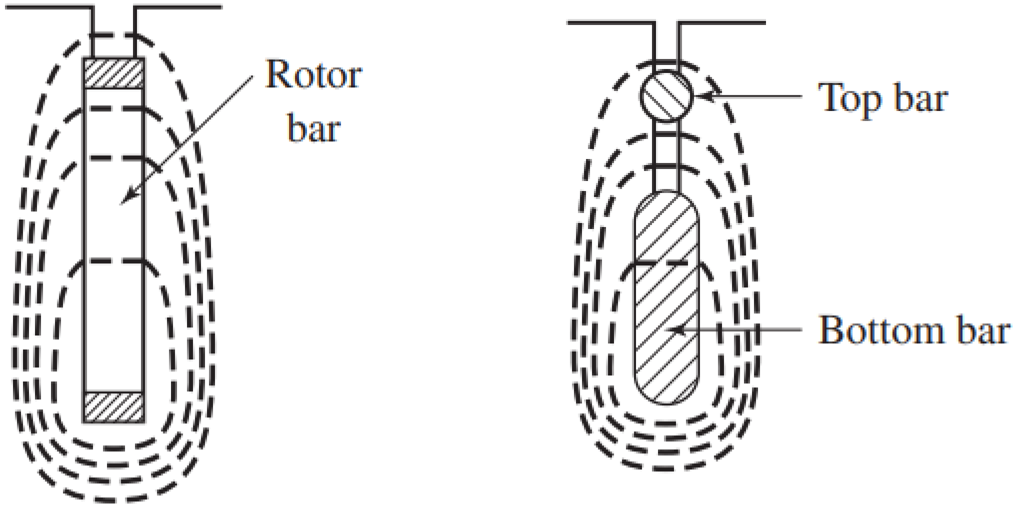
Single cage-low-resistance

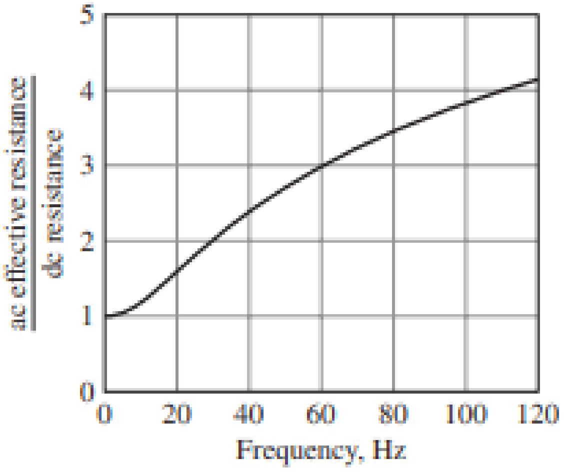
- The full-load slip is low and the full-load efficiency is high.
- $\tau_{max}$ maximum torque usually is well over 2 p.u. and occurs at a small slip.
- $\tau_{int-st}$ at full voltage varies 2 p.u. in small motors to about 1 p.u. in large motors.
- $I_{start}$ is about 5-8 p.u. when started at rated voltage) is the principal disadvantage of this design.
- In sizes below about 7.5 hp these starting currents usually are ok.
- Otherwise, reduced-voltage starting must be used.
- Reduced-voltage starting results in a decrease in starting torque.

# Class B: Normal Starting Torque, low Starting Current

| Motor class | Description |
|---|---|
| A | Normal starting torque, normal starting current |
| B | Normal starting torque, low starting current |
| C | High starting torque, low starting current |
| d | High starting torque, high slip |

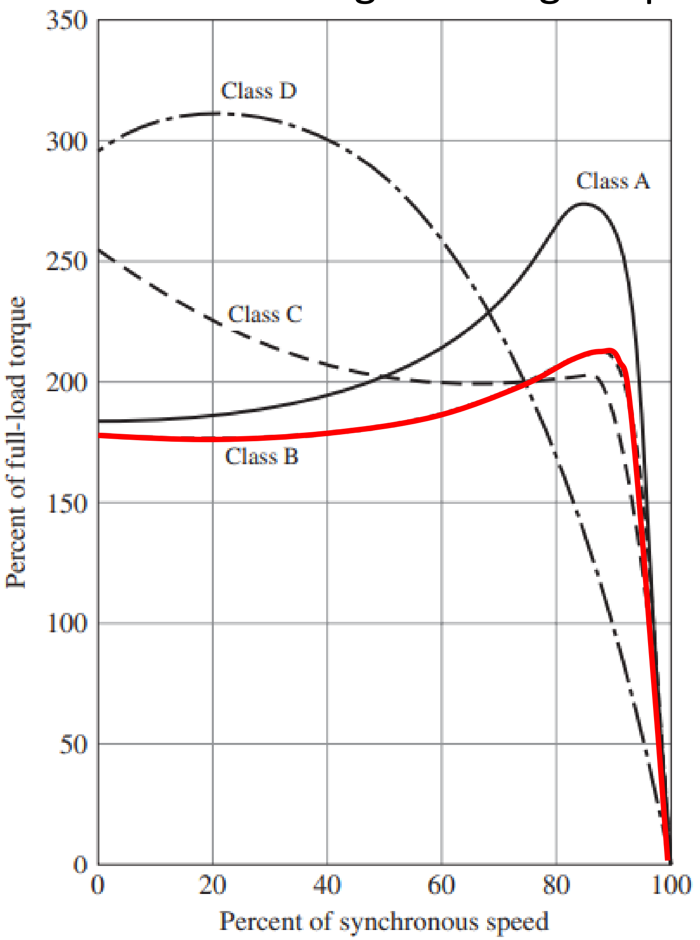


# Class B: Normal Starting Torque, low Starting Current

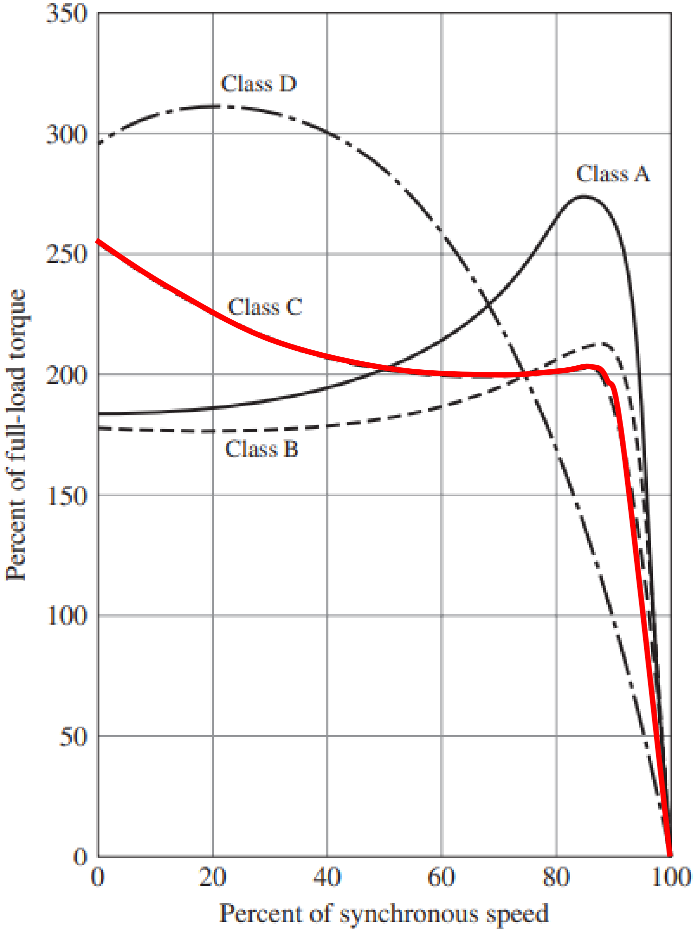
| Motor class | Description |
|---|---|
| A | Normal starting torque, normal starting current |
| B | Normal starting torque, low starting current |
| C | High starting torque, low starting current |
| d | High starting torque, high slip |


- This design has approximately the same starting torque as the class-A design but with 75 percent of the starting current.
- Full-voltage starting, therefore, may be used with larger sizes than with class A.
- The starting current is reduced by designing for relatively high leakage reactance, and the starting torque is maintained by use of a double-cage or deep-bar rotor.
- The full-load slip and efficiency are good, about the same as for the class A design.
- However, the use of high reactance slightly decreases the power factor and decidedly lowers the maximum torque (usually only slightly over 200 percent of full-load torque being obtainable).
- This design is the most common in the 7.5 to 200-hp range of sizes.

# Class C: High Starting Torque, low Starting Current

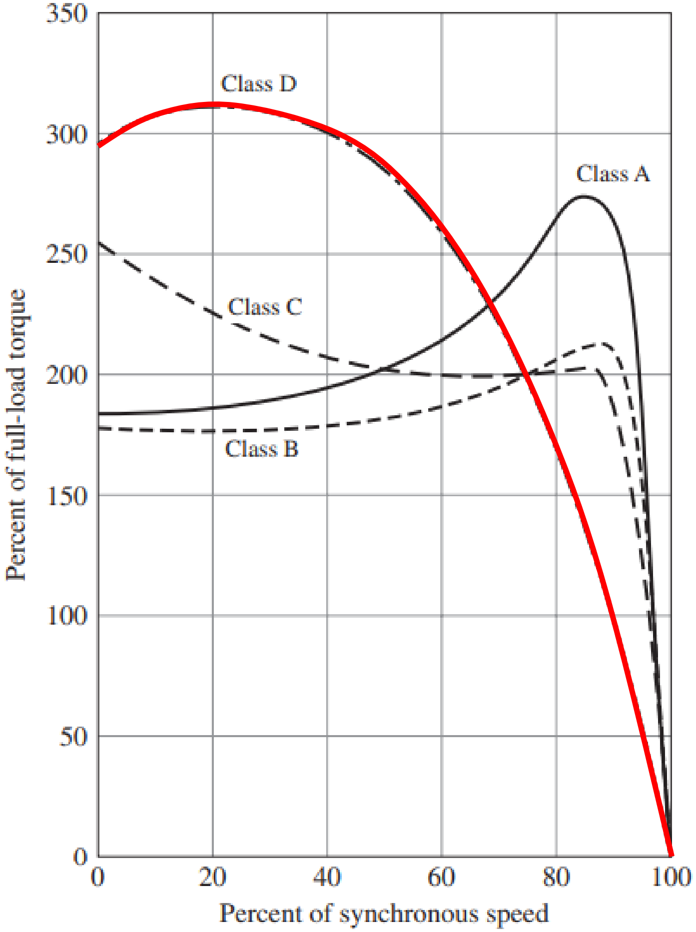
| Motor class | Description |
|---|---|
| A | Normal starting torque, normal starting current |
| B | Normal starting torque, low starting current |
| C | High starting torque, low starting current |
| d | High starting torque, high slip |

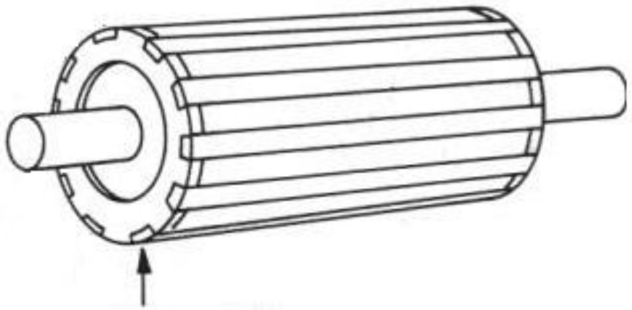



- This design uses a double-cage rotor with higher rotor resistance than the class-B design.
- The result is higher starting torque with low starting current but somewhat lower running efficiency and higher slip than the class-A and class-B designs.
- Typical applications are in driving compressors and conveyers.

# Class D: High Starting Torque, high slip

| Motor class | Description |
|---|---|
| A | Normal starting torque, normal starting current |
| B | Normal starting torque, low starting current |
| C | High starting torque, low starting current |
| d | High starting torque, high slip |



Single cage-high-resistance

- This design usually has a single cage, high-resistance rotor (frequently brass bars).
- It produces very high starting torque at low starting current, high maximum torque at 50 to 100 percent slip, but runs at a high slip at full load (7 to 11 percent) and consequently has low running efficiency.
- Its principal uses are for driving intermittent loads involving high accelerating duty and for driving high-impact loads such as punch presses and shears.

**Exercise 11:** An induction motor with a wound rotor, where the rotor is short-circuited, has a maximum torque of $a$ Nm, a starting torque of $b$ Nm, and a starting current of $c$ A. If a resistance is added to the rotor circuit, how will the maximum torque, starting torque, and starting current of the motor change?

**Exercise 12:** Consider a double cage induction motor.

a) Which cage is more effective during starting?

b) Which cage is more effective during normal operation?

c) Does this effectiveness of the cages occur due to a switch inside the rotor, or is it caused by another phenomenon? If another phenomenon is responsible, explain it briefly in two to three sentences.