LINEAR CONTROL SYSTEMS

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Lecture 8 – Part II

Frequency domain charts

Topics to be covered include:

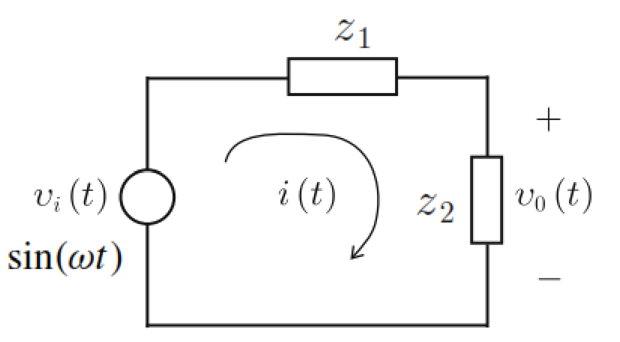
- Bode plot.
- Nichols chart.
- Polar plot.

Frequency domain charts

- Bode plot.
- Nichols chart.
- Polar plot.

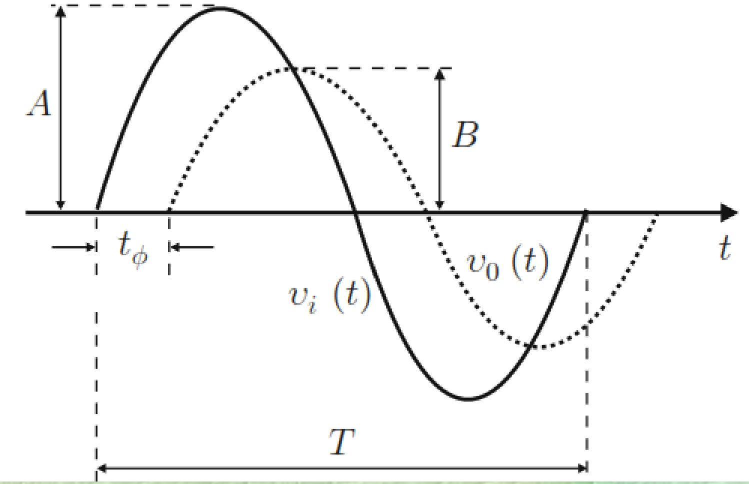
Introduction

The frequency response is the steady-state response of a system to a sinusoidal input where the frequency is varied from zero to infinity.

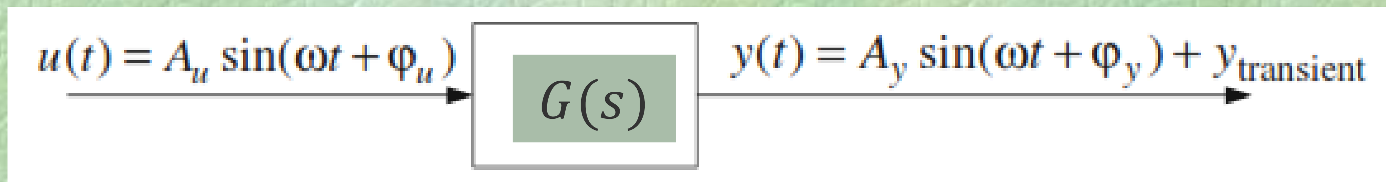


$$v_i(t) = A\sin(\omega t)$$

$$v_0(t) = B\sin(\omega t + \phi)$$



Steady-State Response to a Sinusoidal Input



$$y(t) = y_{\text{steady}}(t) + y_{\text{transient}}(t).$$

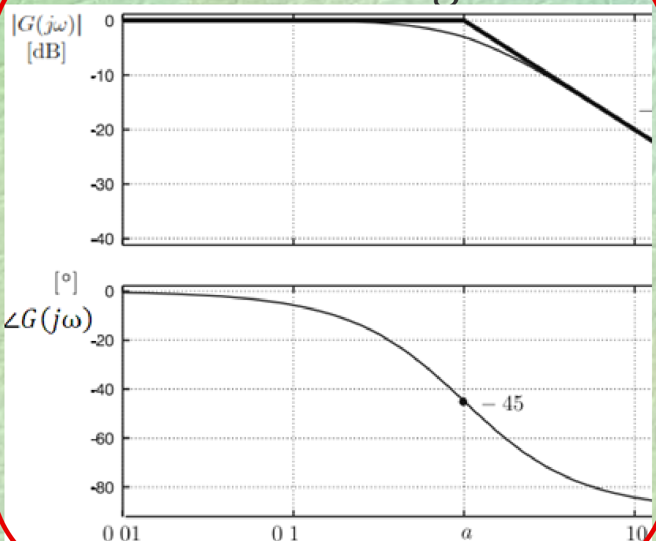
Steady-State Output of the System:

$$y_{\text{steady}}(t) = A_y \sin(\omega t + \varphi_y).$$

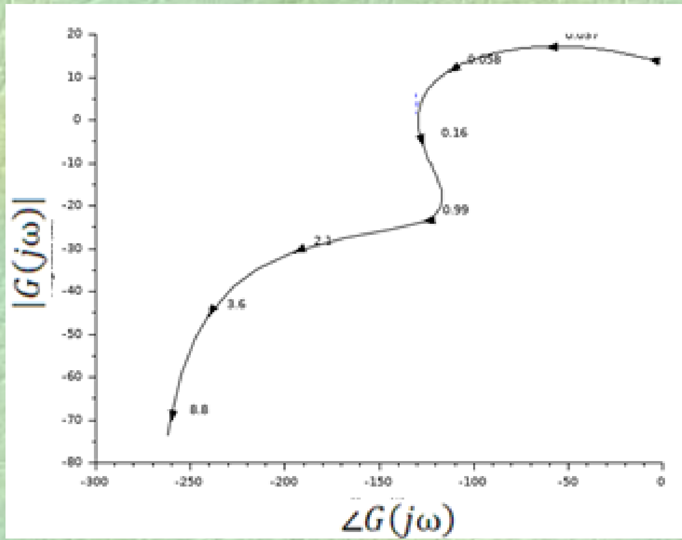
Frequency Domain Representation of Systems

$$|G(j\omega)| = \frac{A_y(\omega)}{A_u(\omega)} \quad \text{and} \quad \angle G(j\omega) = \varphi_y(\omega) - \varphi_u(\omega)$$

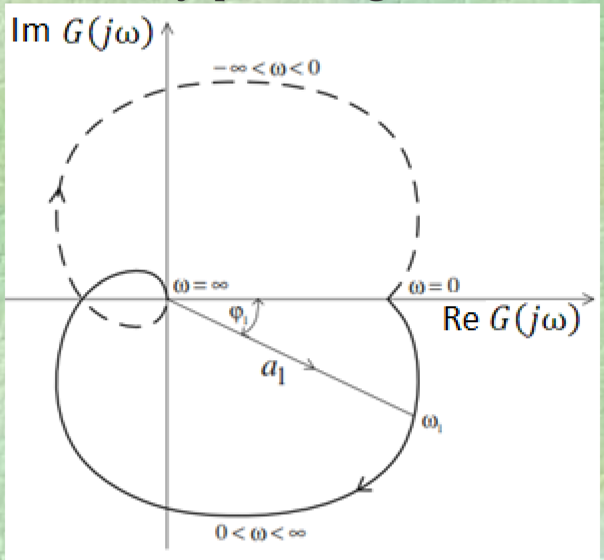
Bode diagram



Nichols diagram



Nyquist diagram



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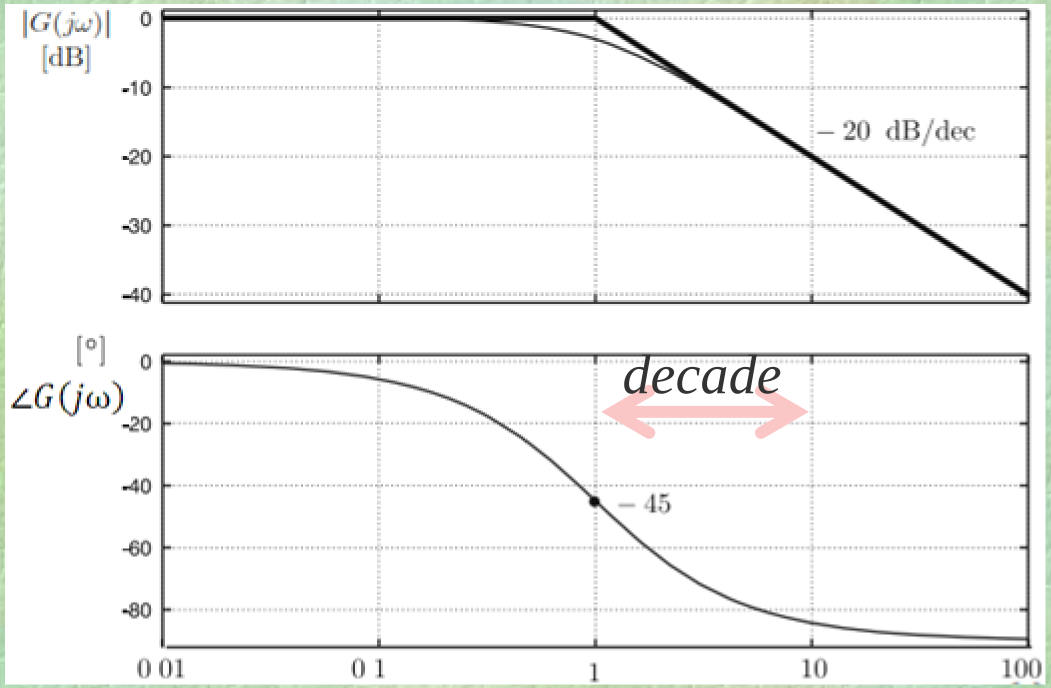
Bode diagram



Magnitude vs. frequency

Phase vs. frequency

Reminder:

$10^z = k, \quad z = \log(k)$

$k_{\mathrm{dB}} = 20\log(k),$

$\log(xy) = \log(x) + \log(y)$

$\log(x/y) = \log(x) - \log(y)$

$\log(x^m) = m\log(x)$

$\log(1) = 0$

Bode diagram

The main factors present in a transfer function

$$G(j\omega) = \frac{1}{0.1j\omega + 1}\frac{1}{2j\omega + 1}$$

$$< G(j\omega) =< \frac{1}{0.1j\omega + 1} +< \frac{1}{2j\omega + 1}$$

$$20\log|G(j\omega)| = 20\log\left|\frac{1}{0.1j\omega + 1}\right| + 20\log\left|\frac{1}{2j\omega + 1}\right|$$

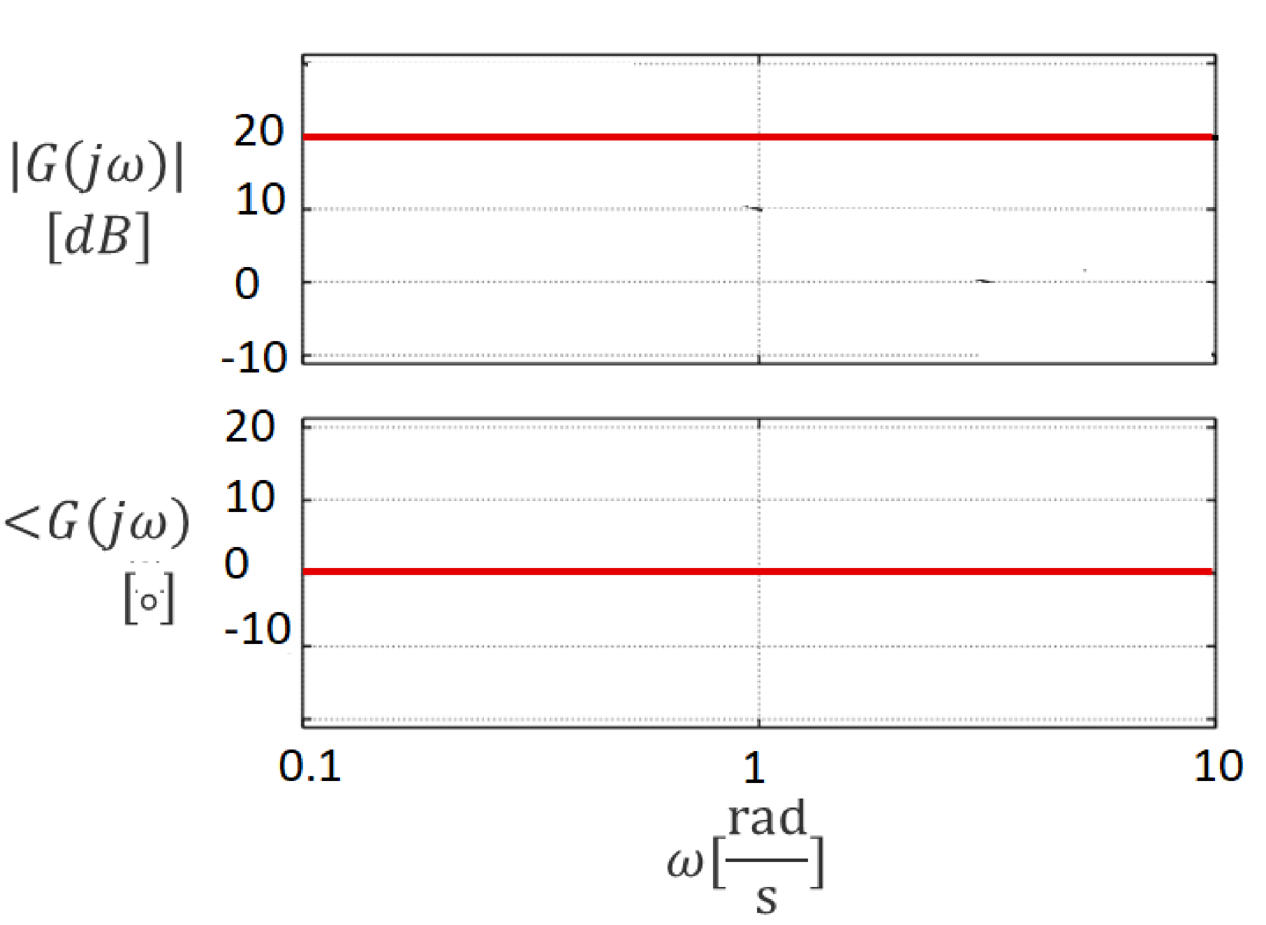
$$k, \quad s, \quad \frac{1}{s}, \quad \frac{1}{\frac{s}{a}+1}, \quad \frac{s}{a}+1$$

$$\frac{s^2 + 2\zeta\omega_n s + \omega_n^2}{\omega_n^2}, \quad \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Bode diagram

Plotting the Bode diagram for the transfer function **G(*s*)=10**

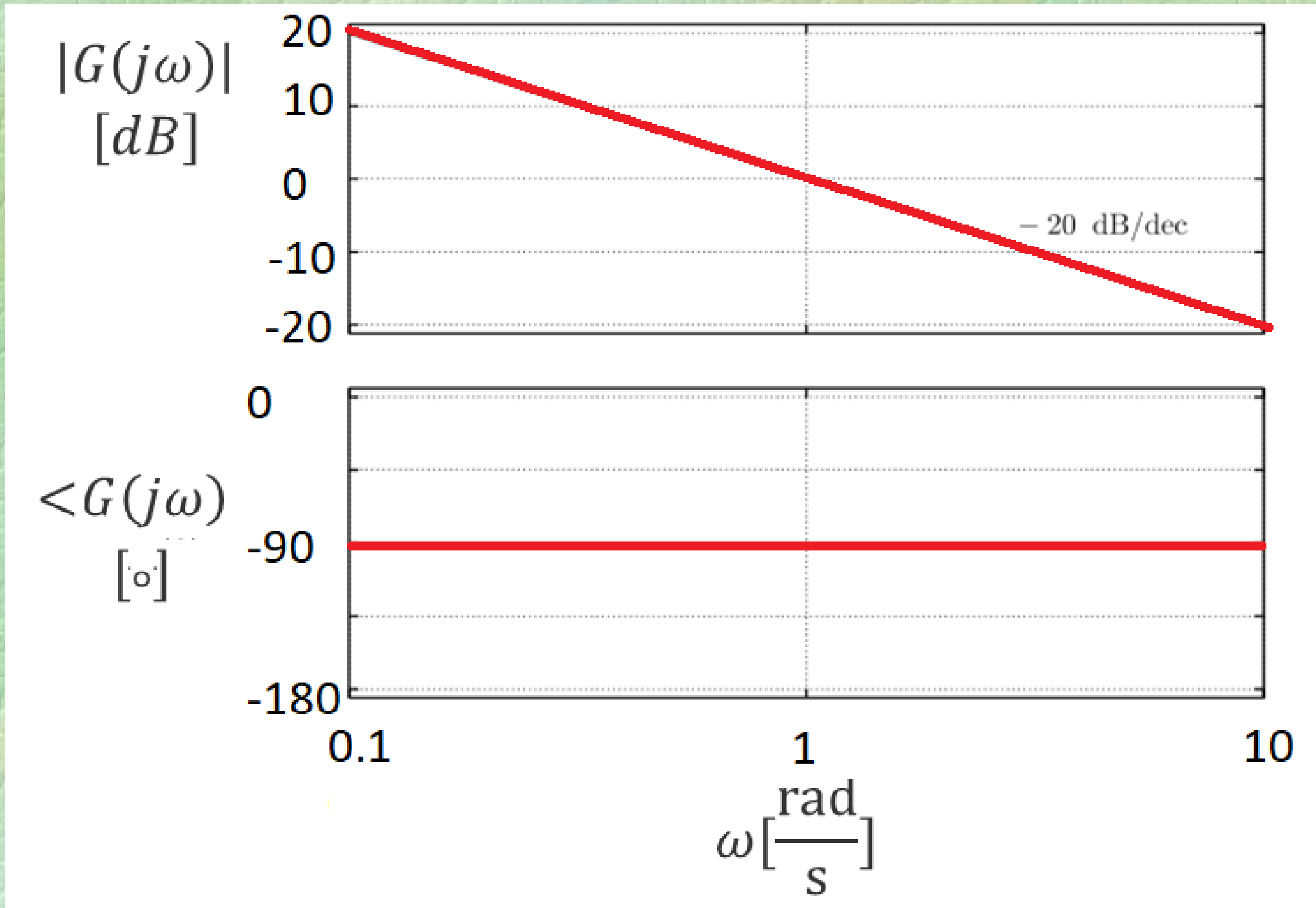
$G(j\omega) = 10$ $\quad < G(j\omega) = 0$ $\quad 20\log|G(j\omega)| = 20$



Bode diagram

Plotting the Bode diagram for the transfer function **G(s)=1/s**

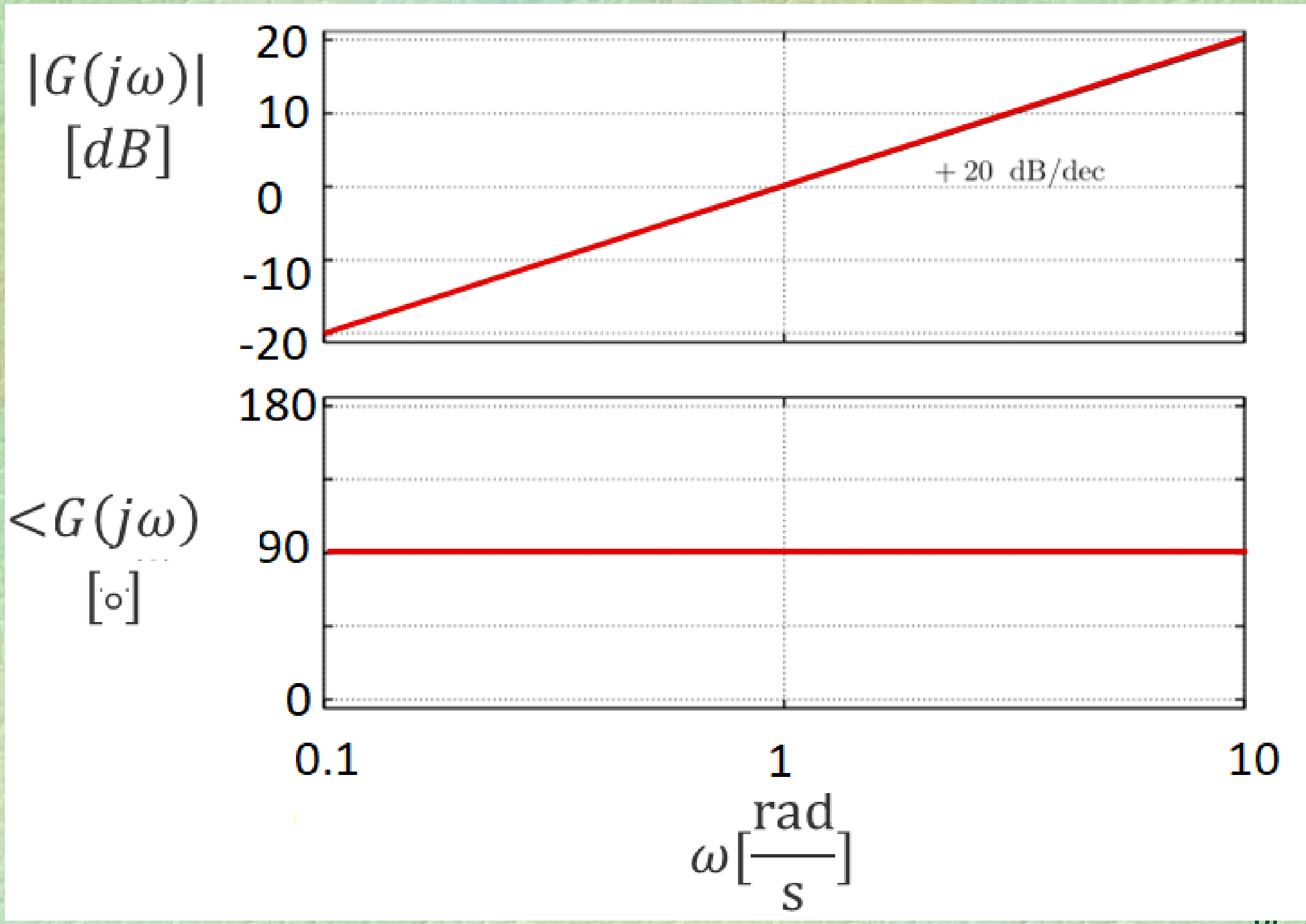
$$G(s) = \frac{1}{s} \quad \rightarrow \quad G(j\omega) = \frac{1}{j\omega}$$



Bode diagram

Plotting the Bode diagram for the transfer function ***G(s)=s***

$$G(s) = s \quad \rightarrow \quad G(j\omega) = j\omega$$



Bode diagram

Plotting the Bode diagram for the transfer function $G(s) = \frac{1}{\frac{s}{a}+1}$

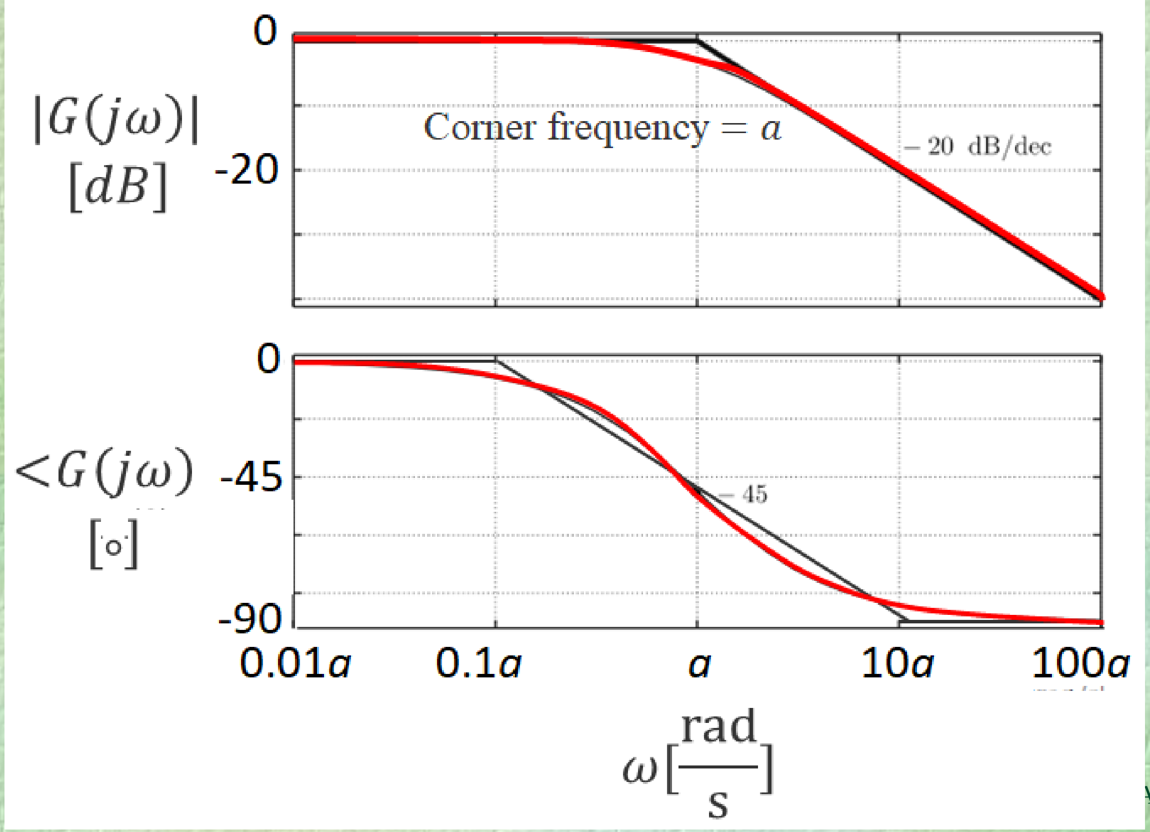
$$G(j\omega) = \frac{1}{1 + j\frac{\omega}{a}}$$

$\omega \ll a \quad G(j\omega) = 1 \quad \begin{cases} 20\log|G| = 0 \\ < G = 0 \end{cases}$

$\omega \gg a \quad G(j\omega) = -j(\frac{a}{\omega}) \quad \begin{cases} 20\log|G| = -20\ db/dec \\ < G = -90 \end{cases}$

$\omega = a \quad G(ja) = \frac{1}{1 + j1}$

$$\begin{cases} 20\log|G| = -3\ db \\ < G = -45 \end{cases}$$



Bode diagram

Plotting the Bode diagram for the transfer function **G(s)=1+s/*a***

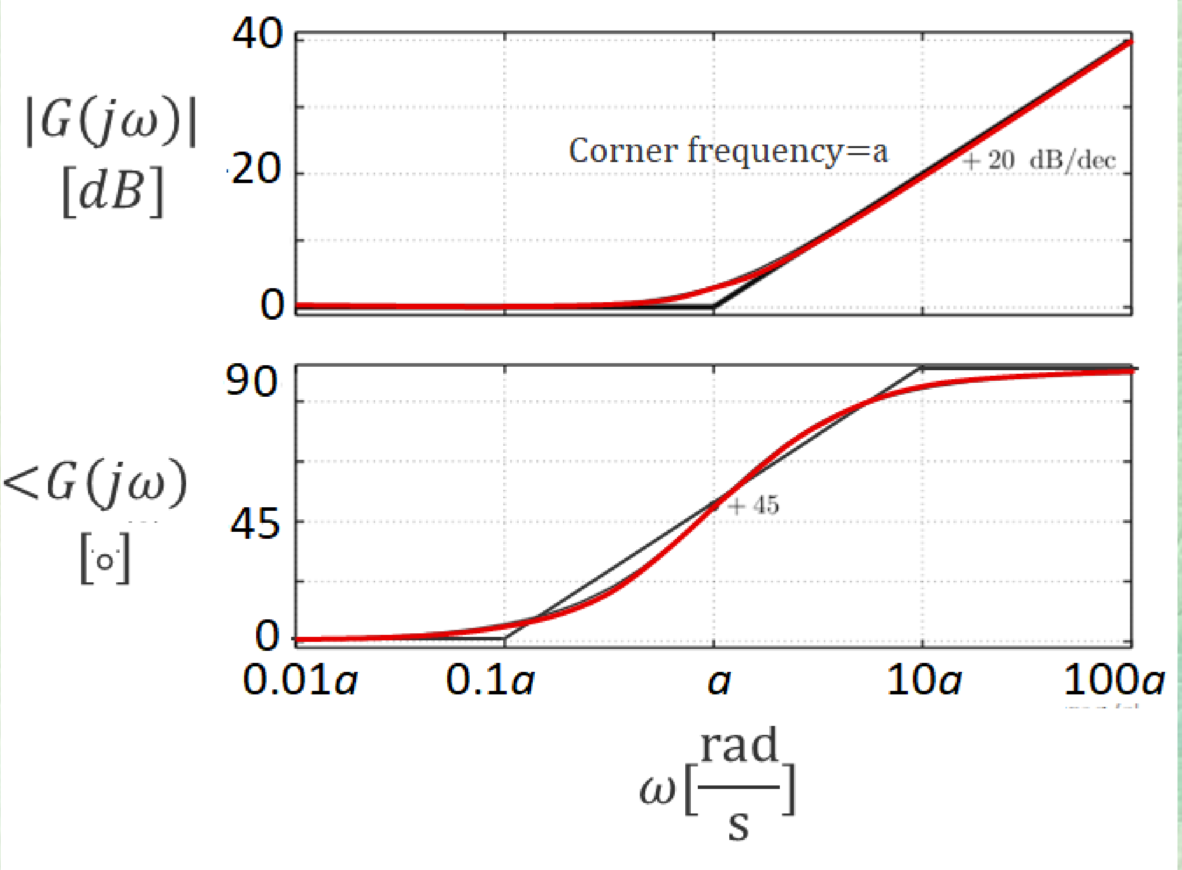
$$G(j\omega) = 1 + j\frac{\omega}{a}$$

$\omega \ll a \qquad G(j\omega) = 1 \qquad \begin{cases} 20\log|G| = 0 \\ < G = 0 \end{cases}$

$\omega \gg a \qquad G(j\omega) = j(\frac{\omega}{a}) \qquad \begin{cases} 20\log|G| = 20\ db/dec \\ < G = 90 \end{cases}$

$\omega = a \qquad G(ja) = 1 + j$

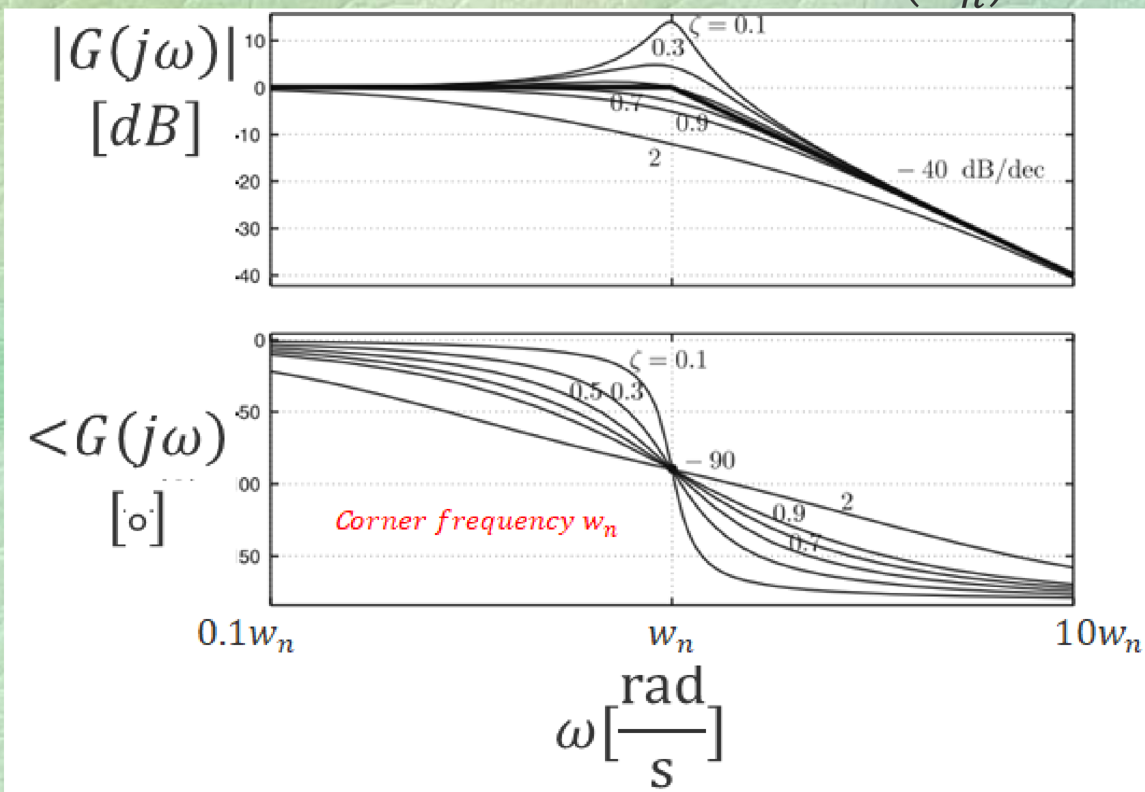
$$\begin{cases} 20\log|G| = 3\ db \\ < G = 45 \end{cases}$$



Bode diagram

Plotting the Bode diagram for the transfer function $G(s) = \frac{w_n^2}{s^2+2\xi w_n s+w_n^2}$

$$G(s) = \frac{1}{\left(\frac{s}{w_n}\right)^2 + \frac{2\xi}{w_n}s + 1}$$



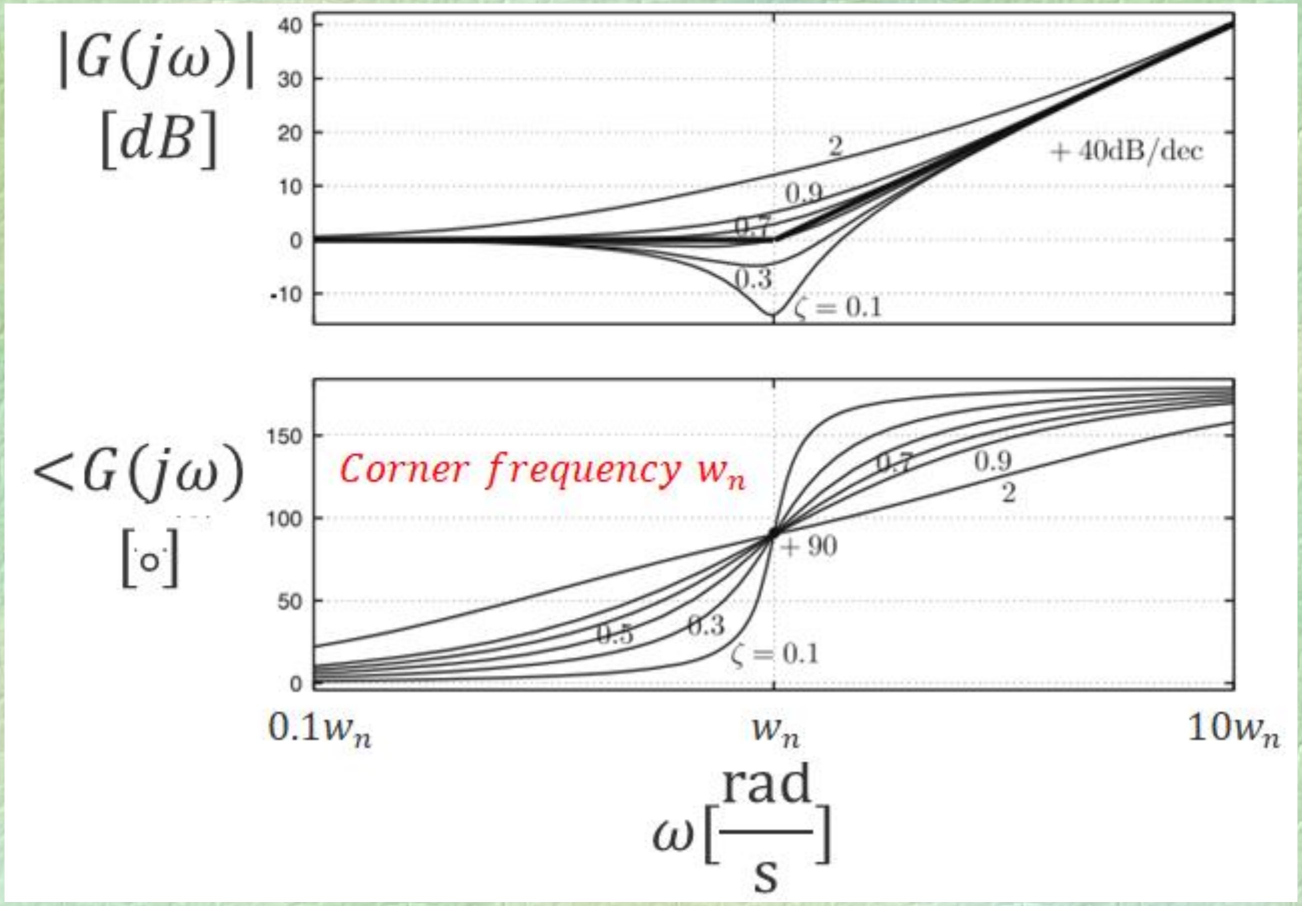
$$\begin{cases} for \quad \omega \prec\prec \omega n \Rightarrow \frac{\omega^2}{\omega_n^{\;2}} \cong 0 \quad \Rightarrow \quad M = -20\log 1 = 0 \quad db \\ for \quad \omega \equiv \omega n \Rightarrow \frac{\omega^2}{\omega_n^{\;2}} = 1 \quad \Rightarrow \quad M = -20\log 2\zeta \quad db \\ for \quad \omega \succ\succ \omega n \Rightarrow \frac{\omega^2}{\omega_n^{\;2}} \succ\succ 0 \Rightarrow \quad M = -40\log(\omega/\omega n) \quad db \end{cases}$$

$$\begin{cases} for \quad \omega \prec\prec \omega n \quad \Rightarrow \quad \frac{\omega^2}{\omega_n^{\;2}} \cong 0 \quad \Rightarrow \quad \phi = 0 \\ for \quad \omega \equiv \omega n \quad \Rightarrow \quad \frac{\omega^2}{\omega_n^{\;2}} = 1 \quad \Rightarrow \quad \phi = -tg^{-1}(2\zeta / 0) = -90 \\ for \quad \omega \succ\succ \omega n \quad \Rightarrow \quad \frac{\omega^2}{\omega_n^{\;2}} \succ\succ 0 \quad \Rightarrow \quad \phi = -180 \end{cases}$$

Bode diagram

Plotting the Bode diagram for the transfer function $G(s) = \frac{s^2+2\xi w_n s+w_n^2}{w_n^2}$

$$G(s) = \left(\frac{s}{w_n}\right)^2 + \frac{2\xi}{w_n}s + 1$$



Bode diagram

Example 1: Draw the Bode diagram for the given system.

$$H(s) = 100\frac{(s+1)}{s^2+110s+1000}$$

Rewrite the transfer function in an appropriate form.

$$H(s) = 100\frac{(s+1)}{s^2+110s+1000} = 100\frac{(s+1)}{(s+10)(s+100)}$$

Extract the components of the transfer function.

$$H(s) = \frac{100}{10\cdot 100}\frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)} = 0.1\frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

- Gain equal to 0.1
- A simple pole at the corner frequency of 10
- A simple pole at the corner frequency of 100
- A simple zero at the corner frequency of 1

Bode diagram

Example 1: Draw the Bode diagram for the given system.

$$H(s) = \frac{100}{10 \cdot 100} \frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)} = 0.1 \frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

Draw the Bode diagram for the main components of the transfer function.

- **Gain equal to 0.1:** a line with a magnitude of -20dB for amplitude and a line with a magnitude of zero for phase.
- **A simple pole at the corner frequency 10:** Magnitude up to the corner frequency (i.e., 10), 0dB, and after that, with a slope of -20dB/dec and phase up to less than 0.1 of the corner frequency (i.e., 1), 0 degrees; at the corner frequency (i.e., 10), -45 degrees; and from 10 times the corner frequency (i.e., 100) onwards, -90 degrees.

Bode diagram

Example 1: Draw the Bode diagram for the given system.

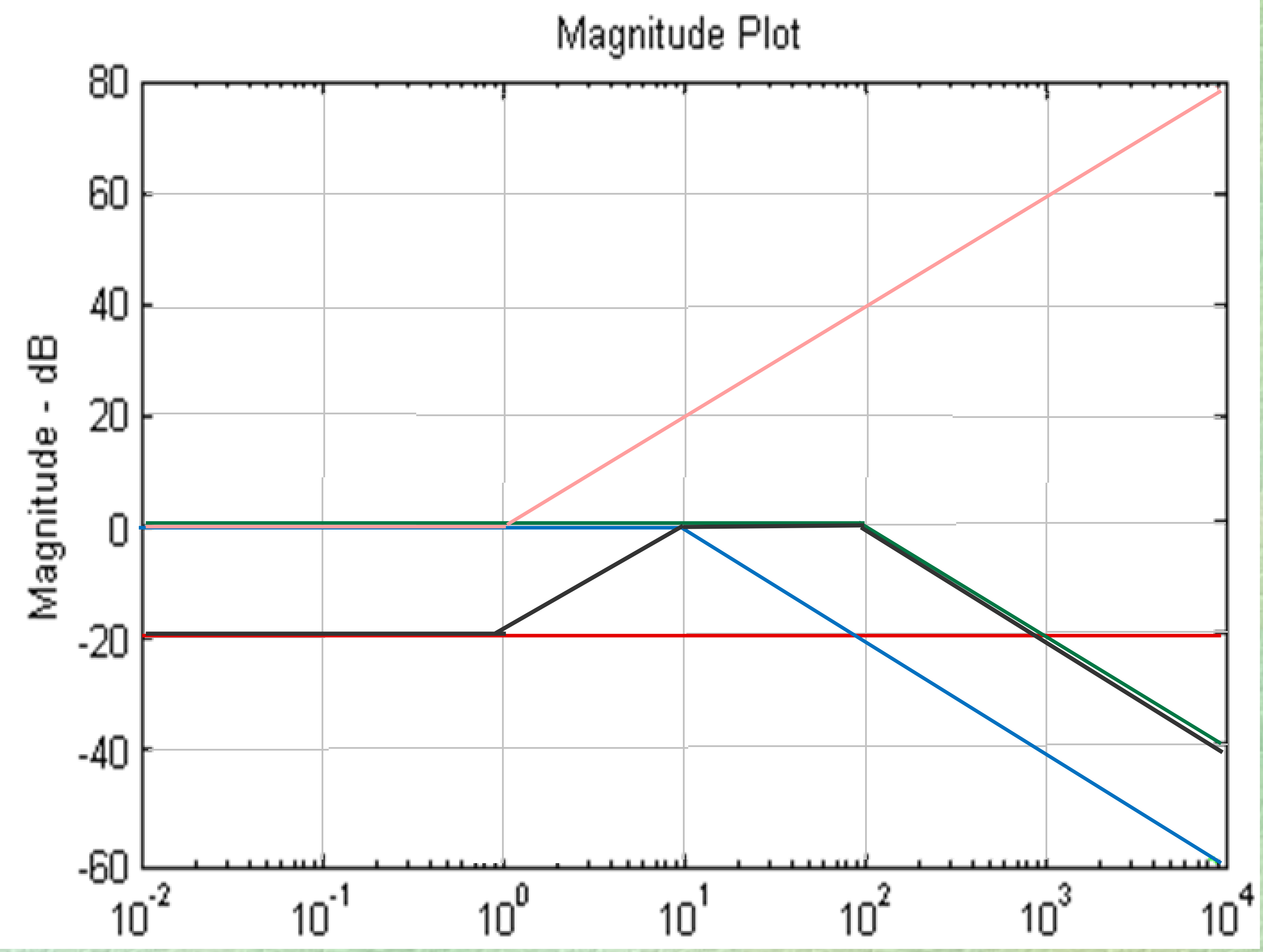
$$H(s)=\frac{100}{10\cdot 100}\frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}=0.1\frac{\frac{s}{1}+1}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

Draw the Bode diagram for the main components of the transfer function.

- **A simple pole at the corner frequency 100:** Magnitude up to the corner frequency (i.e., 100), 0dB, and after that, with a slope of -20dB/dec and phase up to less than 0.1 of the corner frequency (i.e., 10), 0 degrees; at the corner frequency (i.e., 100), -45 degrees; and from 10 times the corner frequency (i.e., 1000) onwards, -90 degrees.

- **A simple zero at the corner frequency 1:** Magnitude up to the corner frequency (i.e., 1), 0dB, and after that, with a slope of +20dB/dec and phase up to less than 0.1 of the corner frequency (i.e., 0.1), 0 degrees; at the corner frequency (i.e., 1), +45 degrees; and from 10 times the corner frequency (i.e., 10) onwards, +90 degrees.

Bode diagram



Gain equal to 0.1

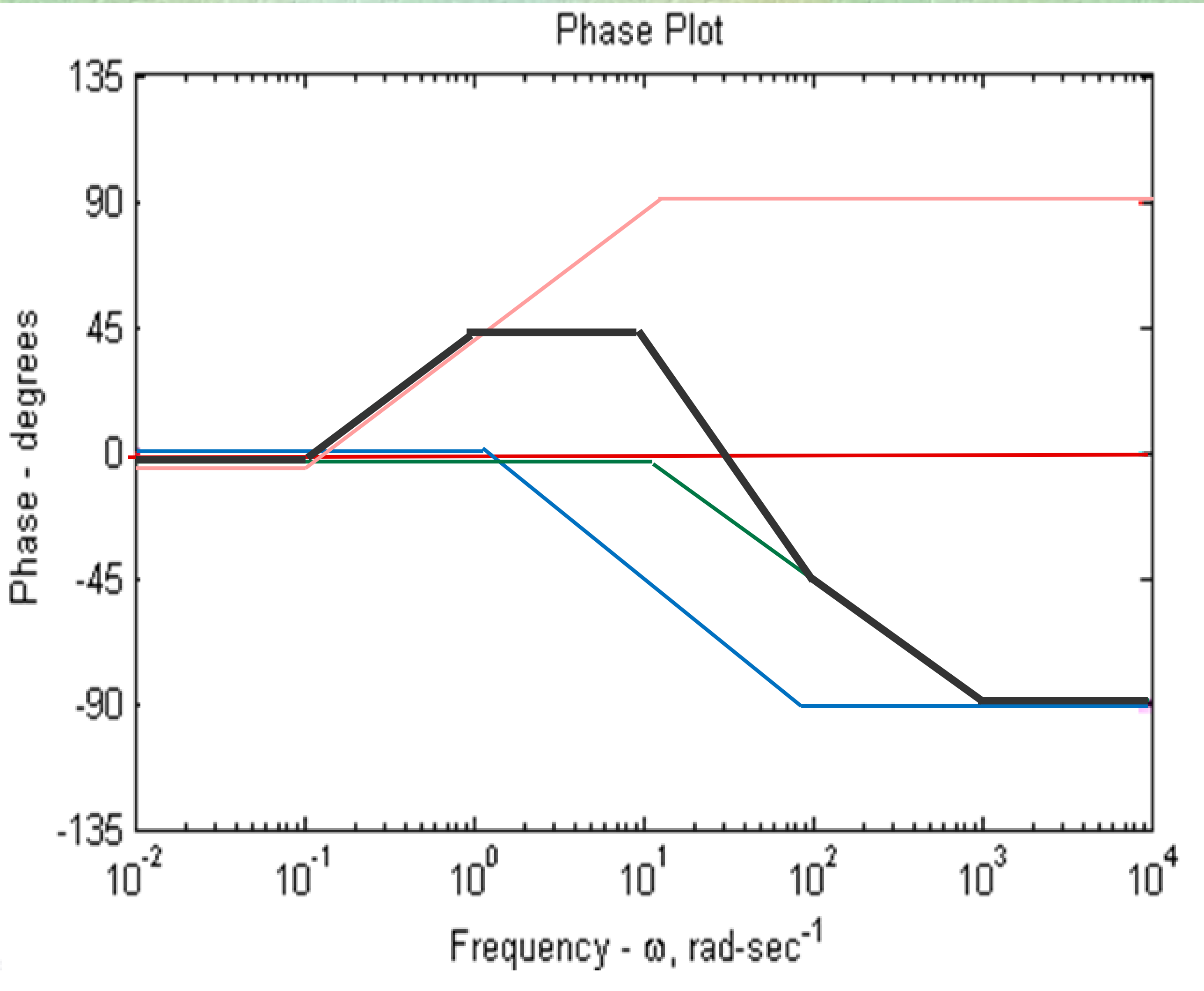
A simple pole at the corner frequency of 10

A simple pole at the corner frequency of 100

A simple zero at the corner frequency of 1

Bode diagram overview

Bode diagram



Gain equal to 0.1

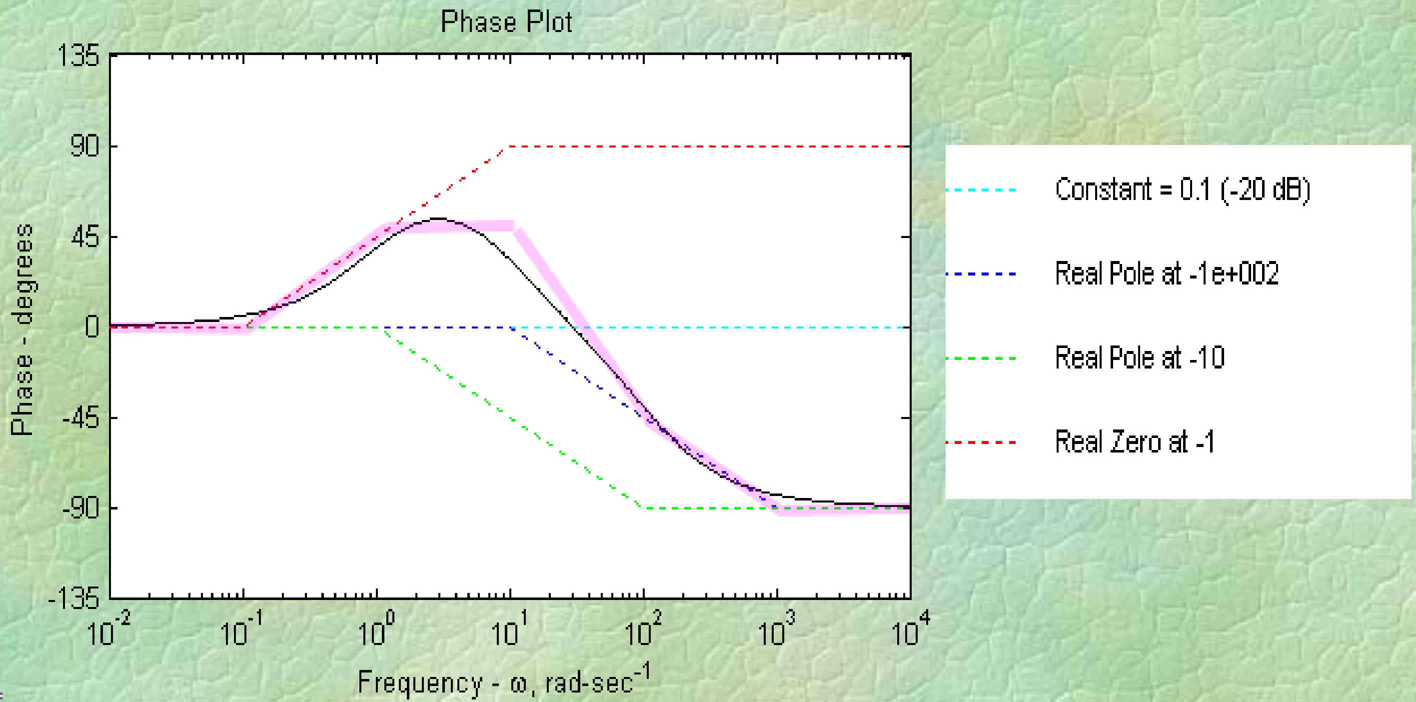
A simple pole at the corner frequency of 10

A simple pole at the corner frequency of 100

A simple zero at the corner frequency of 1

Bode diagram overview

Bode diagram



- Gain is 0.1
- A simple pole at the corner frequency of 10
- A simple pole at the corner frequency of 100
- A simple zero at the corner frequency of 1

Bode diagram

Example 2: Draw the Bode diagram for the given system. $H(s) = 10\dfrac{s+10}{s^2+3s}$

Rewrite the transfer function in an appropriate form.

$$H(s) = 10\frac{10}{3}\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)} = 33.3\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)}$$

Extract the components of the transfer function.

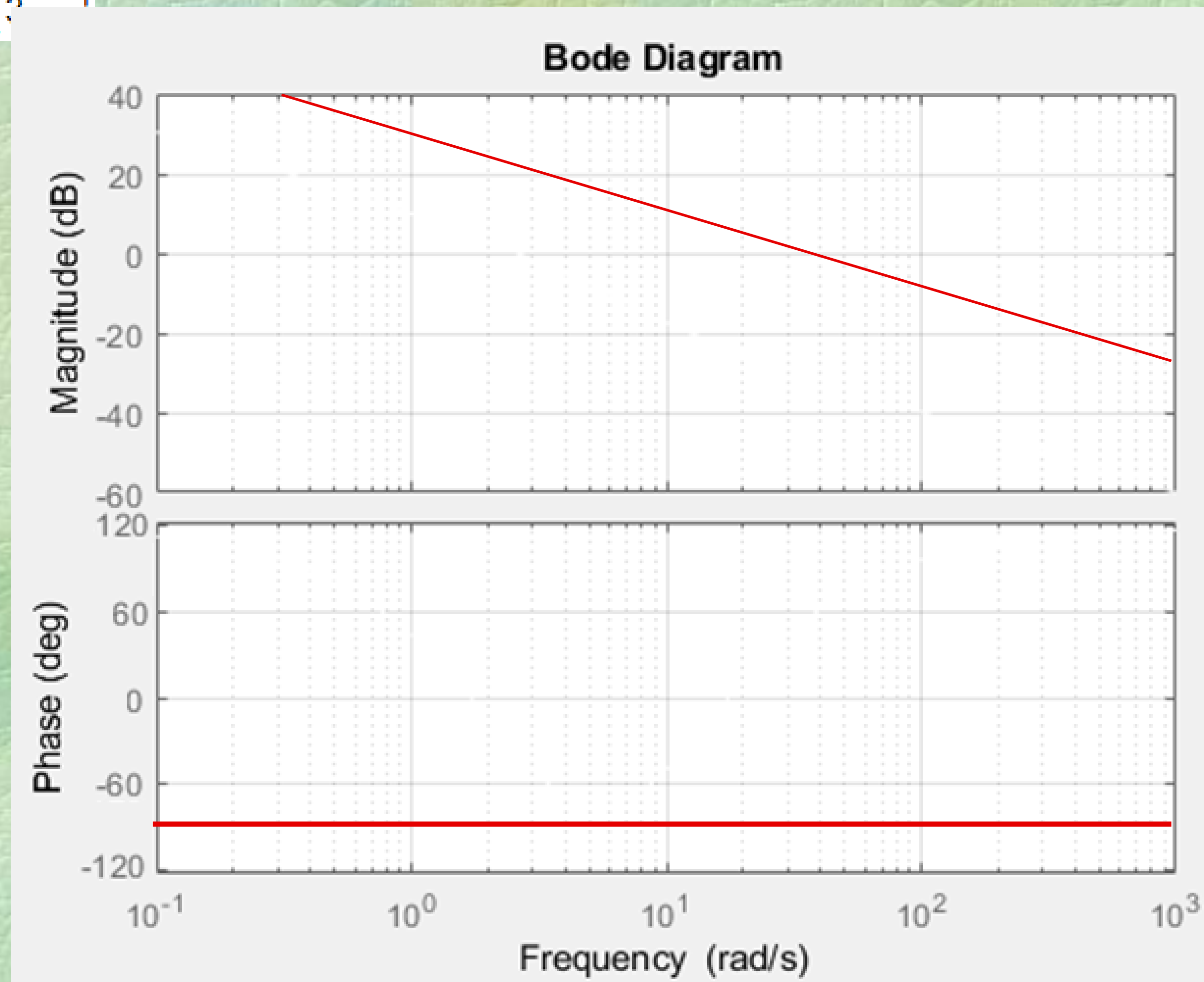
- Gain is 33.3 along with an integrator function.
- A simple zero at the corner frequency of 10.
- A simple pole at the corner frequency of 3.

Bode diagram

$$H(s) = 10\frac{10}{3}\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)} = 33.3\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)}$$

- Gain is 33.3 along with an integrator function.

$$\frac{33.3}{s}$$

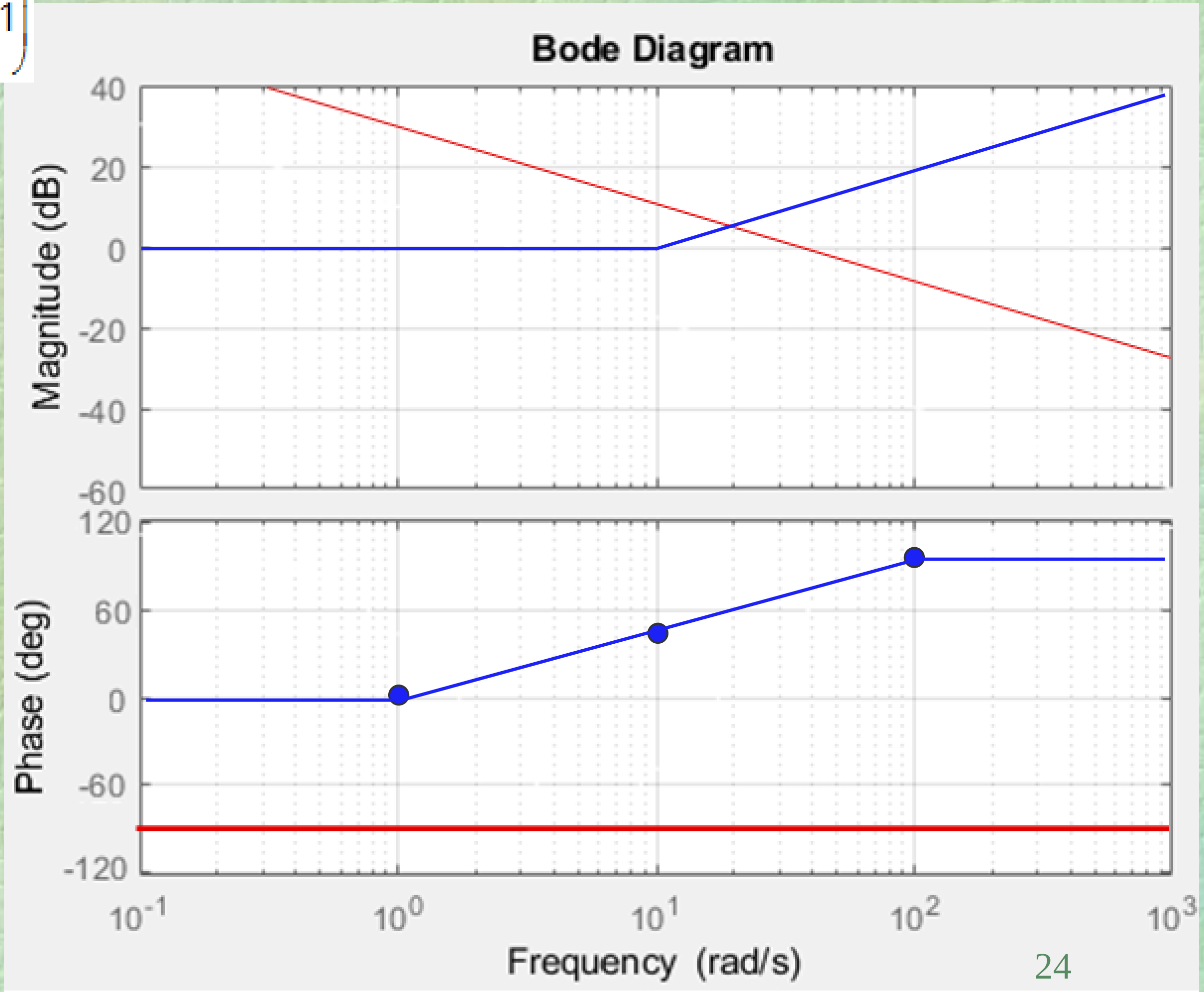


Bode diagram

$$H(s) = 10\frac{10}{3}\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)} = 33.3\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)}$$

- A simple zero at the corner frequency of 10.

$$\frac{s}{10}+1$$

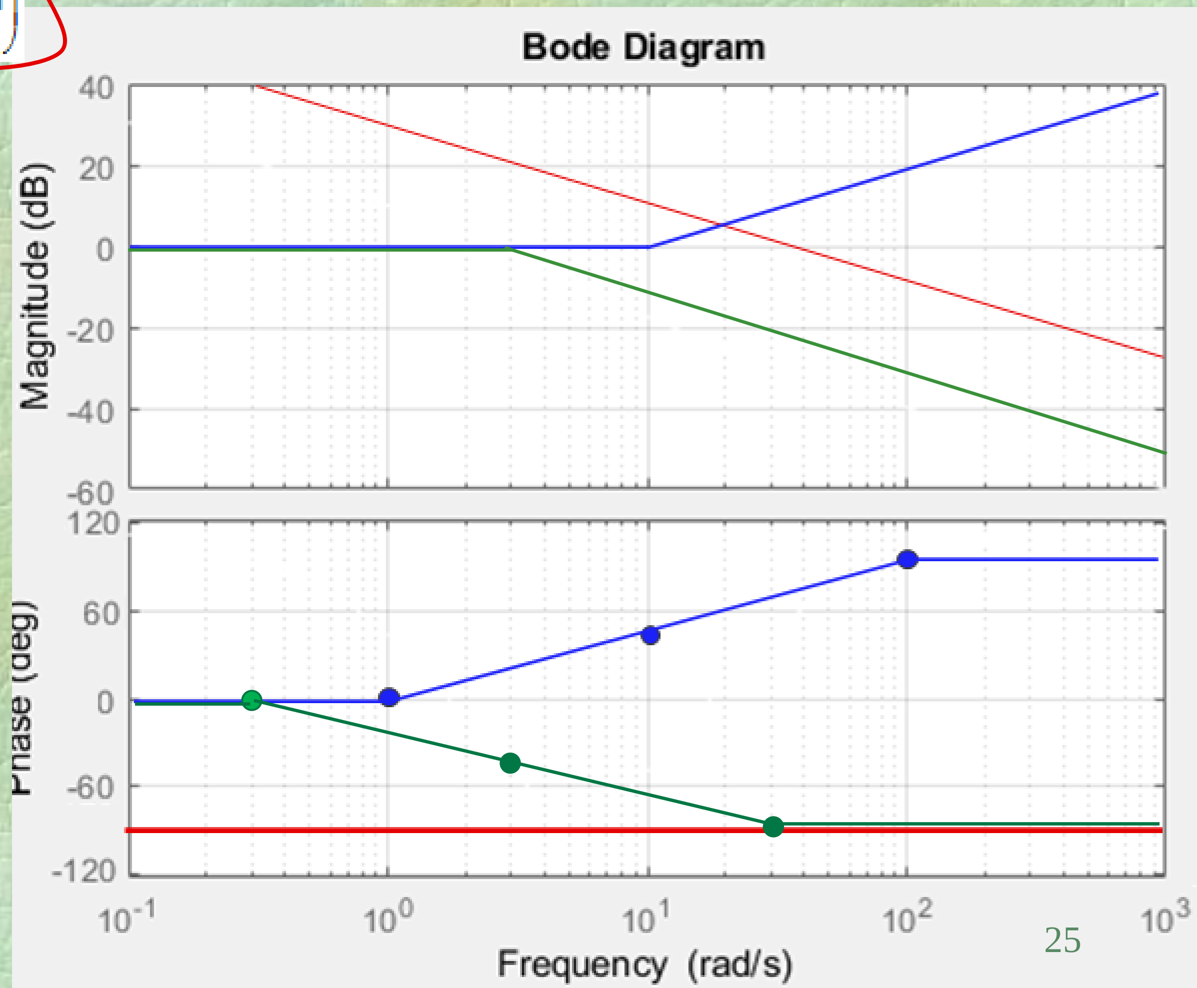


Bode diagram

$$H(s) = 10\frac{10}{3}\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)} = 33.3\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)}$$

- A simple pole at the corner frequency of 3.

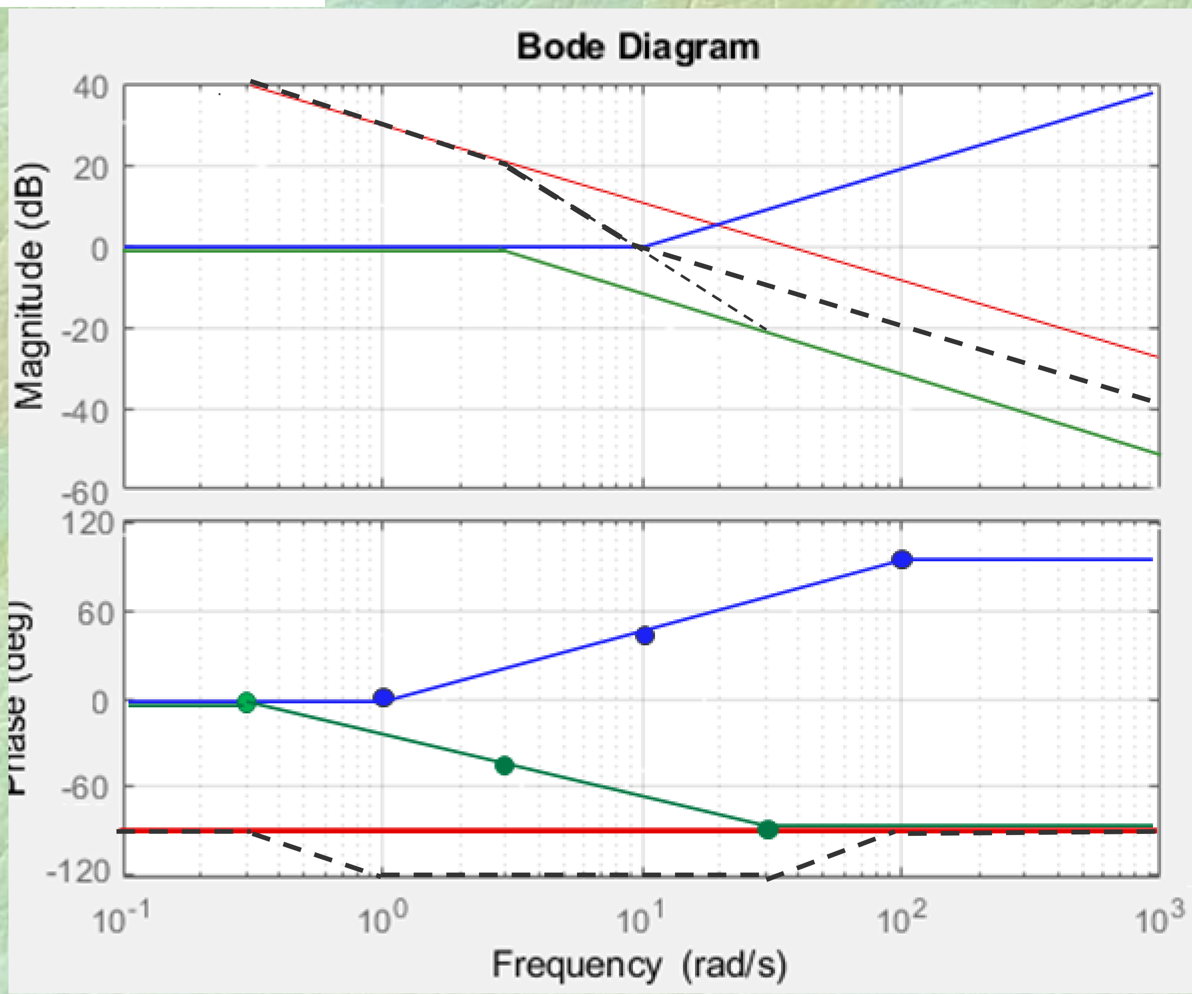
$$\frac{1}{\frac{s}{3}+1}$$



Bode diagram

$$H(s) = 10\frac{10}{3}\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)} = 33.3\frac{\frac{s}{10}+1}{s\left(\frac{s}{3}+1\right)}$$

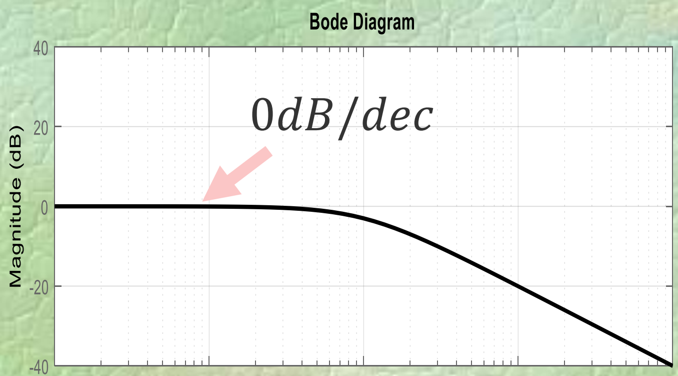
Bode diagram overview



Bode diagram

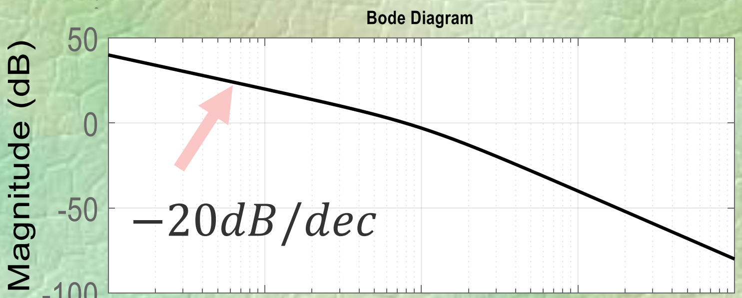
Determining the type of system from the Bode diagram

The slope of the magnitude plot at low frequencies determines the type of the system.



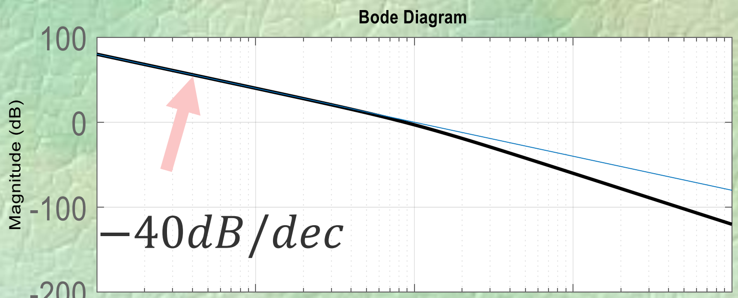
The system is of type 0.

$$G(s) = \frac{1}{s+1}$$



The system is of type 1.

$$G(s) = \frac{1}{s(s+1)}$$



The system is of type 2.

$$G(s) = \frac{1}{s^2(s+1)}$$

Bode diagram

Minimum phase and non-minimum phase systems

- A minimum phase system is a linear time-invariant system whose transfer function has all poles and zeros in the left half of the complex plane.

- A non-minimum phase system is a linear time-invariant system whose transfer function has one or more zeros in the right half of the complex plane.

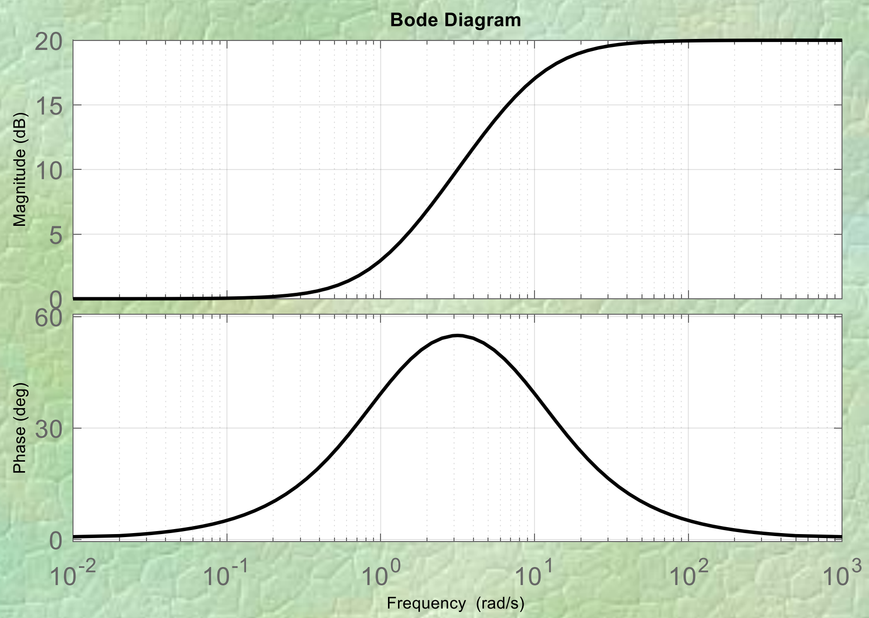
$$G_1(s) = 10\frac{1+s}{10+s} \qquad G_1(jw) = \frac{10\sqrt{1+w^2}}{\sqrt{100+w^2}}\angle(\tan^{-1} w - \tan^{-1}\frac{w}{10})$$

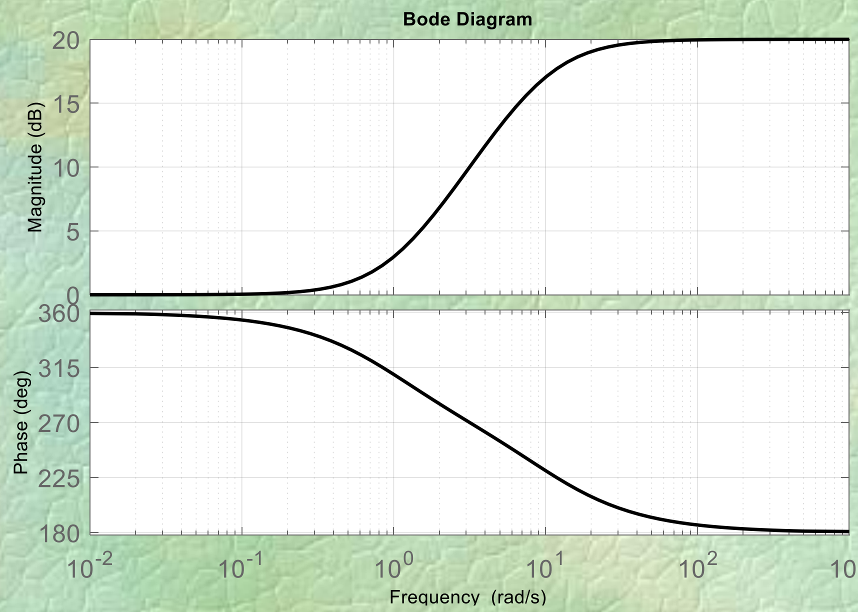
$$G_2(s) = 10\frac{1-s}{10+s} \qquad G_2(jw) = \frac{10\sqrt{1+w^2}}{\sqrt{100+w^2}}\angle(-\tan^{-1} w - \tan^{-1}\frac{w}{10})$$

$$|G_1(j\omega)| = |G_2(j\omega)| \qquad \angle G_1(j\omega) \neq \angle G_2(j\omega)$$

Bode diagram

Minimum phase and non-minimum phase systems





$$G_1(s) = 10\frac{1+s}{10+s}$$

$$G_2(s) = 10\frac{1-s}{10+s}$$

Bode diagram

Minimum phase and non-minimum phase systems

A few remarks:

1.For minimum-phase systems, the phase plot can be determined from the magnitude plot and vice versa. However, this is not possible for non-minimum-phase systems.

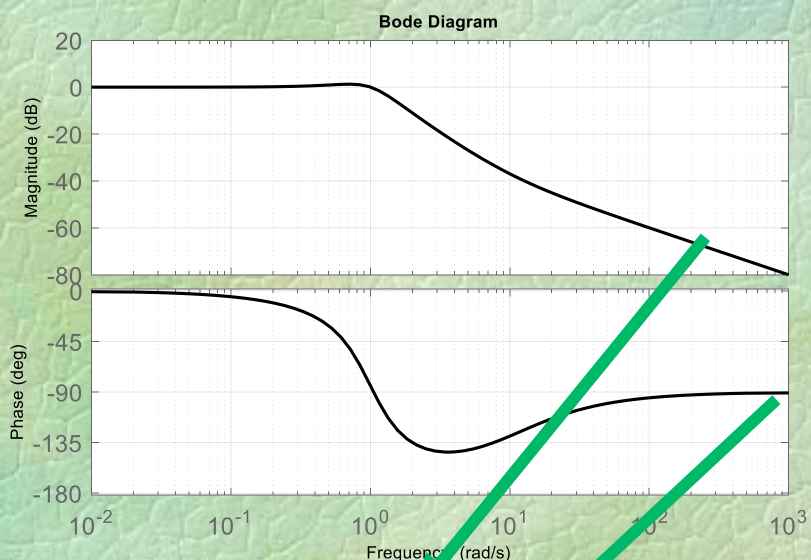
2.In both minimum-phase and non-minimum-phase systems, the slope of the magnitude plot at high frequencies is given by the relation $-20(P-Z)$ dB, where P is the number of poles (degree of the denominator) and Z is the number of zeros (degree of the numerator) of the transfer function.

3.Only in minimum-phase systems, the phase at high frequencies is given by the relation $-90(P-Z)$. This condition does not hold for non-minimum-phase systems.

4.By examining the slope of the magnitude and phase at high frequencies, it is possible to determine whether the system is minimum-phase or non-minimum-phase.

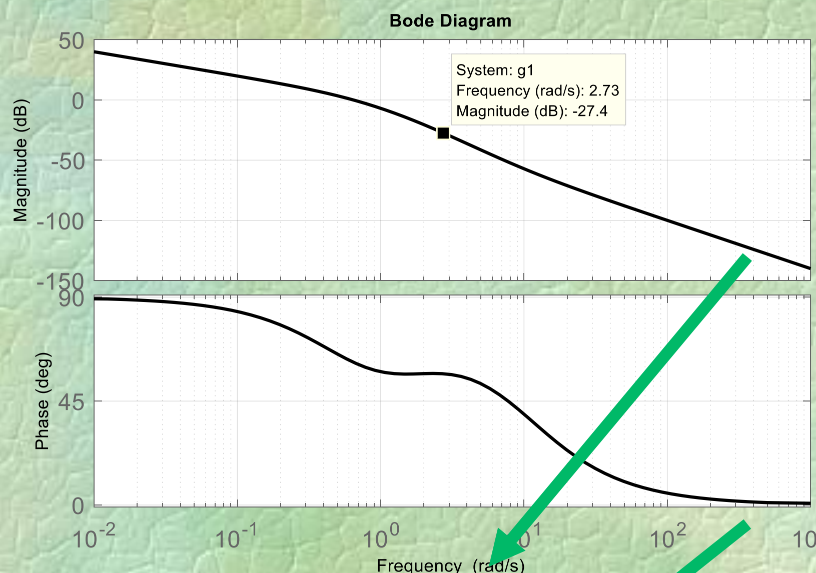
Bode diagram

Example 3: Determine if the following systems are minimum phase or non-minimum phase.



$$-20(P-Z)=-20$$
$$(P-Z)=1$$
$$-90(P-Z)=-90$$

The relationship holds, so the system is minimum phase.



$$-20(P-Z)=-40$$
$$(P-Z)=2$$
$$-90(P-Z)\neq 0$$

The relationship does not hold, so the system is not minimum phase.

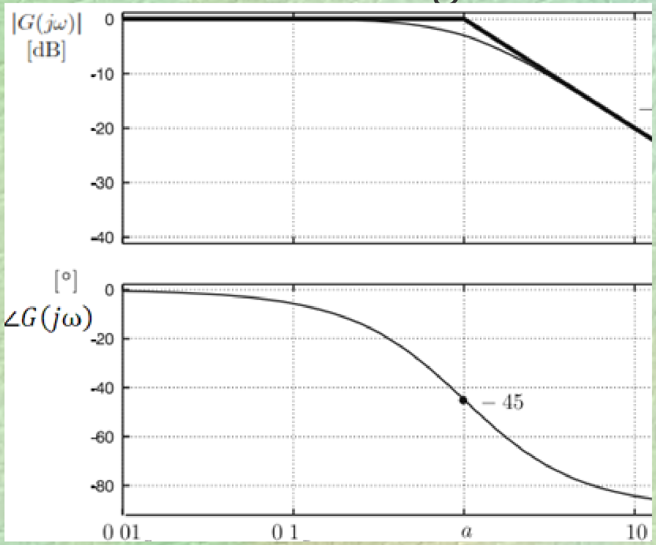
Frequency domain charts

- Bode plot.
- Nichols chart.
- Polar plot.

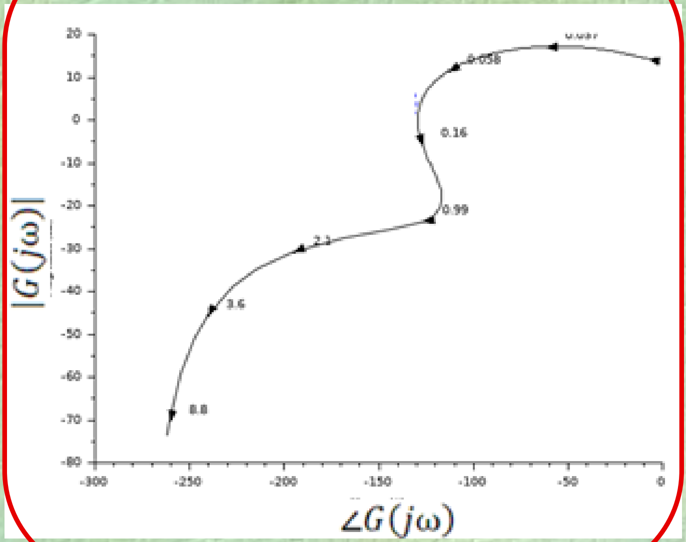
Frequency Domain Representation of Systems

$$|G(j\omega)| = \frac{A_y(\omega)}{A_u(\omega)} \quad \text{and} \quad \angle G(j\omega) = \varphi_y(\omega) - \varphi_u(\omega)$$

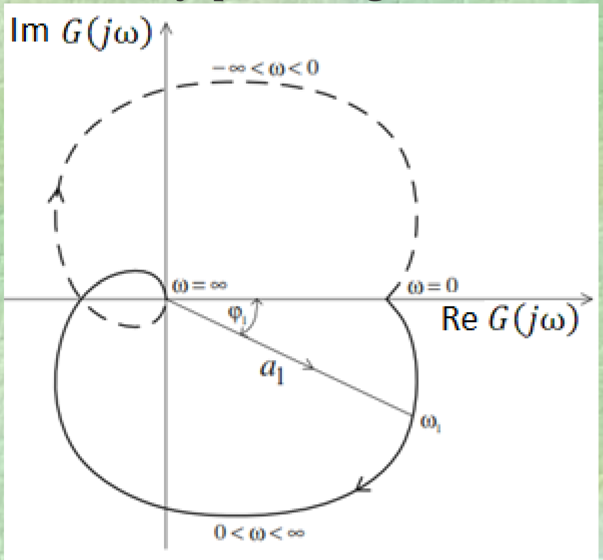
Bode diagram



Nichols diagram



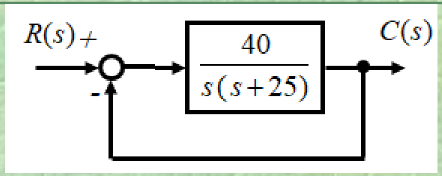
Nyquist diagram

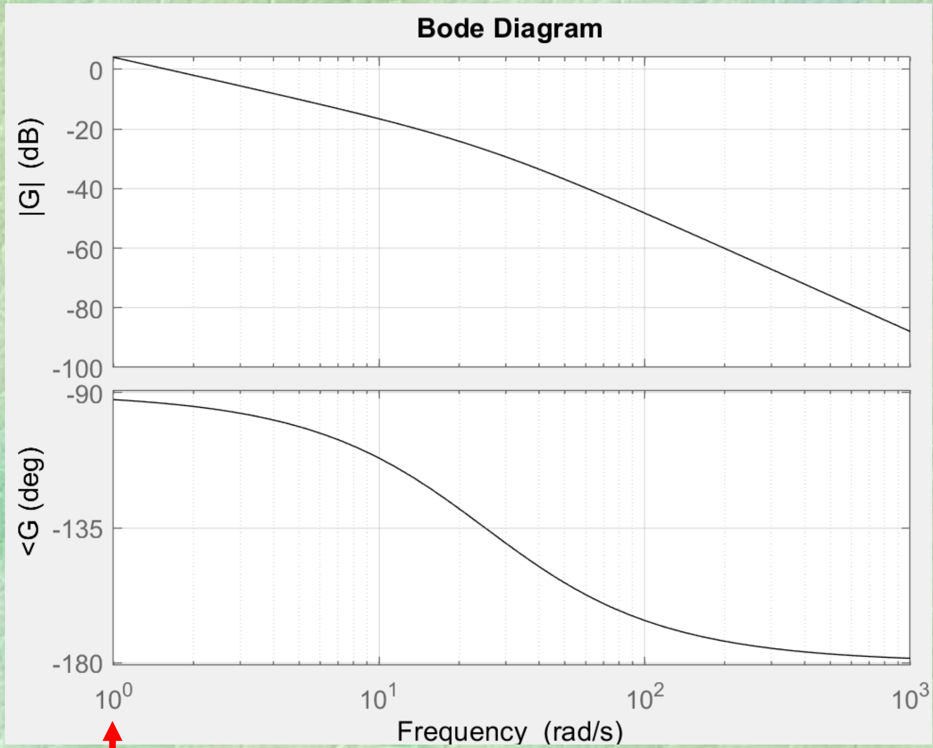


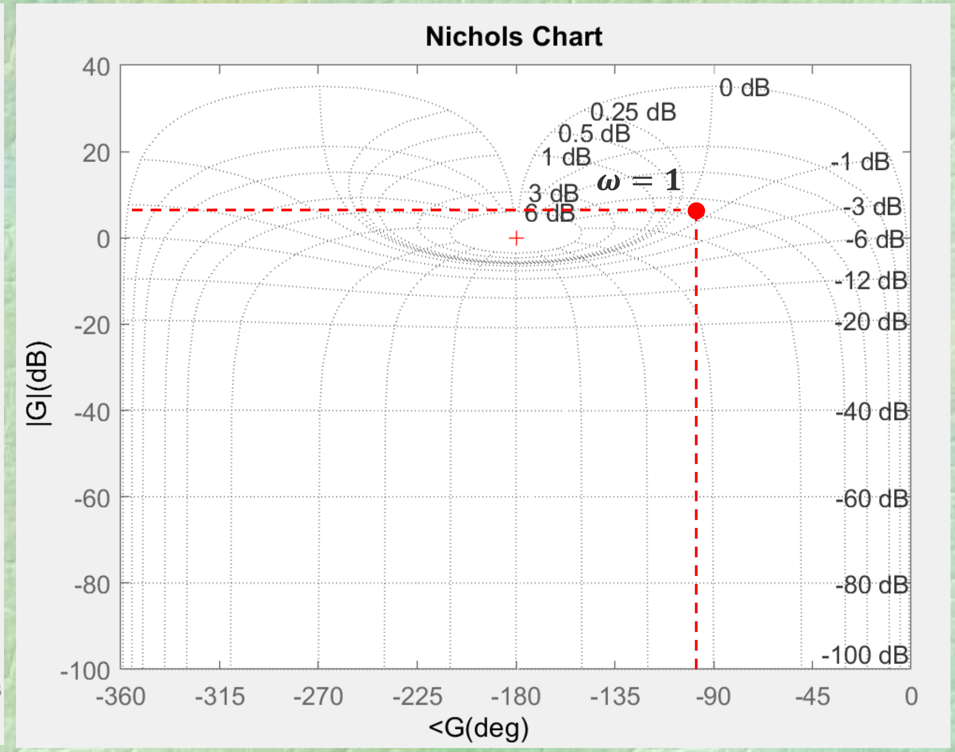
Nichols Diagram (Magnitude-Phase Diagram)

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:



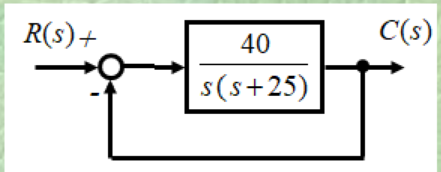


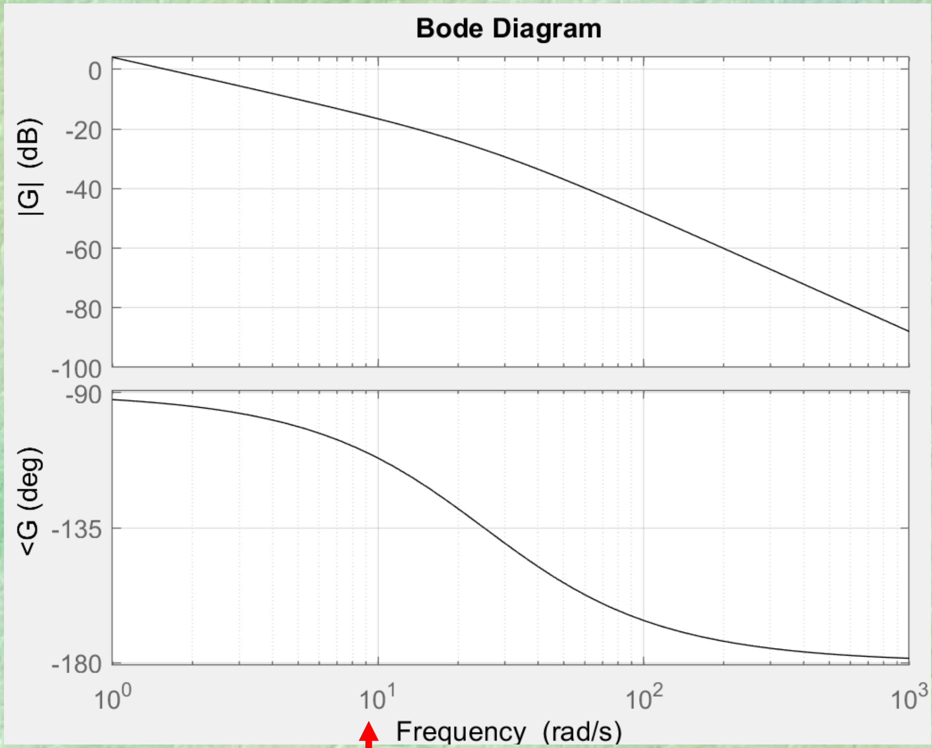


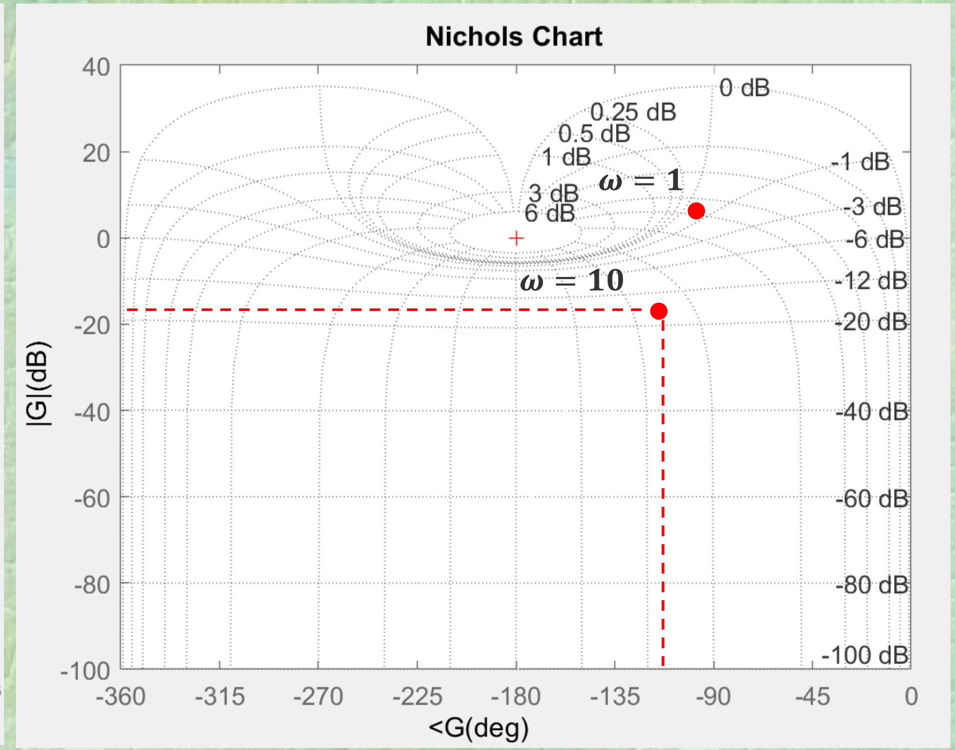
$$\omega = 1\frac{rad}{sec} \quad |G| = ? \quad < G = ?$$

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:



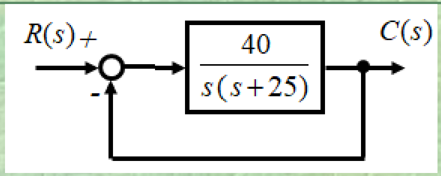


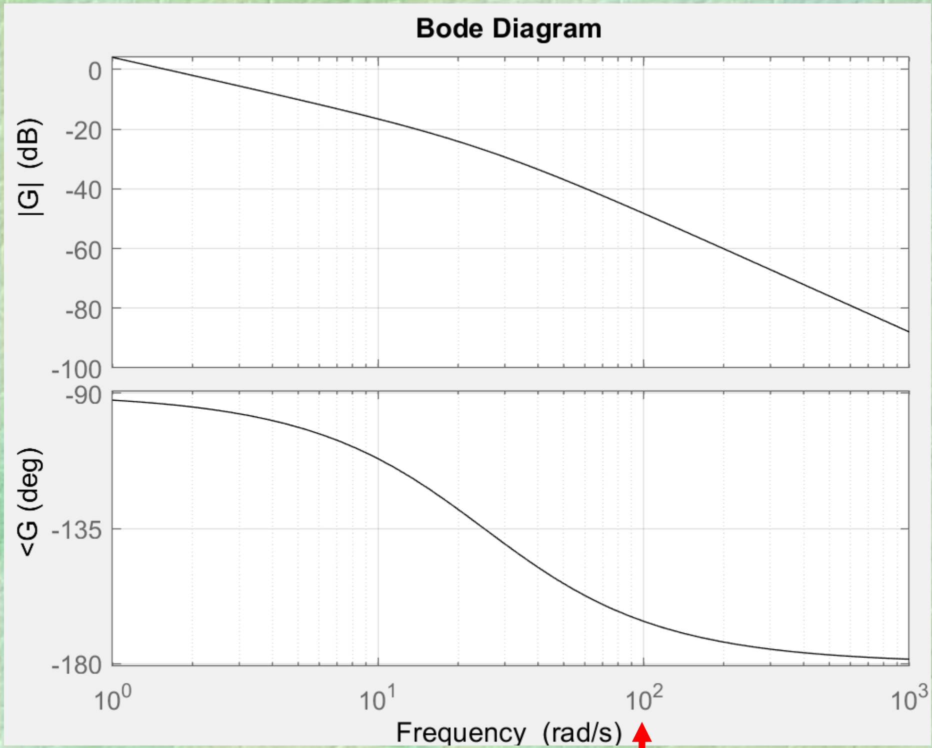


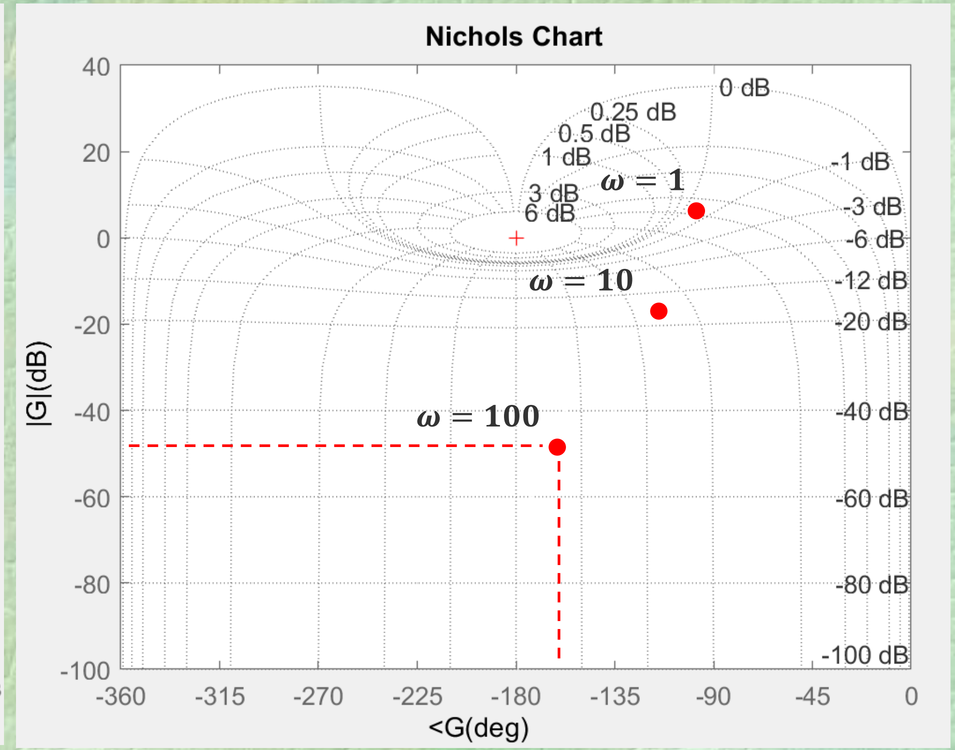
$\omega = 10 \dfrac{rad}{sec}$ $|G| = ?$ $< G = ?$

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:



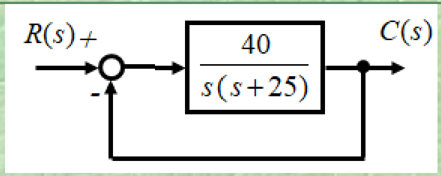


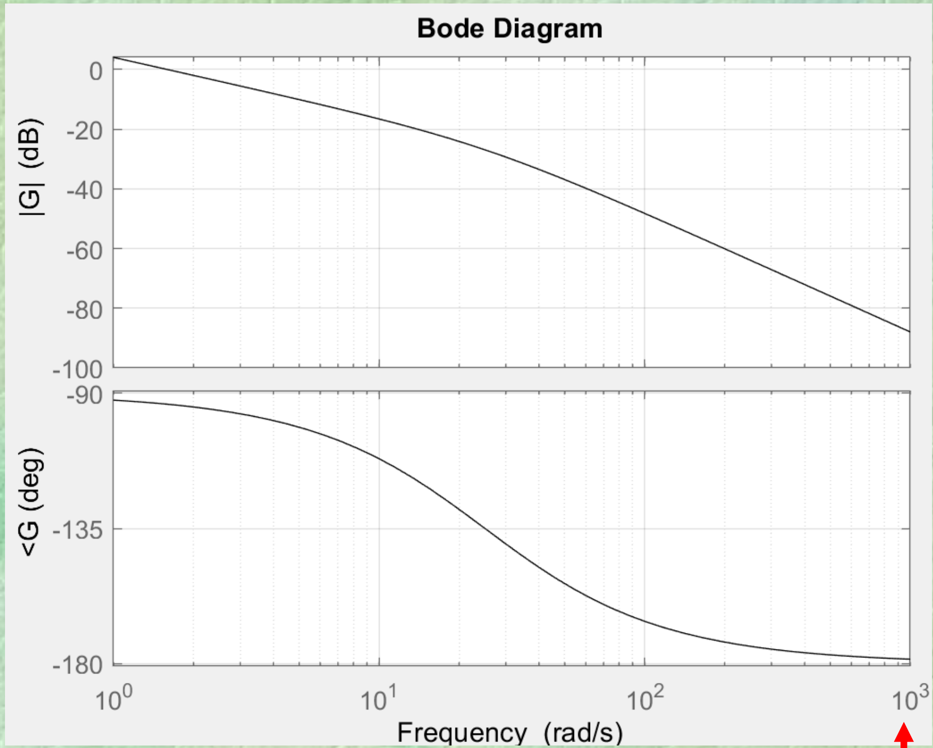


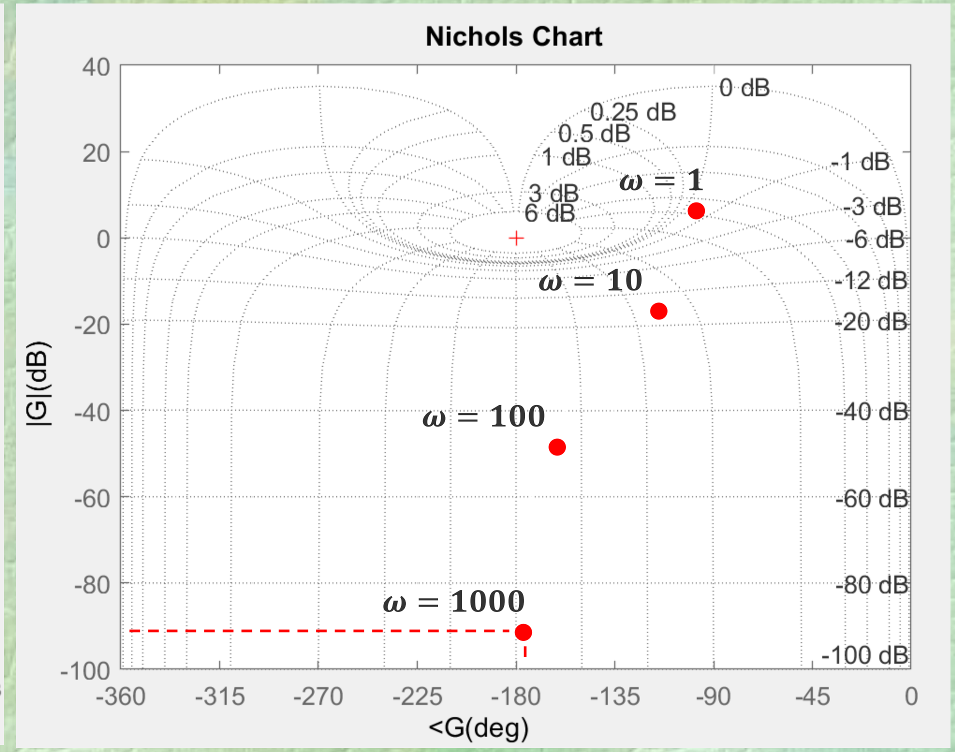
$\omega = 100\dfrac{rad}{sec}$ $\quad |G| = ? \quad < G = ?$

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:





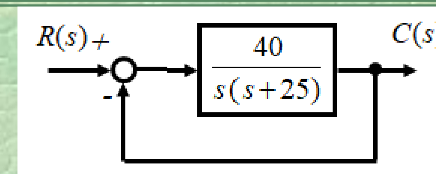


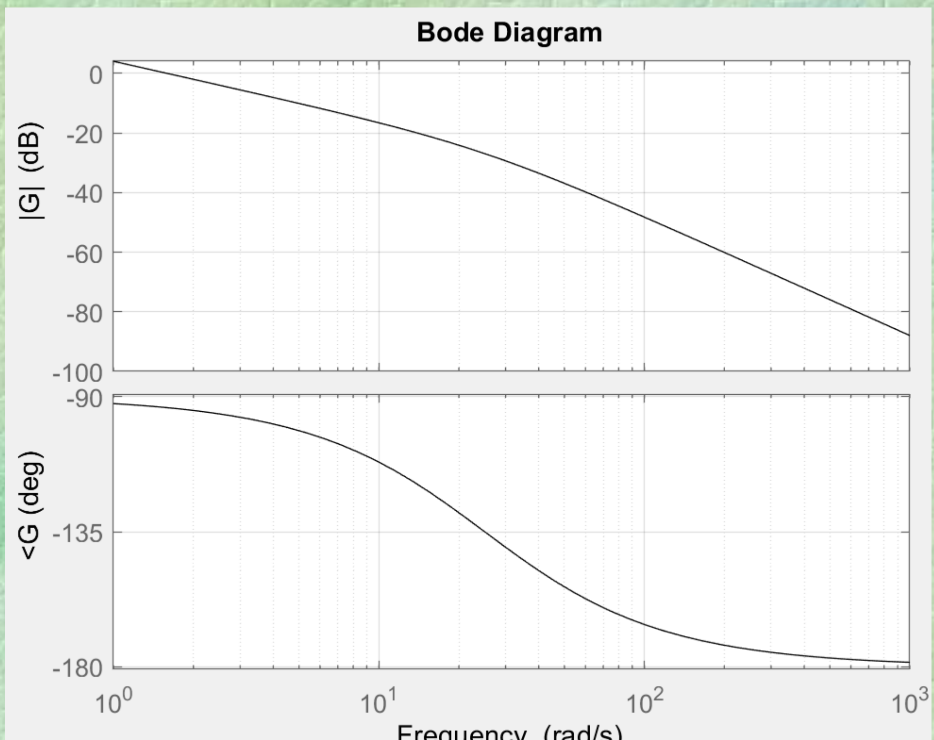
$$\omega = 1000\frac{rad}{sec} \quad |G| = ? \quad < G = ?$$

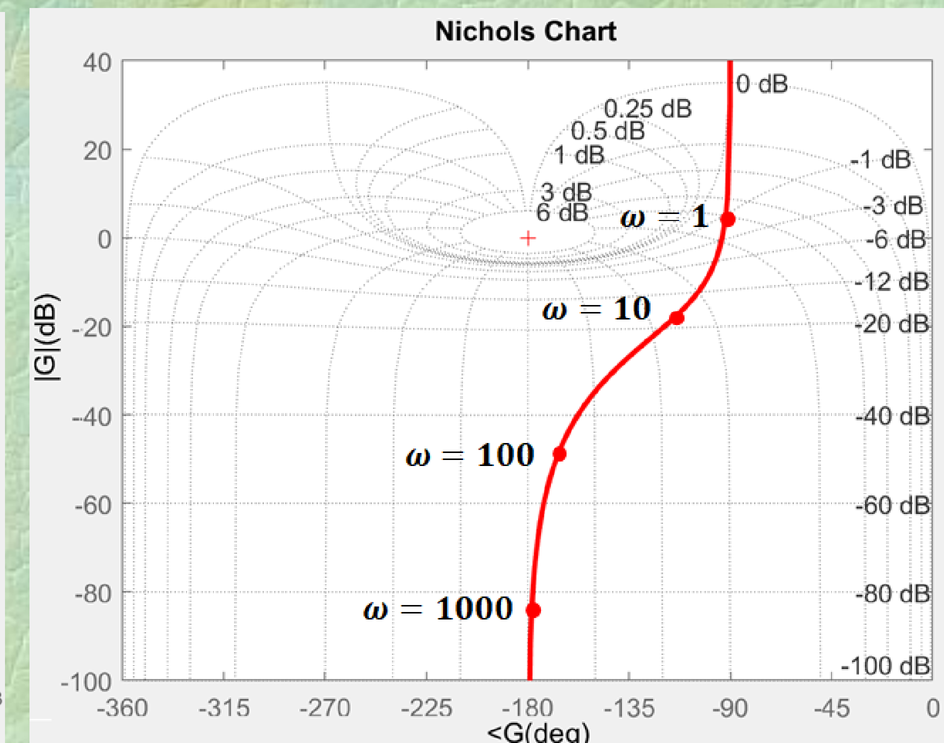
Dr. Ali Karimpour Aug 2024

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:

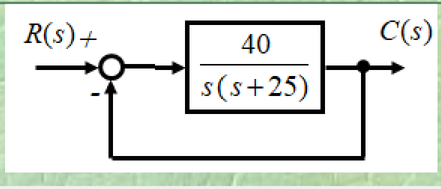


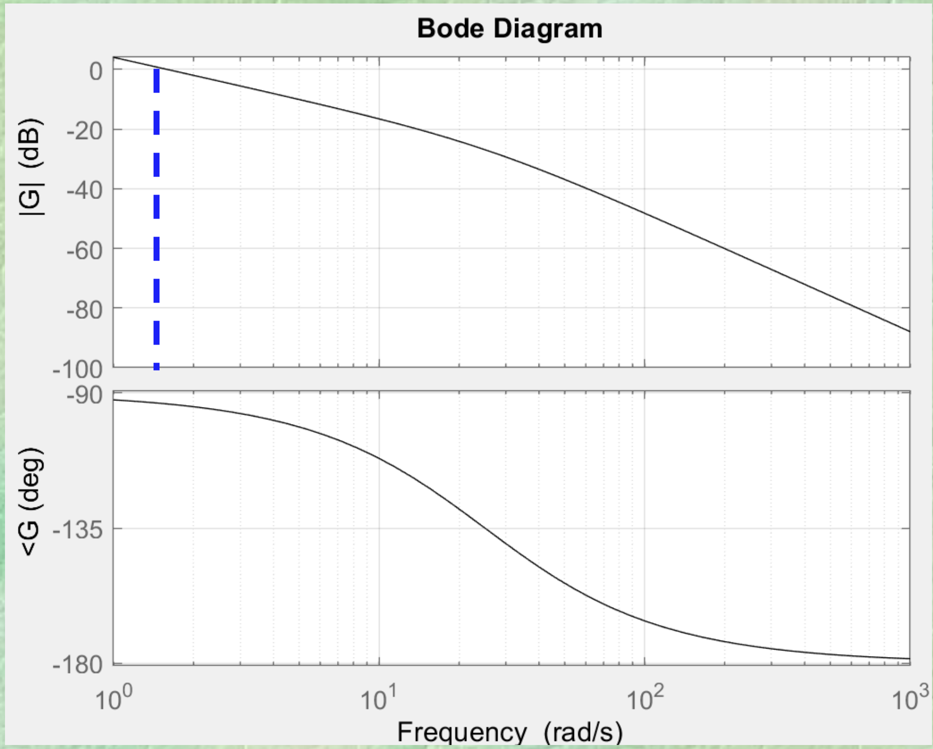


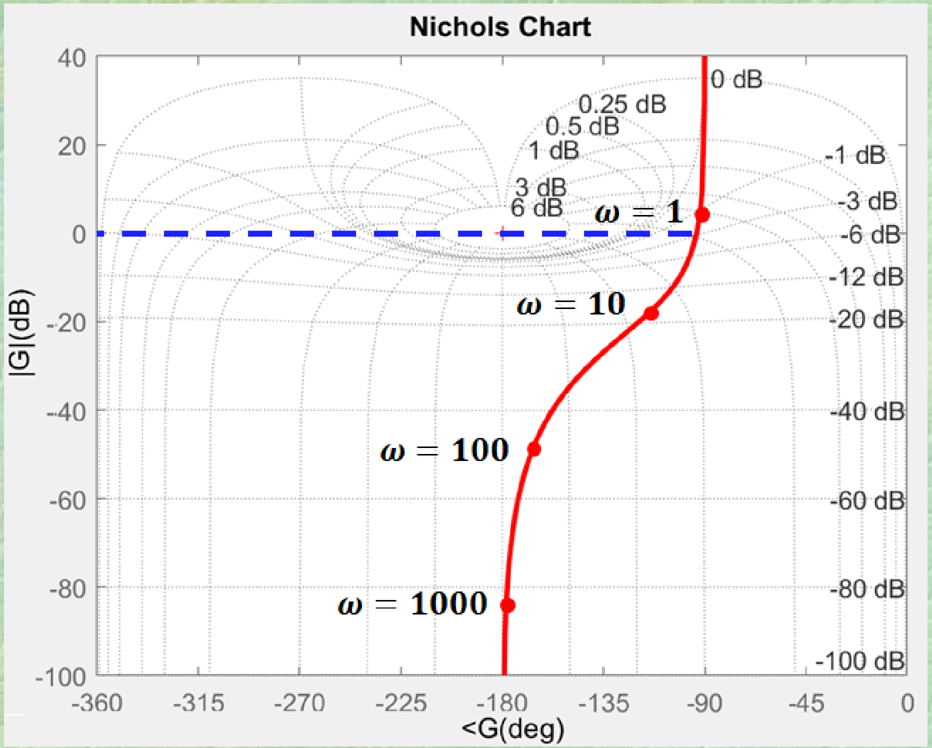


How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:







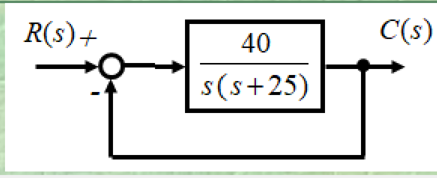
1- Gain crossover frequency ω_c : The frequency at which the magnitude plot intersects the 0 dB line (the magnitude of the transfer function equals 1).

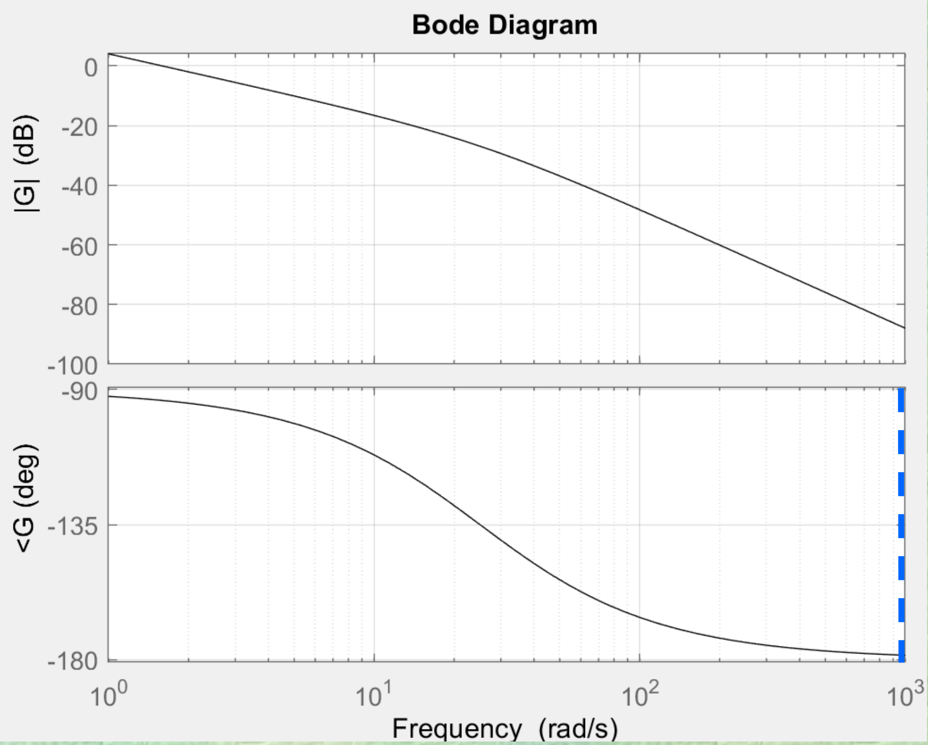
$\omega_c = 1.5$

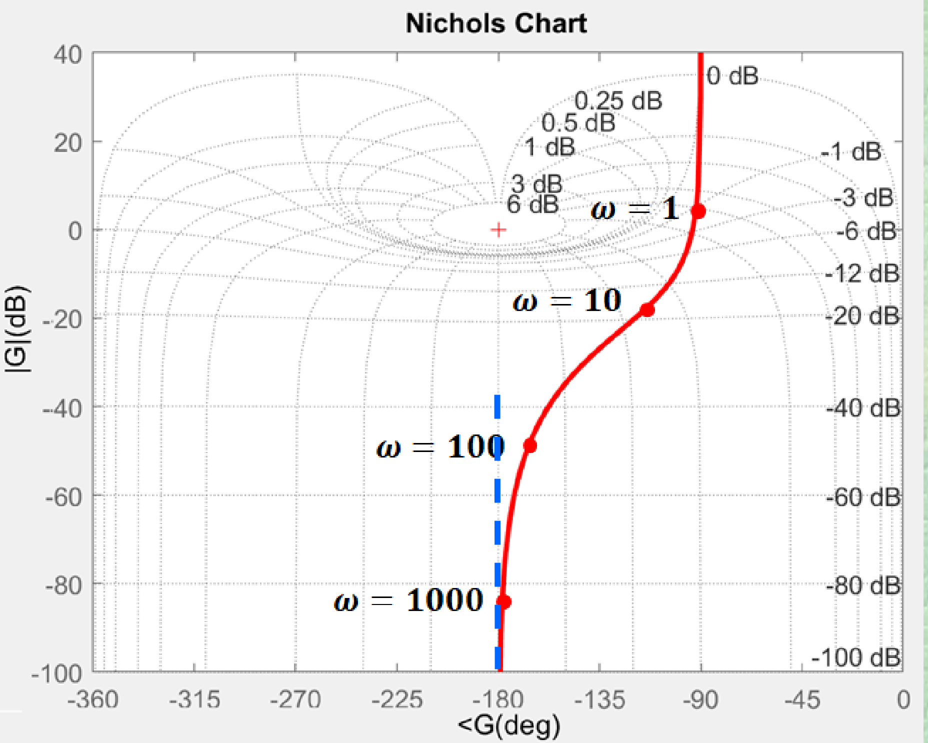
Which is more precise, Bode or Nichols? Why?

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:







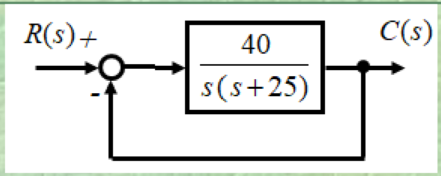
Phase crossover frequency ω_{180} : The frequency at which the phase plot intersects the -180 degree line.

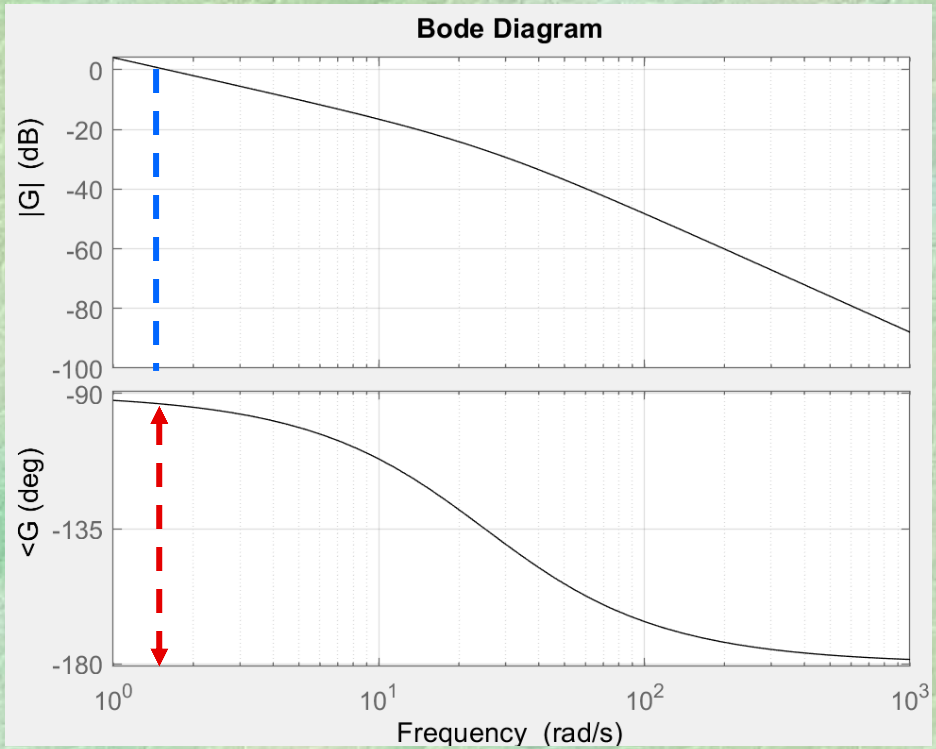
$$\omega_{180} > 1000$$

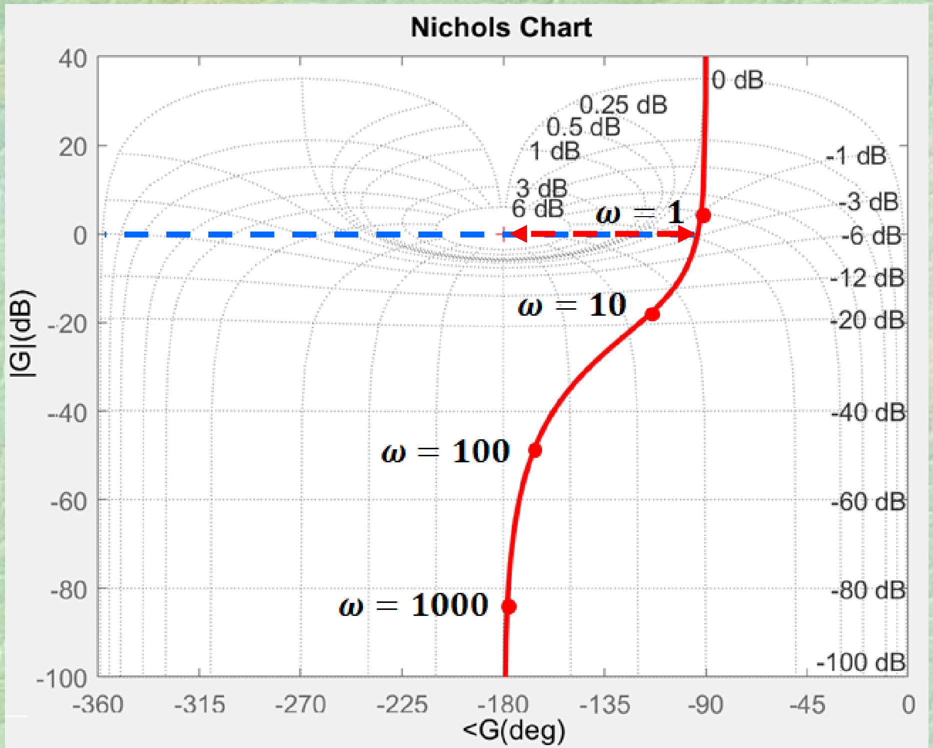
Which is more precise, Bode or Nichols? Why?

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:







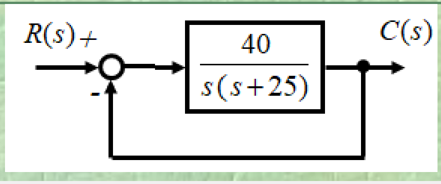
Phase Margin (*GM*): The phase margin is the difference between the system phase at the gain crossover frequency and -180 degree.

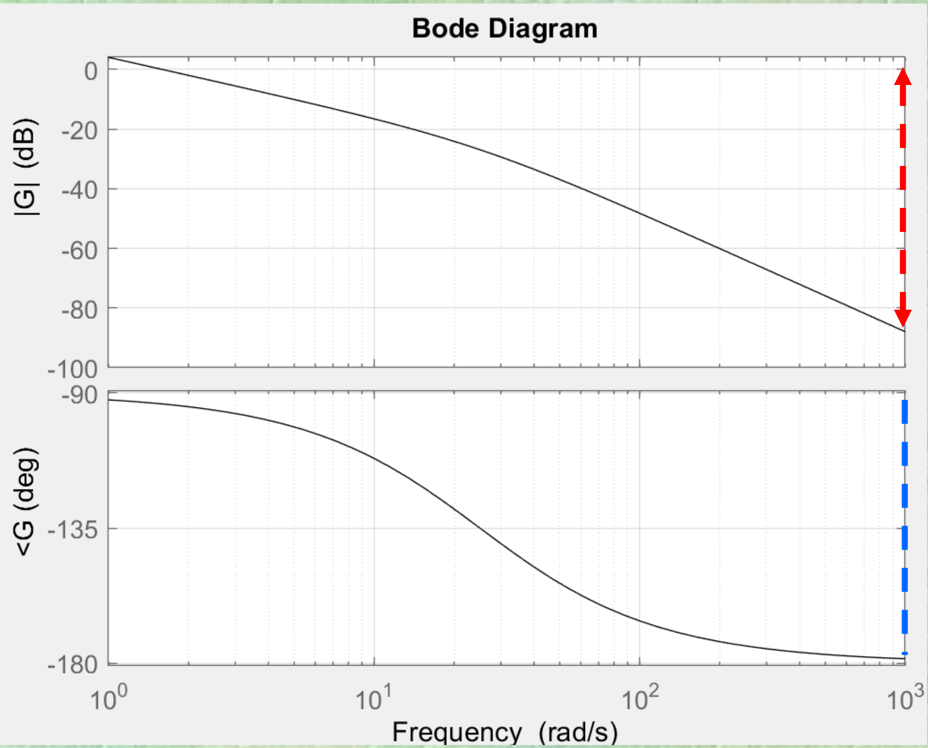
$$PM = 80°$$

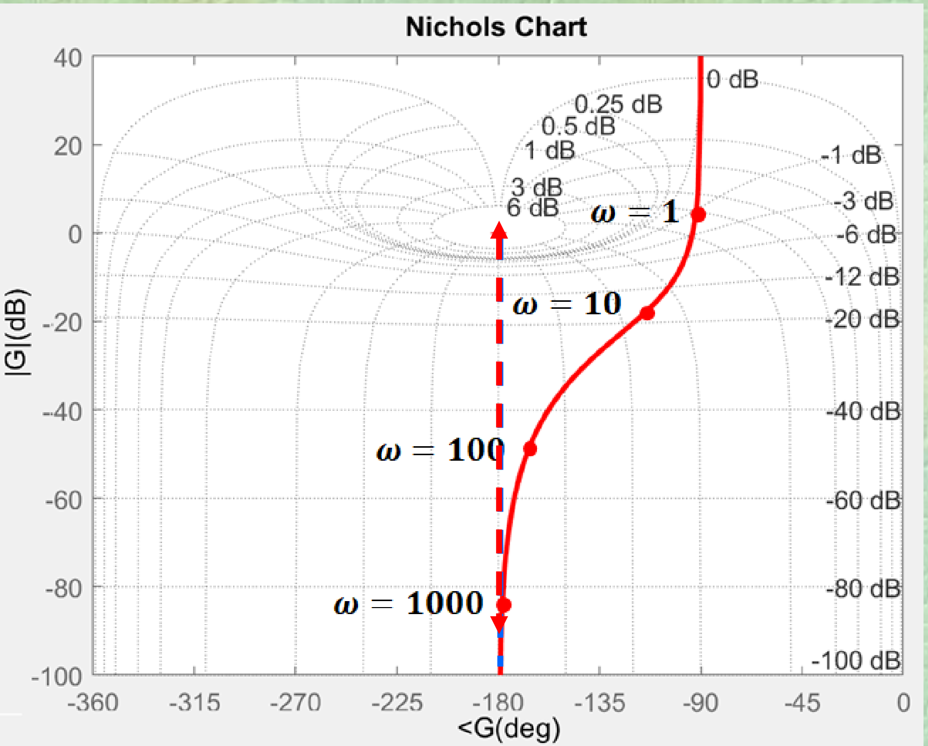
Which is more precise, Bode or Nichols? Why?

How to plot a Nichols diagram (magnitude-phase diagram)

Bode and Nichols diagrams for:





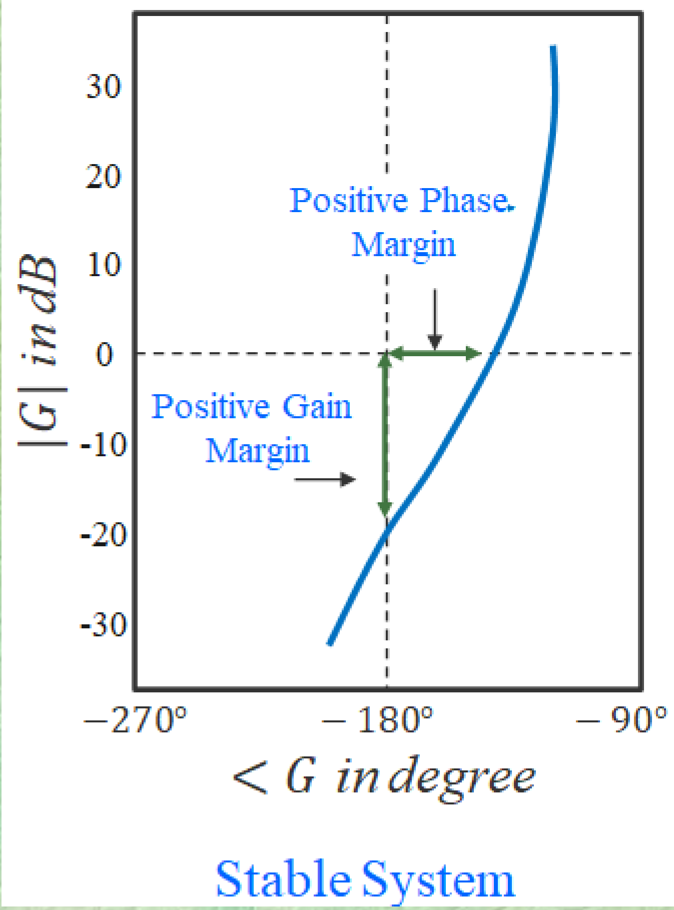


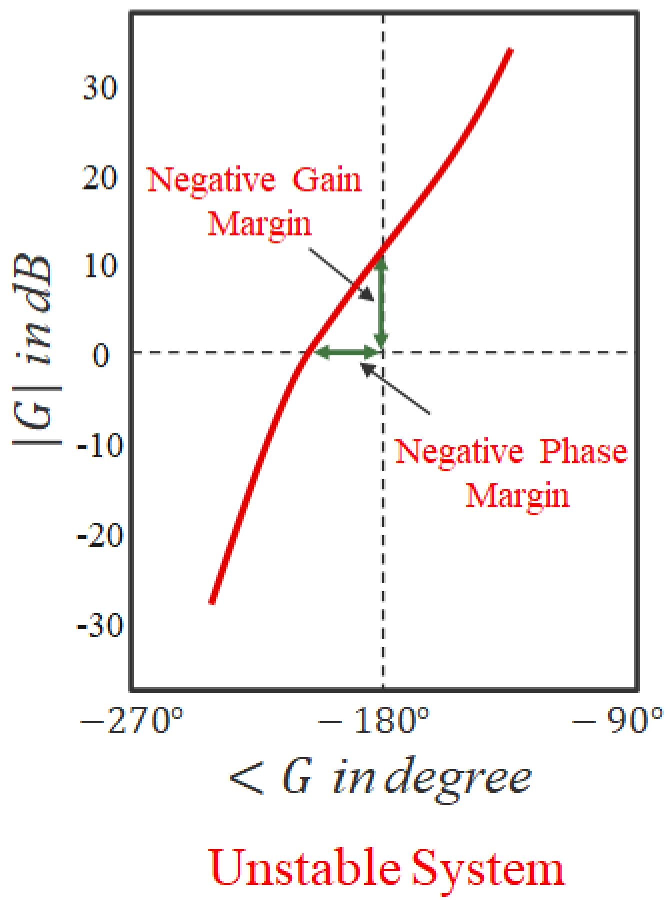
Gain Margin (*GM*): The gain margin is the difference between the system gain at the phase crossover frequency and 0 dB line.

$$GM > 80\ dB$$

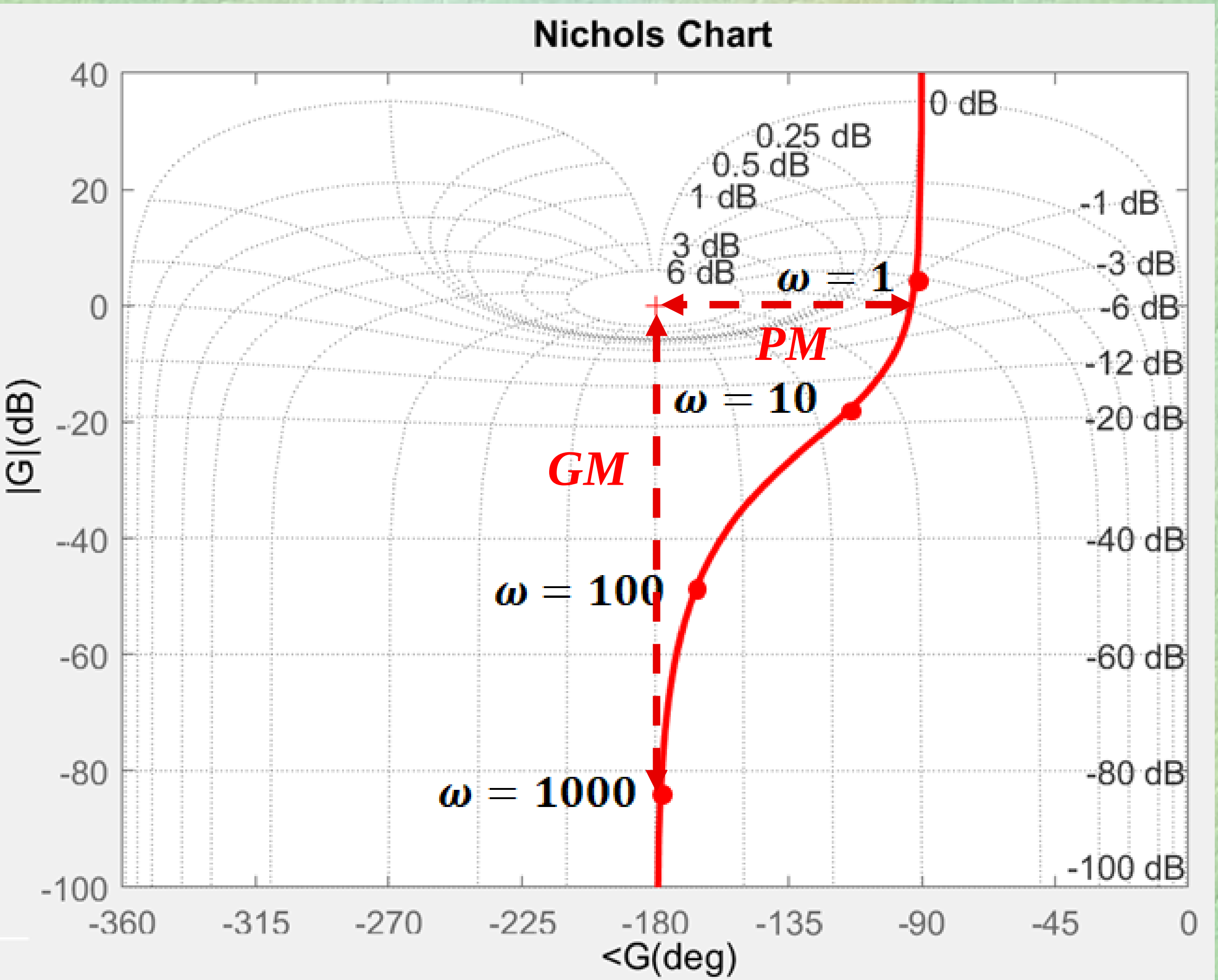
Which is more precise, Bode or Nichols? Why?

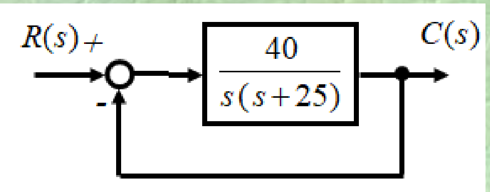
Stability Analysis from the Nichols Diagram (Magnitude-Phase Diagram)





Benefits of the Nichols Diagram (Magnitude-Phase Diagram)





Which is more precise, Bode or Nichols? Why?

Nichols is a single diagram, but Bode consists of two.

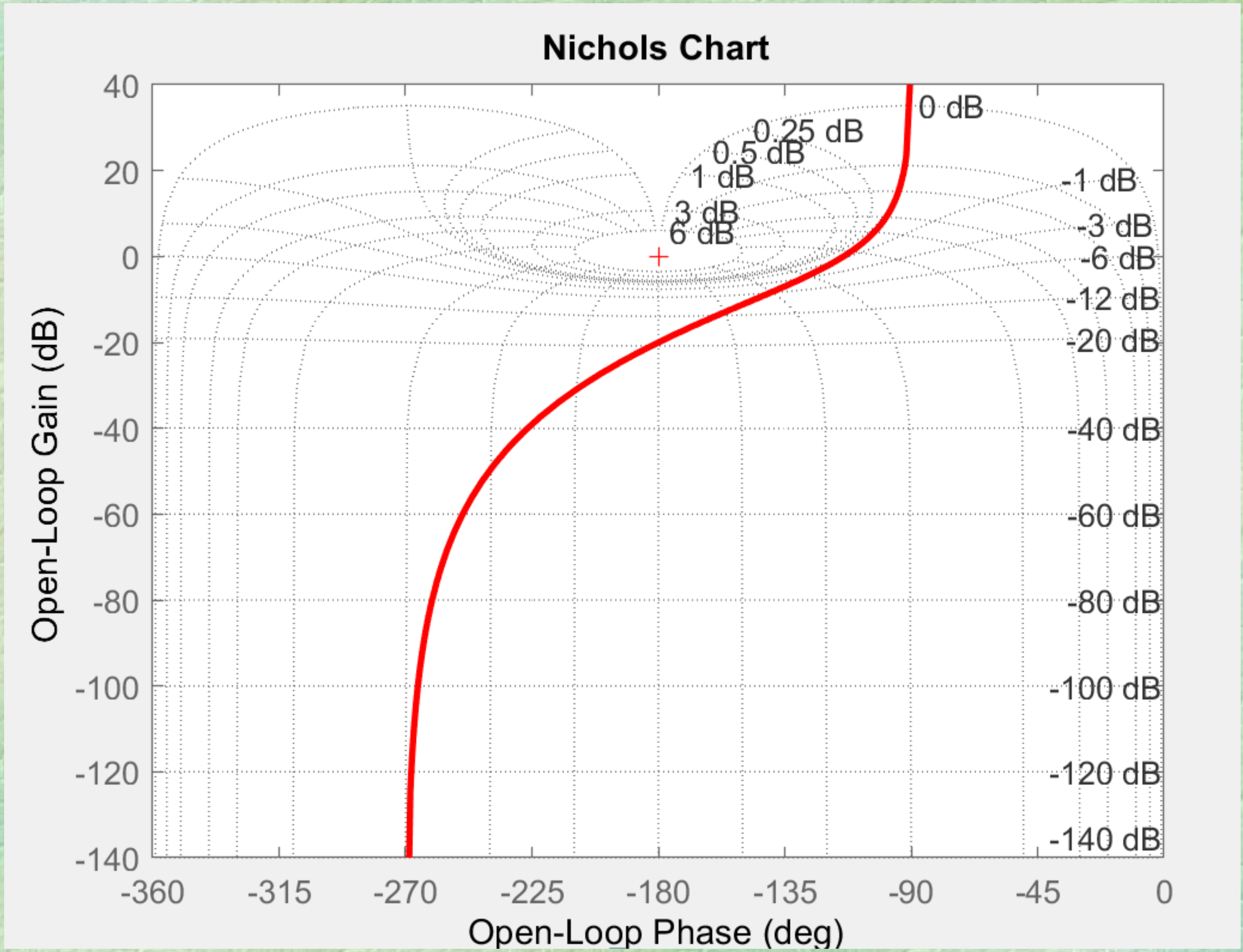
The improved system is easier to see.

Nichols provides the magnitude of the closed-loop transfer function.

Nichols provides the bandwidth of the closed-loop system.

Information from the Nichols Diagram (Magnitude-Phase Diagram)

The Nichols diagram of a minimum-phase system.



1- Stability?

2- Phase Margin?

3- Gain Margin?

4- Maximum magnitude of the closed-loop transfer function?

5- Open-loop Bandwidth?

6- Close-loop Bandwidth?

7- Type of syste?

Information from the Nichols Diagram (Magnitude-Phase Diagram)

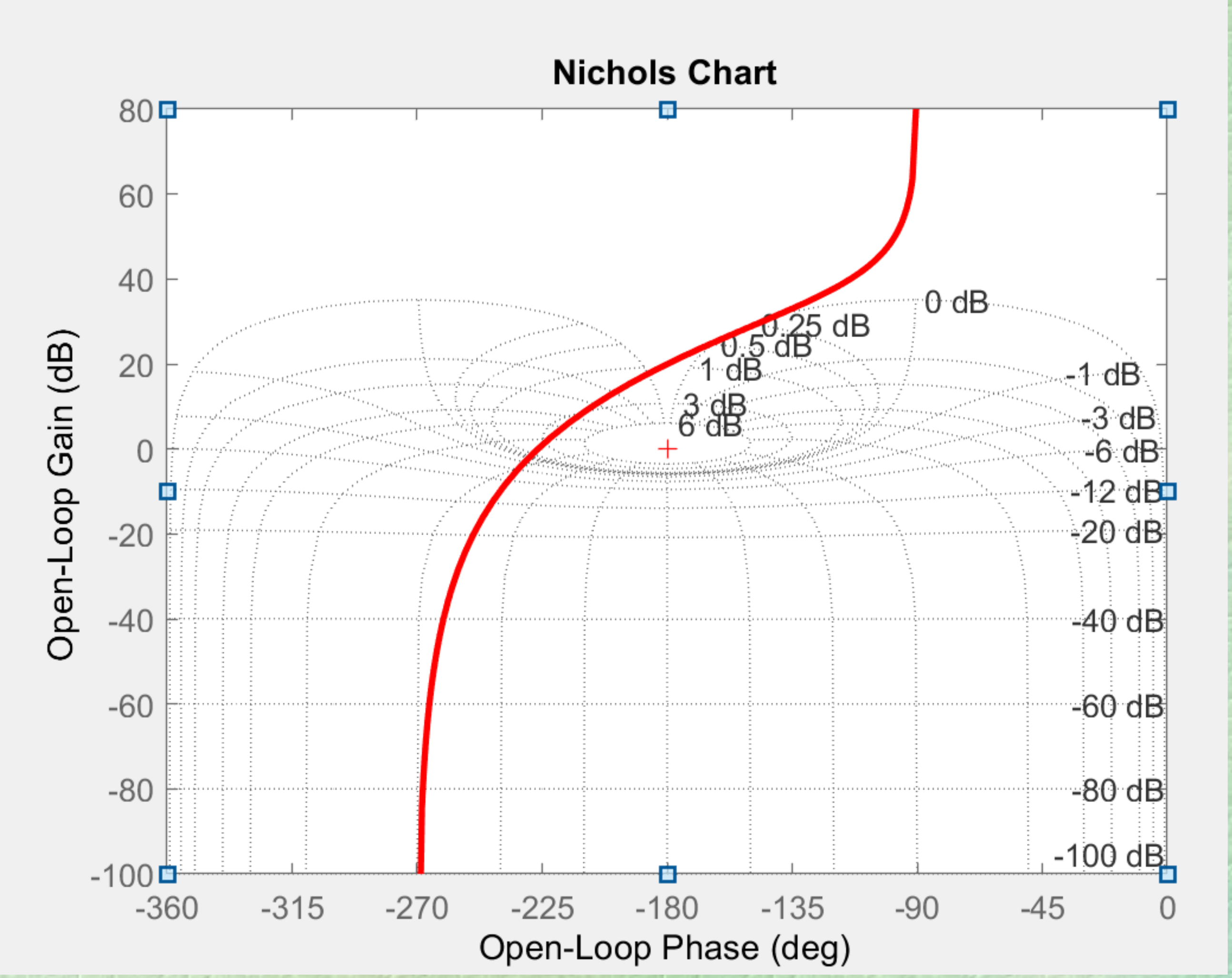
The Nichols diagram of a minimum-phase system.

1- Stability?

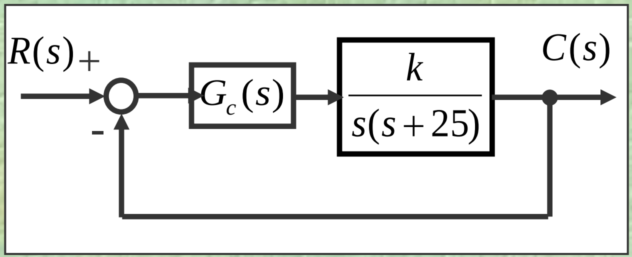
2- Phase Margin?

3- Gain Margin?

4- Type of syste?



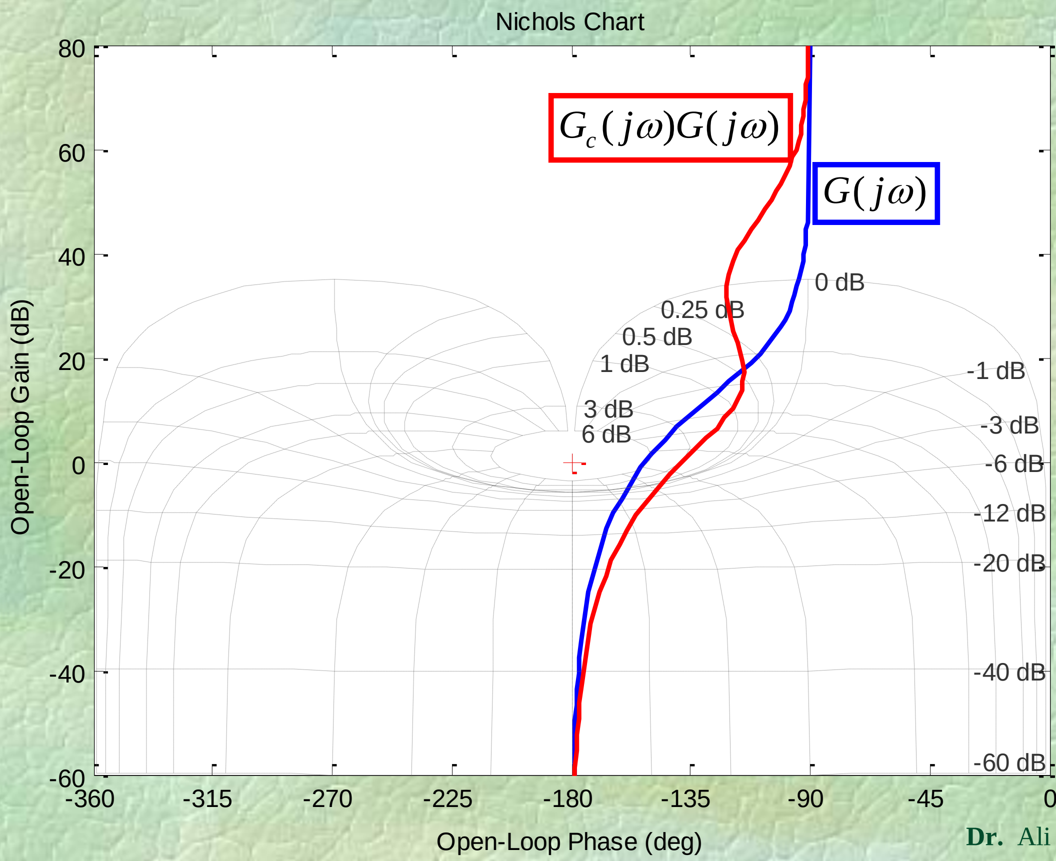
Information from the Nichols Diagram (Magnitude-Phase Diagram)



Blue chart is original system $G(s) = \frac{k}{s(s+25)}$.

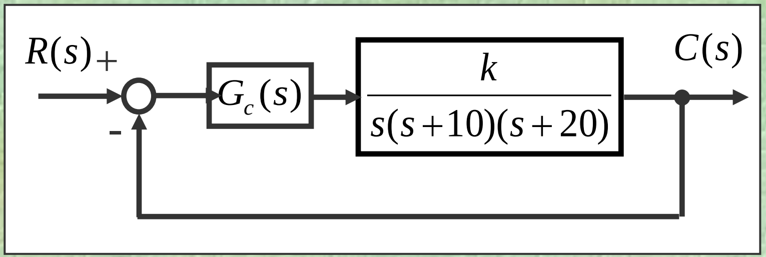
Red chart is compensated system $G(s)G_c(s)$

Compare the stability of the original and the compensated systems.



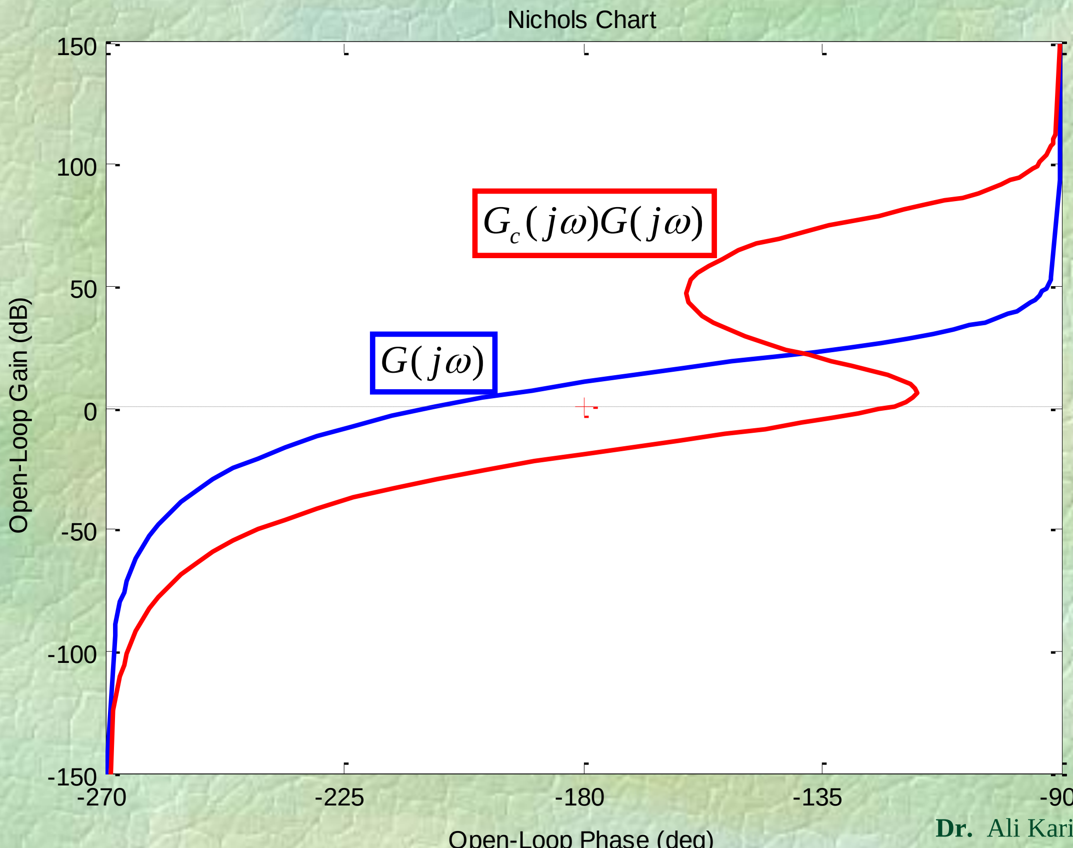
Information from the Nichols Diagram (Magnitude-Phase Diagram)

Blue chart is original system $G(s) = \dfrac{k}{s(s+10)(s+20)}$.



Red chart is compensated system $G(s)G_c(s)$

Compare the stability of the original and the compensated systems.



Exercise

The Nichols diagram of a minimum-phase system.

1- Stability?

2- Phase Margin?

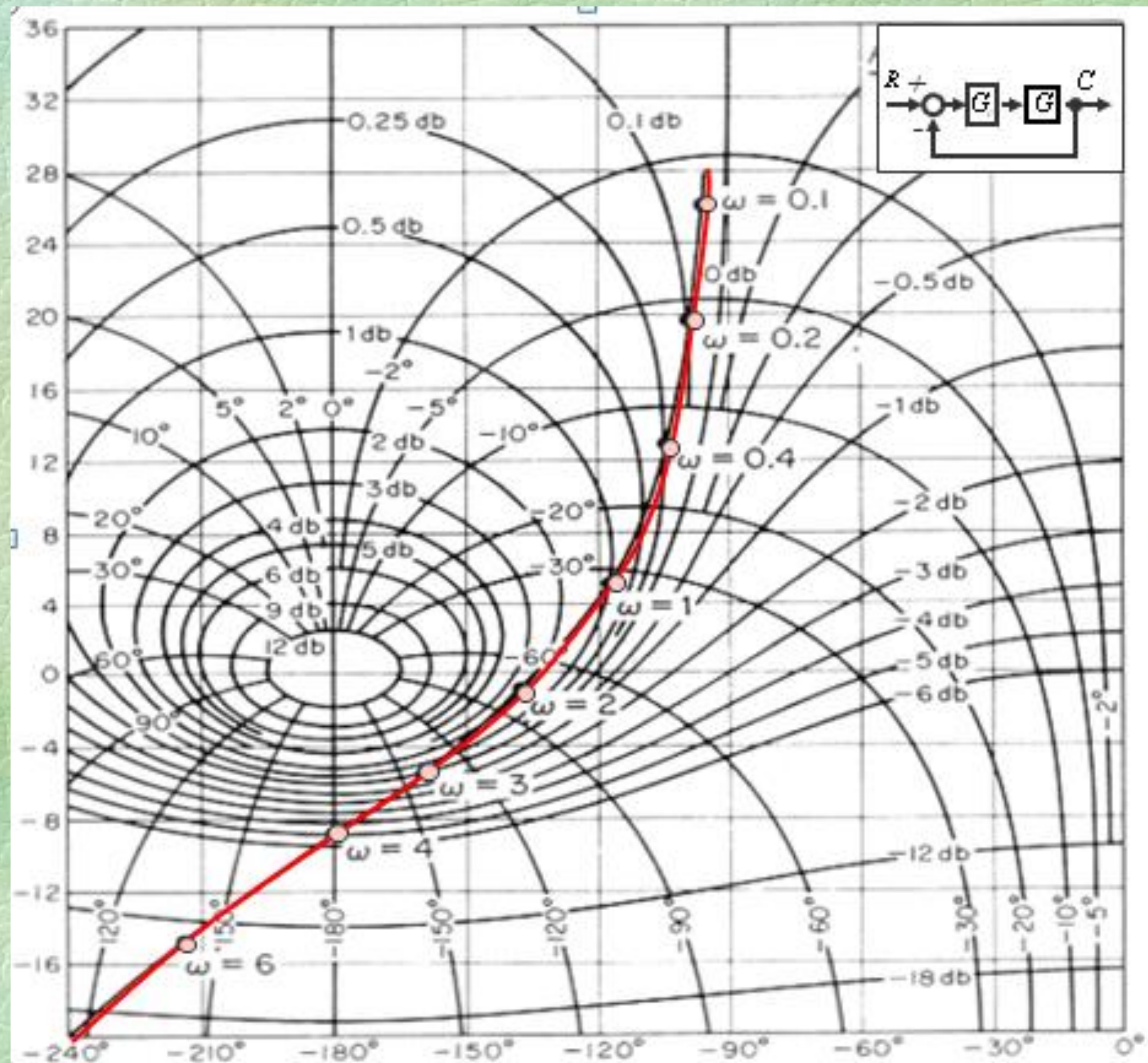
3- Gain Margin?

4- Maximum magnitude of the closed-loop transfer function?

5- Open-loop Bandwidth?

6- Close-loop Bandwidth?

7- Type of syste?



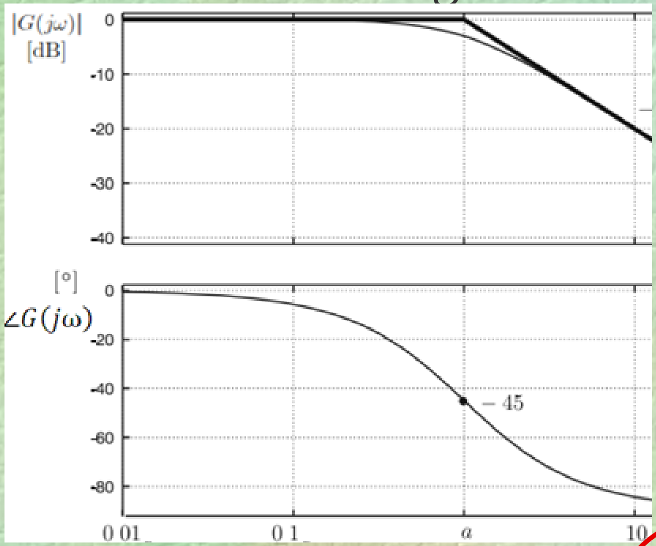
Frequency domain charts

- Bode plot.
- Nichols chart.
- Polar plot.

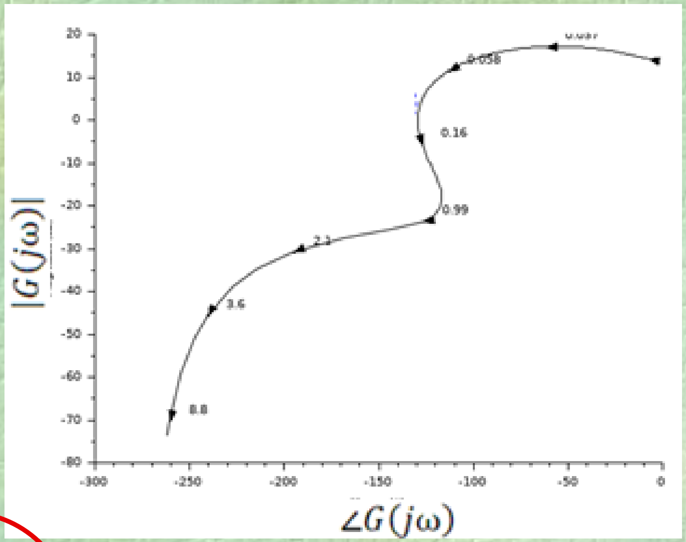
Frequency Domain Representation of Systems

$$|G(j\omega)| = \frac{A_y(\omega)}{A_u(\omega)} \quad \text{and} \quad \angle G(j\omega) = \varphi_y(\omega) - \varphi_u(\omega)$$

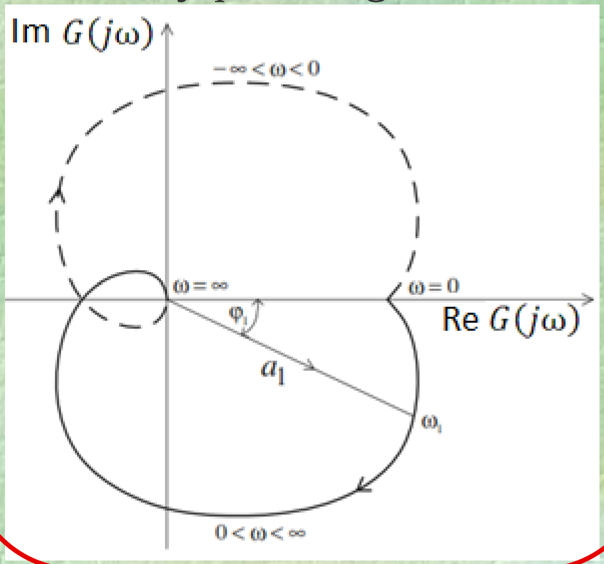
Bode diagram



Nichols diagram



Nyquist diagram



How to Plot the Nyquist Diagram and Analyze Stability

Plotting the Nyquist diagram for the transfer function **G(s)=1/s**

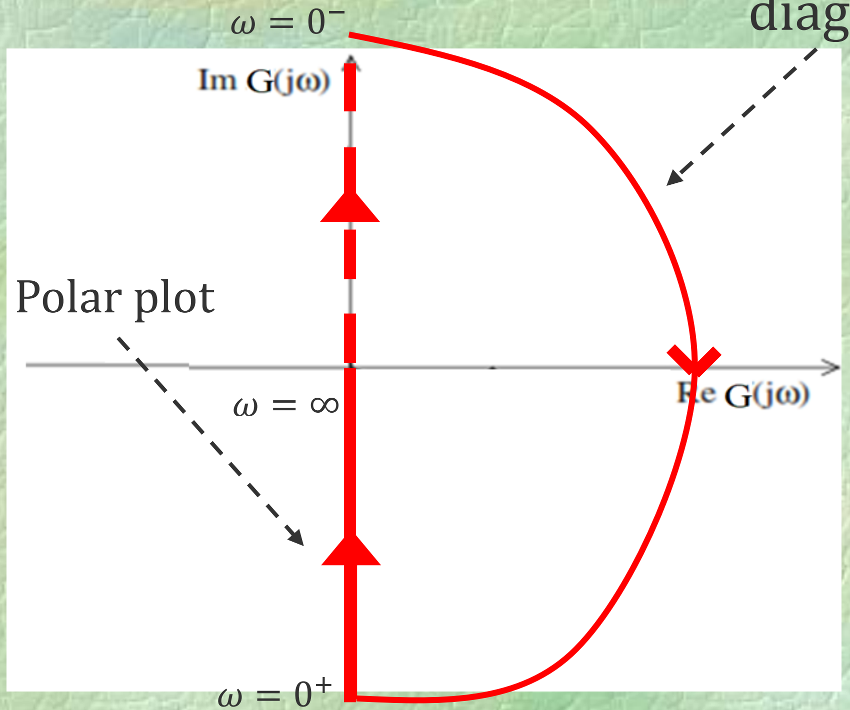
$$G(s) = \frac{1}{s} \quad \rightarrow \quad G(j\omega) = \frac{1}{j\omega} = -j\frac{1}{\omega}$$

$\omega = 0^+ \rightarrow G(j0^+) = \infty < -90$

$\omega = 1 \rightarrow G(j1) = -1j$

$\omega = 10 \rightarrow G(j10) = -0.1j$

$\omega < 0$ The reflection of the polar plot curve with respect to the horizontal axis



Obtaining the closed Nyquist curve (connecting the curve from $\omega=0^-$ to $\omega=0^+$ in a clockwise direction)

How to Plot the Nyquist Diagram and Analyze Stability

Algorithm for Using the Nyquist Criterion in Stability Detection

Step 1: Convert the denominator of the closed-loop transfer function to the form

$$1+kf(s)=0$$

Step 2: Plot the polar curve for $f(s)$.

Starting point: $\omega \longrightarrow 0^+$ $f(j0^+)$

Determining the intersection points of the polar plot with the real axis.

$$Im\{f(j\omega)\} = 0$$

Final point: $\omega \longrightarrow \infty$ $f(j\infty)$

Determining the intersection points of the polar plot with the imaginary axis.

$$Re\{f(j\omega)\} = 0$$

Step 3: The reflection of the polar plot curve with respect to the horizontal axis.

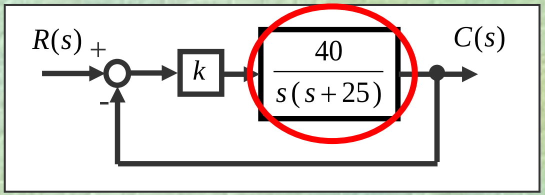
Step 4 (just if type of $f(s)$ is more than zero): Connecting the curve from $\omega=0^-$ to $\omega=0^+$ in a clockwise direction according to type of $f(s)$.

Step 5: Determine the number of RHP poles of the closed-loop transfer function (Z) by the encirclements of -1 (N_{-1}) and the number of RHP poles of f(s) (P) as:

$$Z = N_{-1} + P$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 4: Analyze the stability of the system using the Nyquist method for k>0.



Step 1: Convert the denominator of the closed-loop transfer function to the form

$$1+kf(s)=0$$

$$T(s)=\frac{40k}{s^2+25s+40k}$$

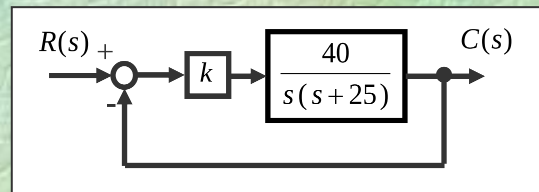
مخرج $T(s) = s^2+25s+40k=0$

$$1+k\frac{40}{s^2+25s}=0$$

$$f(s)=\frac{40}{s(s+25)}$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 4: Analyze the stability of the system using the Nyquist method for k>0.



Step 2: Plot the polar curve for $f(s) = \frac{1}{s(s+25)}$.

$$f(j\omega) = \frac{40}{j\omega(j\omega+25)}$$

$\omega \longrightarrow 0^+$ $\quad f(j0) = \infty < -90$ $\quad$ Starting point

$\omega \longrightarrow \infty$ $\quad f(j\infty) = 0$ $\quad$ Final point

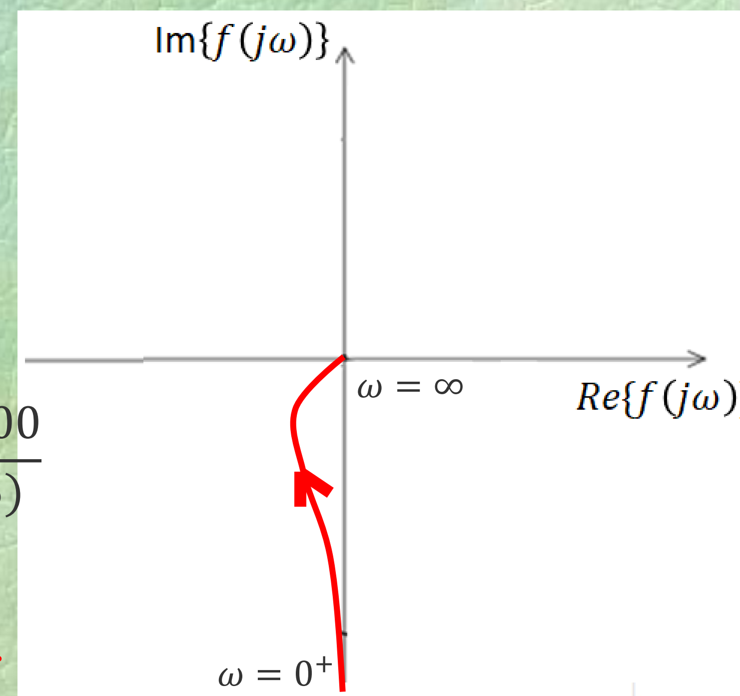
$$f(j\omega) = \frac{40}{j\omega(j\omega+25)} \frac{-j\omega(-j\omega+25)}{-j\omega(-j\omega+25)} = \frac{-40\omega - j1000}{\omega(\omega^2+625)}$$

$$Re\{f(j\omega)\} = \frac{-40\omega}{\omega\,(\omega^2+625)} = 0 \rightarrow \omega = 0$$

Intersection with the imaginary axis.

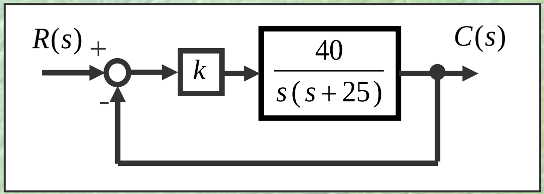
$$Im\{f(j\omega)\} = \frac{-1000}{\omega\,(\omega^2+625)} = 0$$

Intersection with real axis



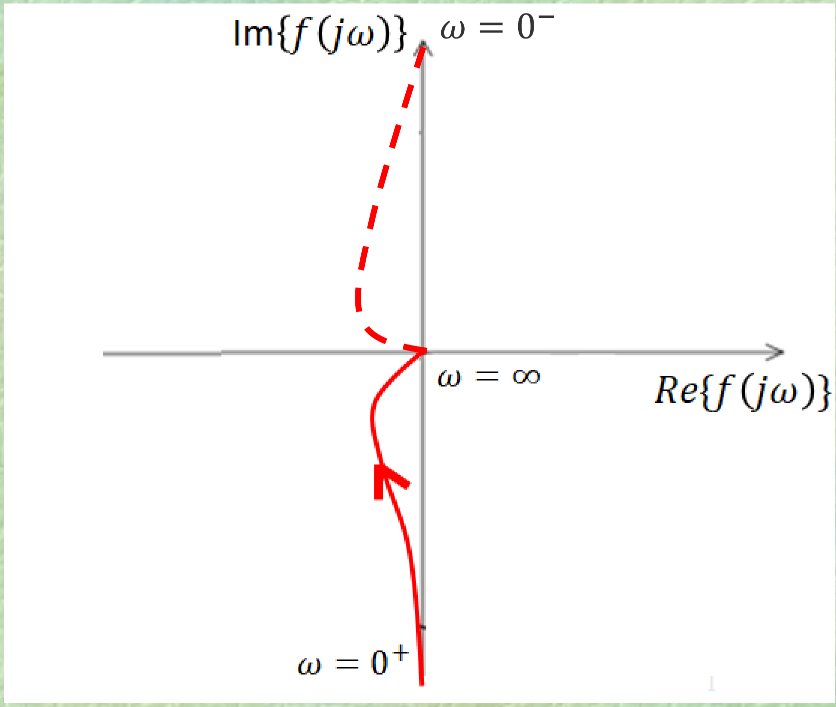
How to Plot the Nyquist Diagram and Analyze Stability

Example 4: Analyze the stability of the system using the Nyquist method for k>0.



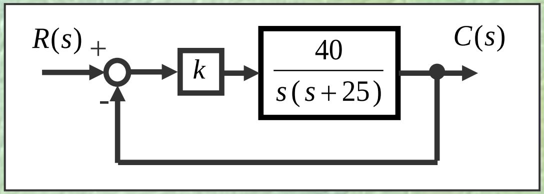
Step 3: The reflection of the polar plot curve with respect to the horizontal axis.

$\omega < 0$ reflection of the polar plot

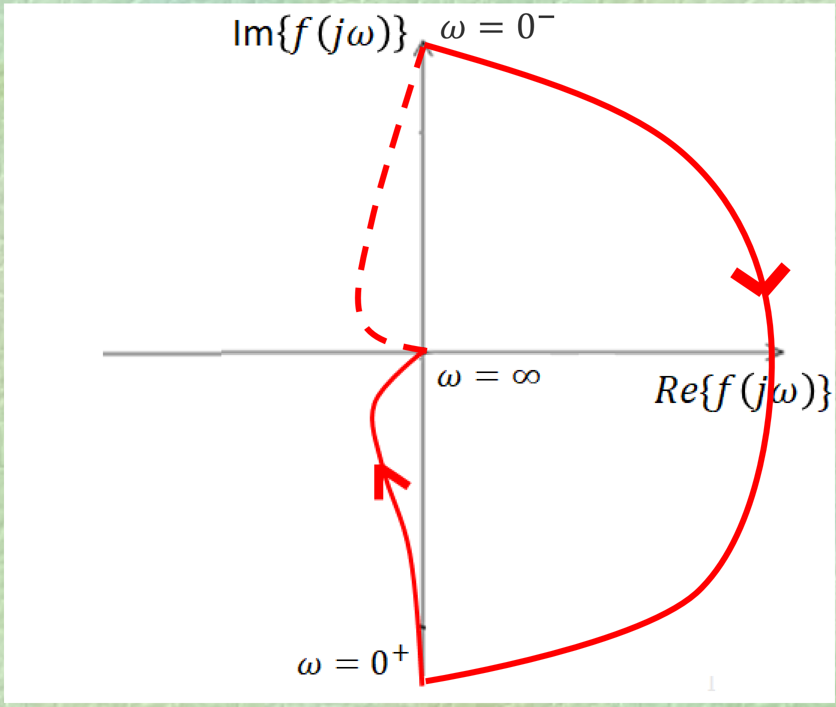


How to Plot the Nyquist Diagram and Analyze Stability

Example 4: Analyze the stability of the system using the Nyquist method for k>0.

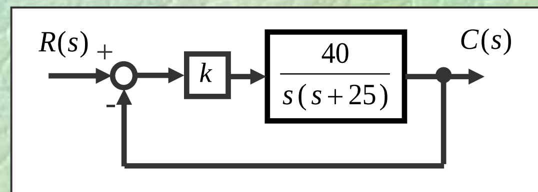


Step 4 (just if type of *f*(*s*) is more than zero): Connecting the curve from $\omega=0^-$ to $\omega=0^+$ in a clockwise direction according to type of $f(s)$.

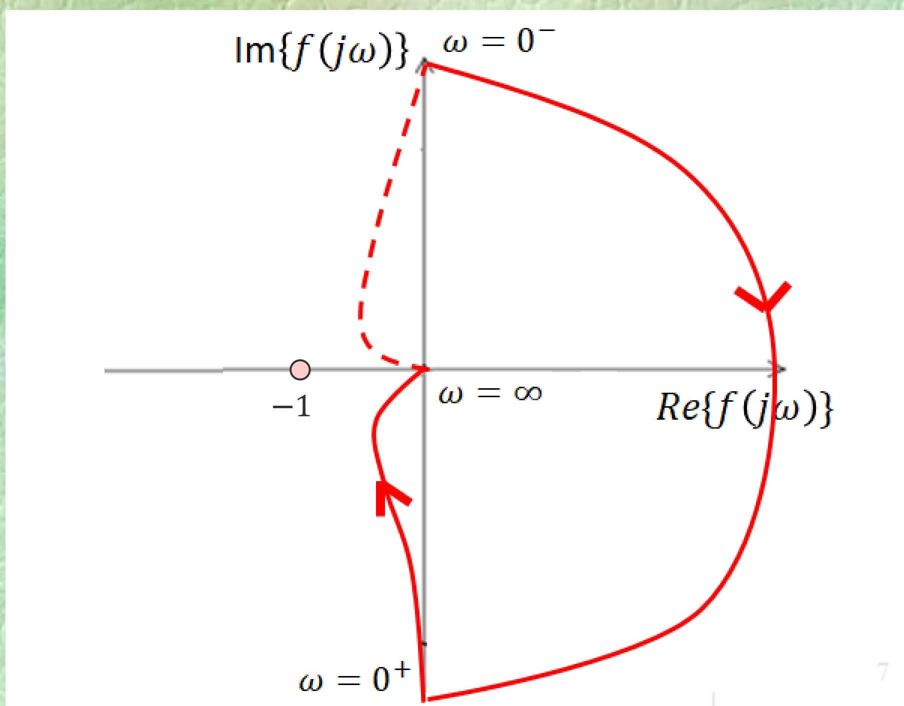


How to Plot the Nyquist Diagram and Analyze Stability

Example 4: Analyze the stability of the system using the Nyquist method for k>0.



Step 5: Determine the number of RHP poles of the closed-loop transfer function (Z) by the encirclements of −1 (N_{-1}) and the number of RHP poles of f(s) (P) as:



$$Z = N_{-1} + P = N_{-1}$$

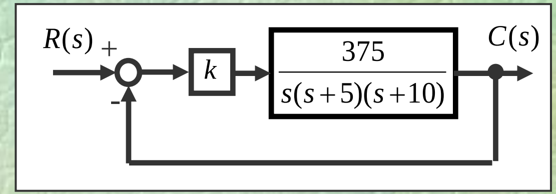
$P \rightarrow 0$

$$Z = N_{-1} = 0$$

The system is **stable** for all k>0.

How to Plot the Nyquist Diagram and Analyze Stability

Example 5: Analyze the stability of the system using the Nyquist method for k>0.



Step 1: Convert the denominator of the closed-loop transfer function to the form

$1+kf(s)=0$

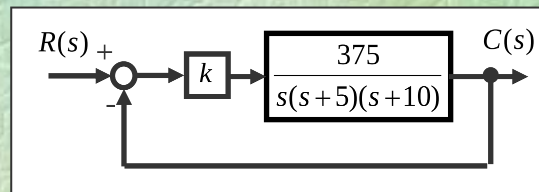
Denominator of the closed-loop transfer function is:

$$1 + k\frac{375}{s(s+5)(s+10)} = 0$$

$$f(s) = \frac{375}{s(s+5)(s+10)}$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 5: Analyze the stability of the system using the Nyquist method for k>0.



Step 2: Plot the polar curve for $f(s) = \frac{375}{s(s+5)(s+10)}$.

$$f(j\omega) = \frac{375}{j\omega(j\omega+5)(j\omega+10)}$$

$\omega \longrightarrow 0^+$ $\quad f(j0) = \infty < -90$ $\quad$ Starting point

$\omega \longrightarrow \infty$ $\quad f(j\infty) = 0$ $\quad$ Final point

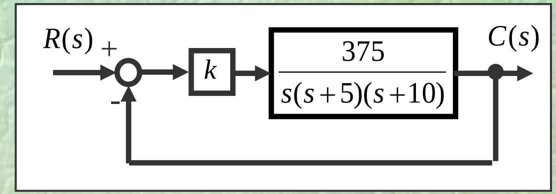
$$f(j\omega) = \frac{375}{j\omega(j\omega+5)(j\omega+10)} \frac{-j\omega(-j\omega+5)(-j\omega+10)}{-j\omega(-j\omega+5)(-j\omega+10)} = \frac{-375[15\omega + j(50-\omega^2)]}{\omega\,(\omega^2+25)(\omega^2+100)}$$

$Im\{f(j\omega)\} = 0$ $\quad (50-\omega^2) = 0$ $\quad \omega = \pm 7.07$ $\quad f(j7.07) = -0.5$ $\quad$ Intersection with real axis

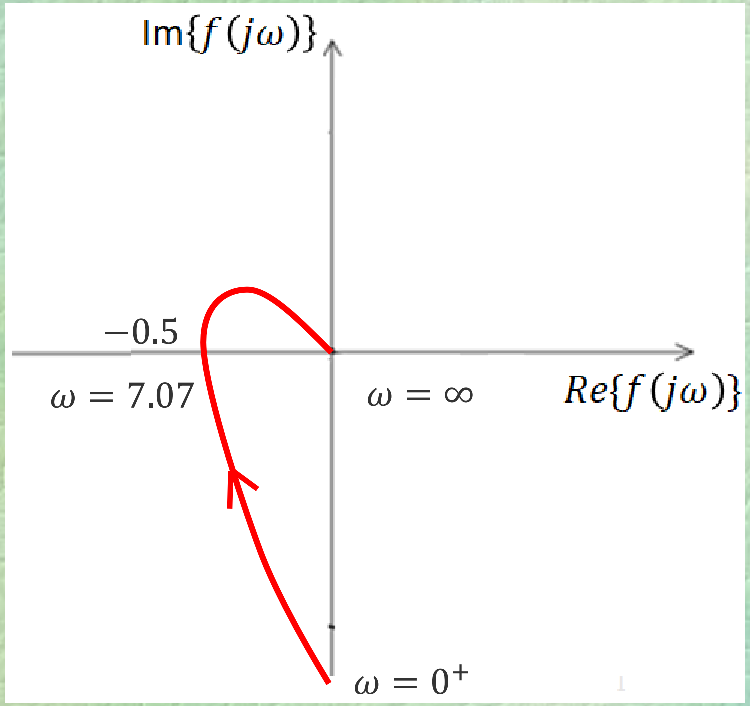
$Re\{f(j\omega)\} = 0$ $\quad$ No intersection with imaginary axis

How to Plot the Nyquist Diagram and Analyze Stability

Example 5: Analyze the stability of the system using the Nyquist method for k>0.



Step 2: Plot the polar curve for $f(s) = \frac{375}{s(s+5)(s+10)}$.



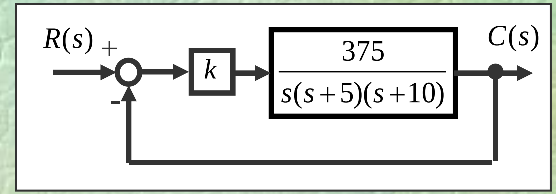
$\omega \longrightarrow 0^+ \qquad f(j0) = \infty < -90$ Starting point

$\omega \longrightarrow \infty \qquad f(j\infty) = 0$ Final point

$\omega = \pm 7.07 \qquad f(j7.07) = -0.5$ Intersection with real axis

How to Plot the Nyquist Diagram and Analyze Stability

Example 5: Analyze the stability of the system using the Nyquist method for k>0.



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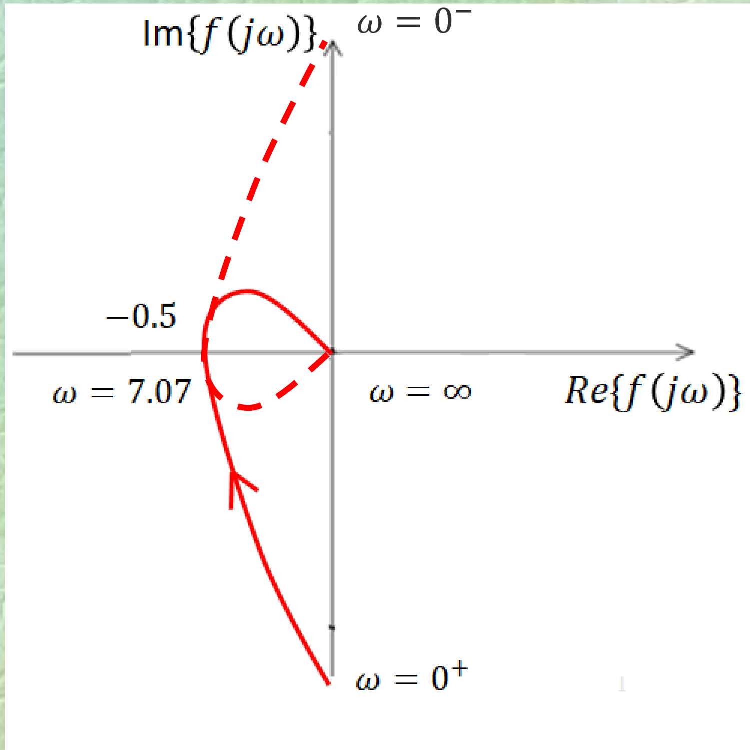

How to Plot the Nyquist Diagram and Analyze Stability

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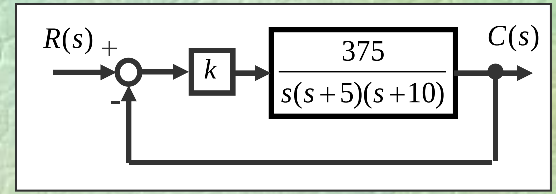

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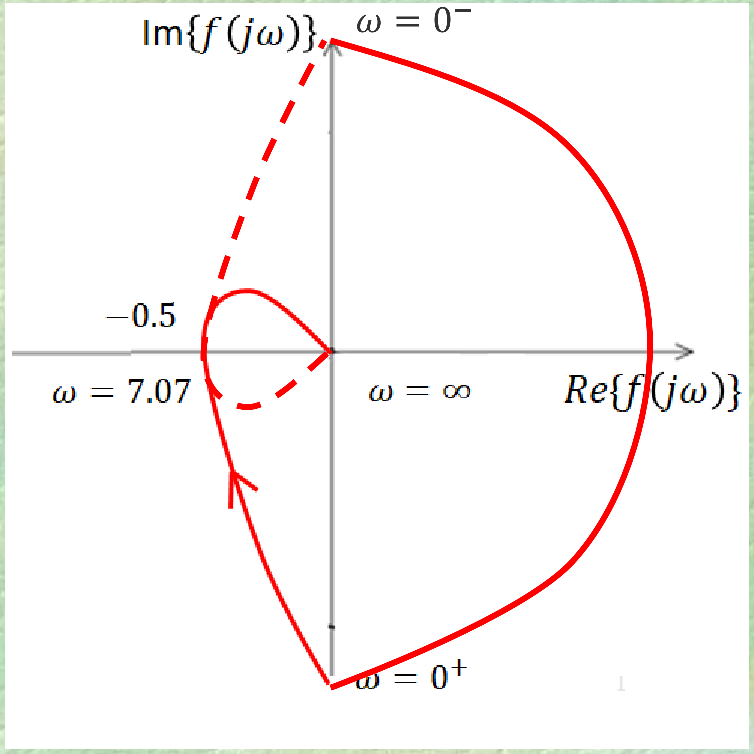

How to Plot the Nyquist Diagram and Analyze Stability

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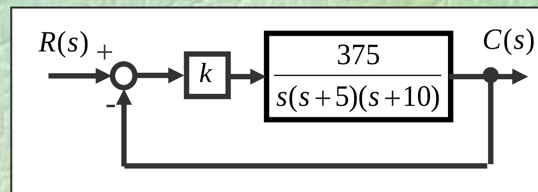

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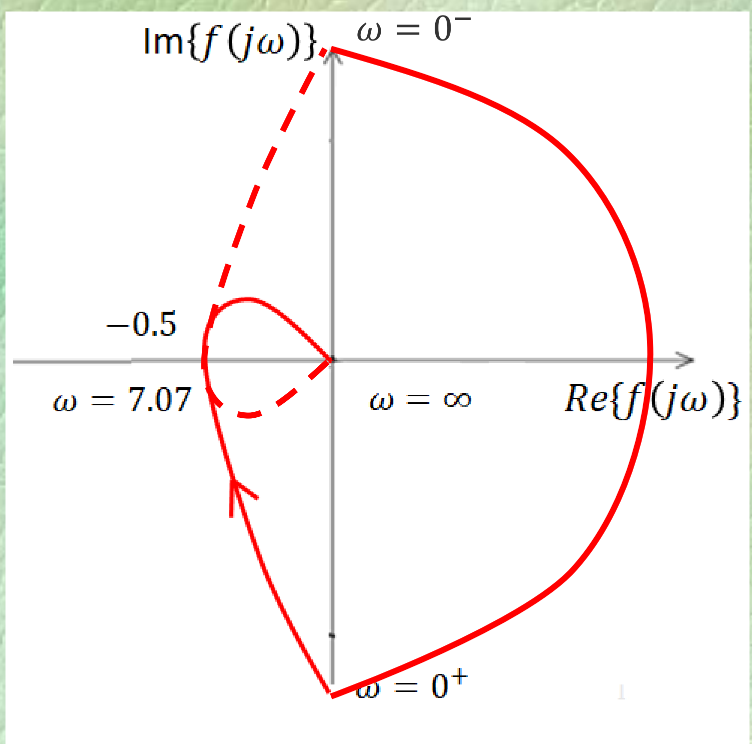
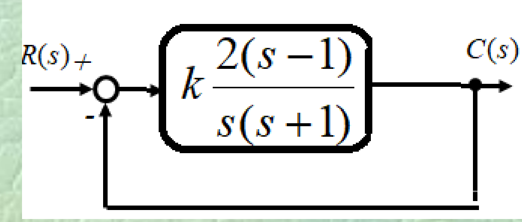


$$Z = N_{-1} + P \quad (P = 0) \quad = N_{-1}$$

$$Z = N_{-1} = \begin{cases} 0 & \text{The closed loop system is stable for } 0<k<2 \\ 2 & \text{The closed loop system is unstable } k>2 \text{ and there is 2 RHP poles.} \end{cases}$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 6: Analyze the stability of the system using the Nyquist method for k>0.



Step 1: Convert the denominator of the closed-loop transfer function to the form

$$1+kf(s)=0$$

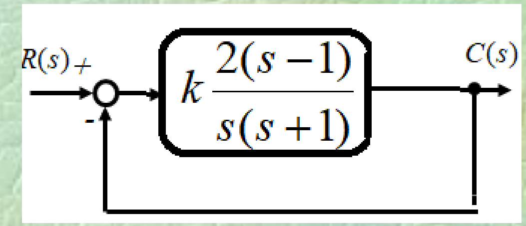
Denominator of the closed-loop transfer function is:

$$1+k\frac{2(s-1)}{s(s+1)}=0$$

$$f(s)=\frac{2(s-1)}{s(s+1)}$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 6: Analyze the stability of the system using the Nyquist method for k>0.



Step 2: Plot the polar curve for $f(s) = \frac{2(s-1)}{s(s+1)}$.

$$f(j\omega) = \frac{2(j\omega - 1)}{j\omega(j\omega + 1)}$$

$\omega \longrightarrow 0^+$ $\quad f(j0) = \infty < 90$ $\quad$ Starting point

$\omega \longrightarrow \infty$ $\quad f(j\infty) = 0$ $\quad$ Final point

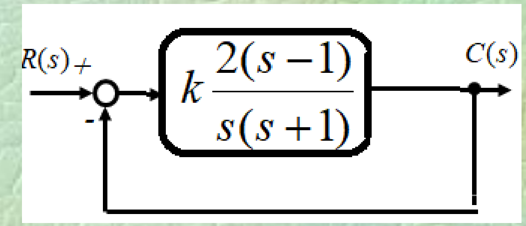
$$f(j\omega) = \frac{2(j\omega - 1)}{j\omega(j\omega + 1)} \frac{-j\omega(-j\omega + 1)}{-j\omega(-j\omega + 1)} = \frac{4\omega^2 - 2j\omega(\omega^2 - 1)}{\omega^2(\omega^2 + 1)} = \frac{4\omega - 2j(\omega^2 - 1)}{\omega(\omega^2 + 1)}$$

$Im\{f(j\omega)\} = 0 \quad (\omega^2 - 1) = 0 \quad \omega = \pm 1 \quad f(j1) = \frac{4}{1(1+1)} = 2$ $\quad$ Intersection with real axis

$Re\{f(j\omega)\} = 0$ $\quad$ No intersection with imaginary axis

How to Plot the Nyquist Diagram and Analyze Stability

Example 6: Analyze the stability of the system using the Nyquist method for k>0.

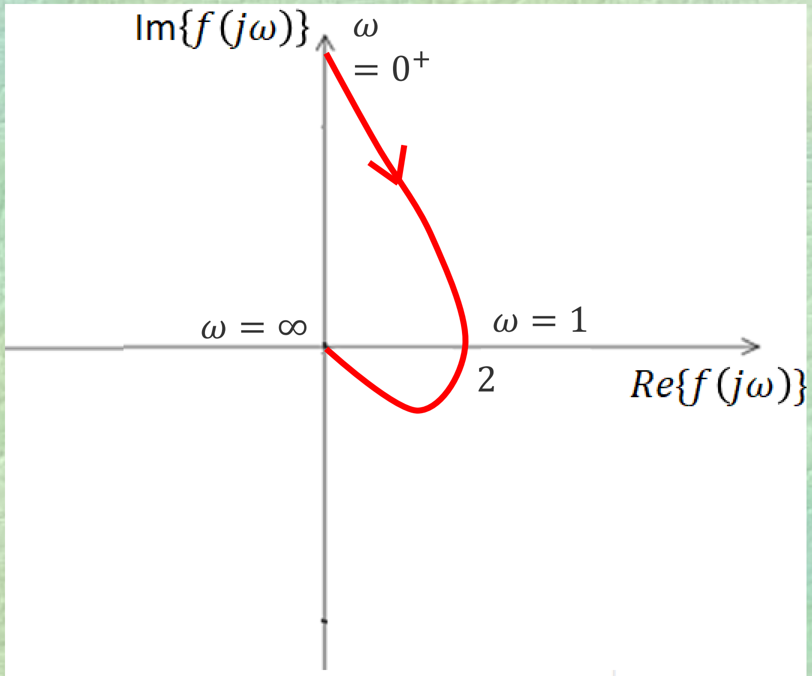


Step 2: Plot the polar curve for $f(s)=\frac{2(s-1)}{s(s+1)}$.

$\omega \longrightarrow 0^+ \quad f(j0)=\infty < 90$ Starting point

$\omega \longrightarrow \infty \quad f(j\infty)=0$ Final point

$f(j1)=\frac{4}{1(1+1)}=2$ Intersection with real axis



How to Plot the Nyquist Diagram and Analyze Stability

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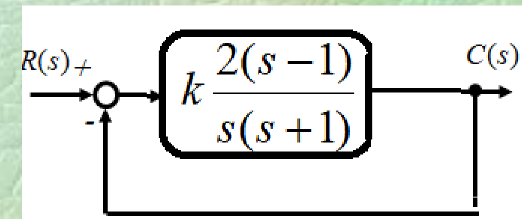

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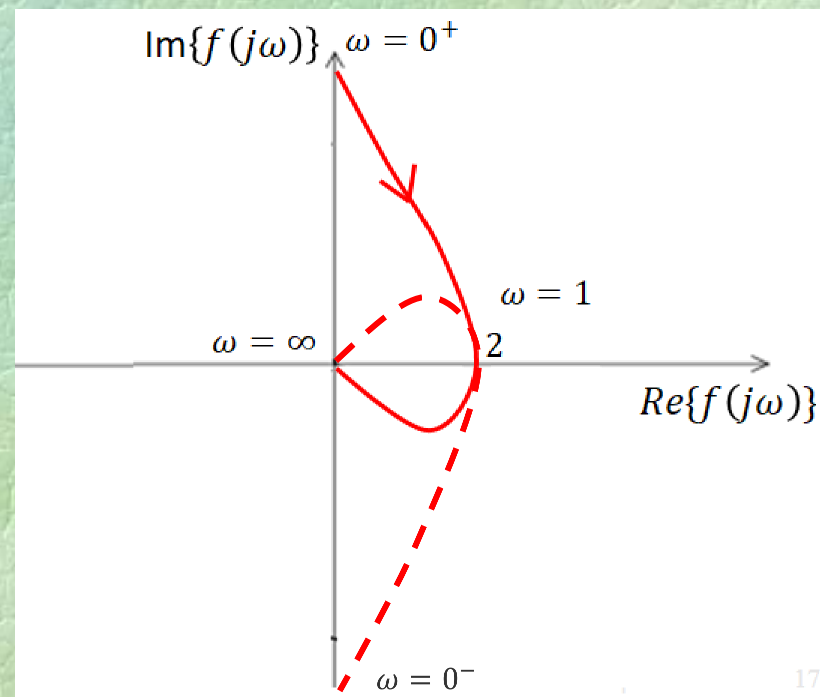
$\omega < 0$ reflection of the polar plot



How to Plot the Nyquist Diagram and Analyze Stability

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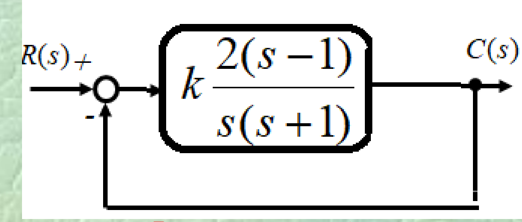

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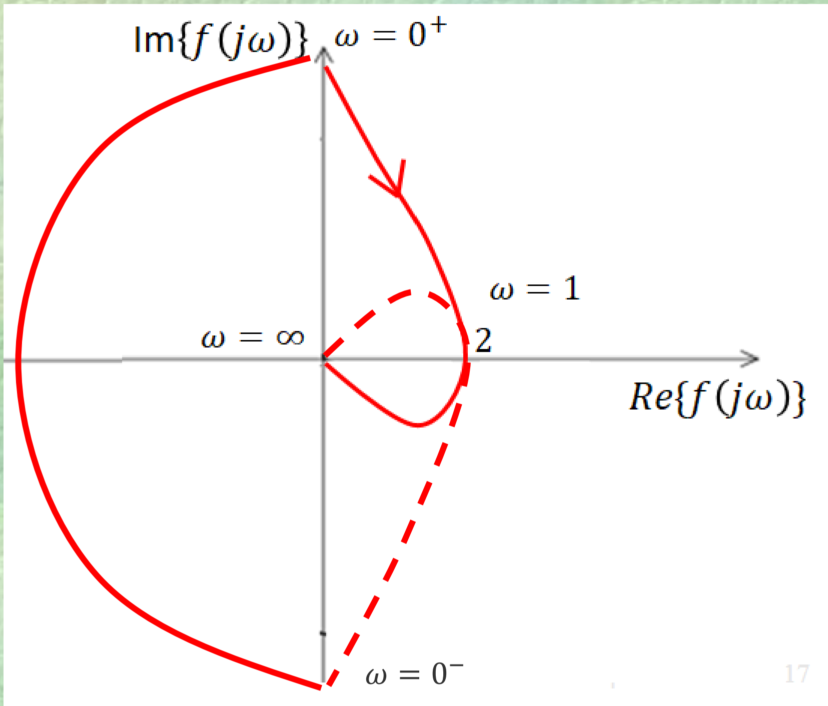

How to Plot the Nyquist Diagram and Analyze Stability

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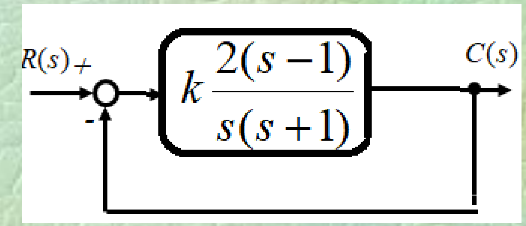

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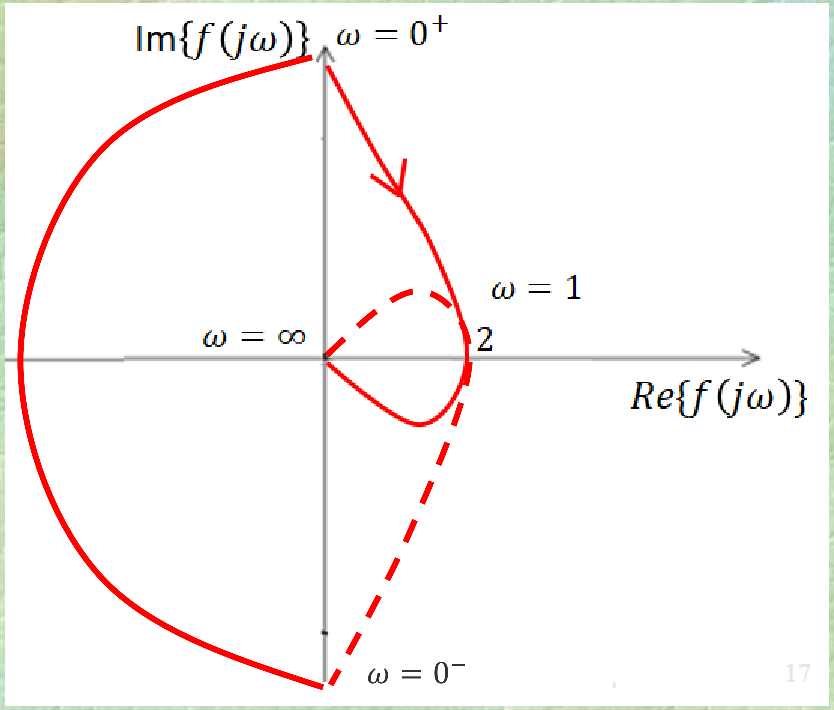


$$Z = N_{-1} + P \quad = N_{-1}$$

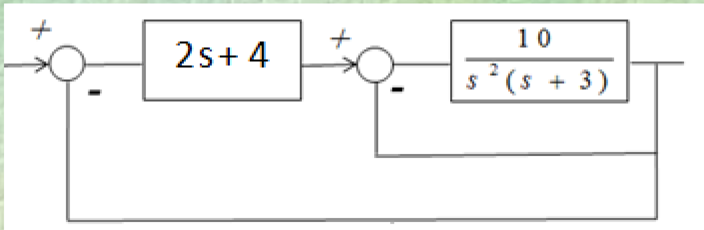
$P \to 0$

$$Z = N_{-1} = 1$$

The closed-loop system is **unstable** for all k>0 and has one right-half plane pole.

How to Plot the Nyquist Diagram and Analyze Stability

Example 7: Discuss the system stability using the Nyquist method.



Step 1: Convert the denominator of the closed-loop transfer function to the form

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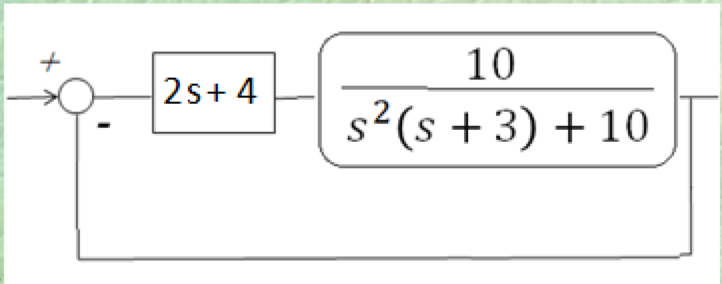


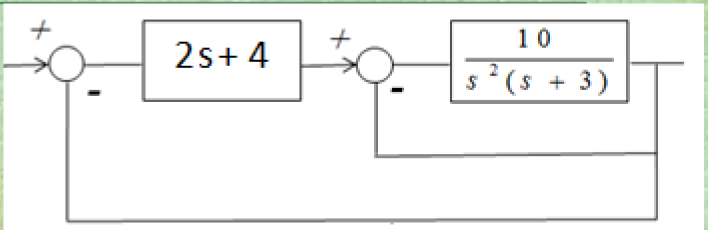
Denominator of the closed-loop transfer function is:

$$1+k\,\frac{20(s+2)}{s^3+3s^2+10}=0 \qquad k=1$$

$$f(s)=\frac{20(s+2)}{s^3+3s^2+10}$$

How to Plot the Nyquist Diagram and Analyze Stability

Example 7: Discuss the system stability using the Nyquist method.



Step 2: Plot the polar curve for $f(s) = \frac{20(s+2)}{s^3+3s^2+10}$.

$$f(j\omega) = \frac{20(j\omega + 2)}{10 - 3\omega^2 - j\omega^3}$$

$\omega \longrightarrow 0 \qquad f(j0) = 4$ Starting point

$\omega \longrightarrow \infty \qquad f(j\infty) = 0$ Final point

$$f(j\omega) = \frac{20(j\omega + 2)}{10 - 3\omega^2 - j\omega^3} \frac{10 - 3\omega^2 + j\omega^3}{10 - 3\omega^2 + j\omega^3} = \frac{400 - 20\omega^4 - 120\omega^2 + j(20\omega(10 - \omega^2))}{(10 - 3\omega^2)^2 + \omega^6}$$

$Im\{f(j\omega)\} = 0 \qquad 20\omega(10 - \omega^2) = 0 \qquad \omega = 0, \pm\sqrt{10} \qquad f(j\sqrt{10}) = -2$ Intersection with real axis

$\mathrm{Re}\{f(j\omega)\} = 0 \qquad 400 - 20\omega^4 - 120\omega^2 = 0 \qquad \omega = \pm 1.54 \qquad f(j1.54) = 10.84j$ Intersection with imaginary axis

How to Plot the Nyquist Diagram and Analyze Stability

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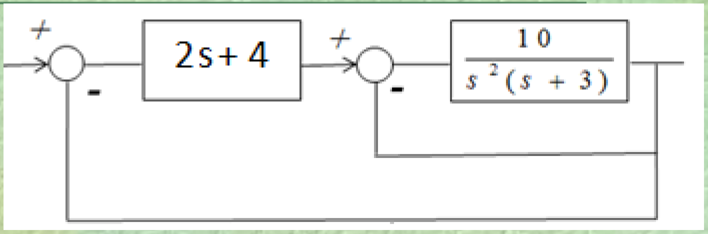

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$\omega \longrightarrow 0 \qquad f(j0) = 4$ Starting point

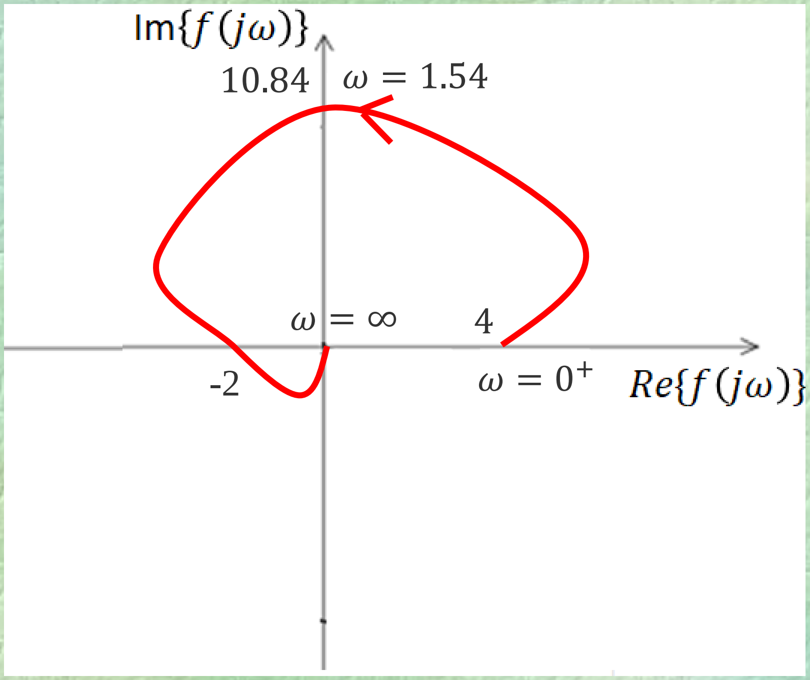
$\omega \longrightarrow \infty \qquad f(j\infty) = 0$ Final point

$\omega = 0, \pm\sqrt{10}$

$f\left(j\sqrt{10}\right) = -2$ Intersection with real axis

$\omega = \pm 1.54$

$f(j1.54) = 10.84j$ Intersection with imaginary axis



How to Plot the Nyquist Diagram and Analyze Stability

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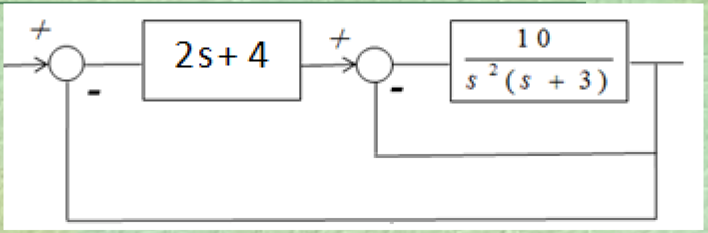

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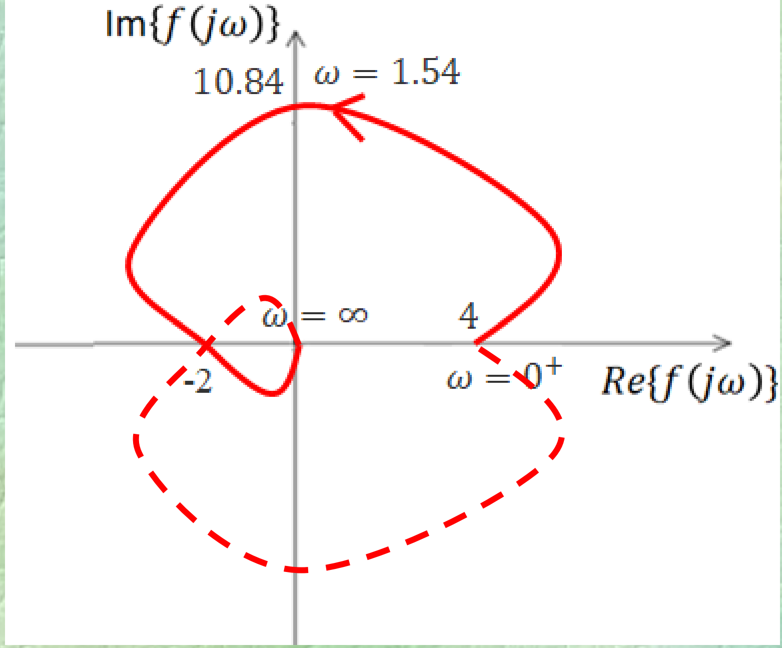

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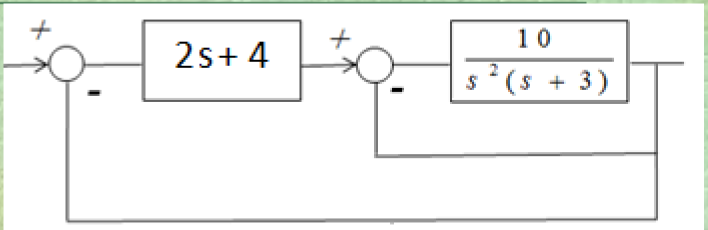
How to Plot the Nyquist Diagram and Analyze Stability



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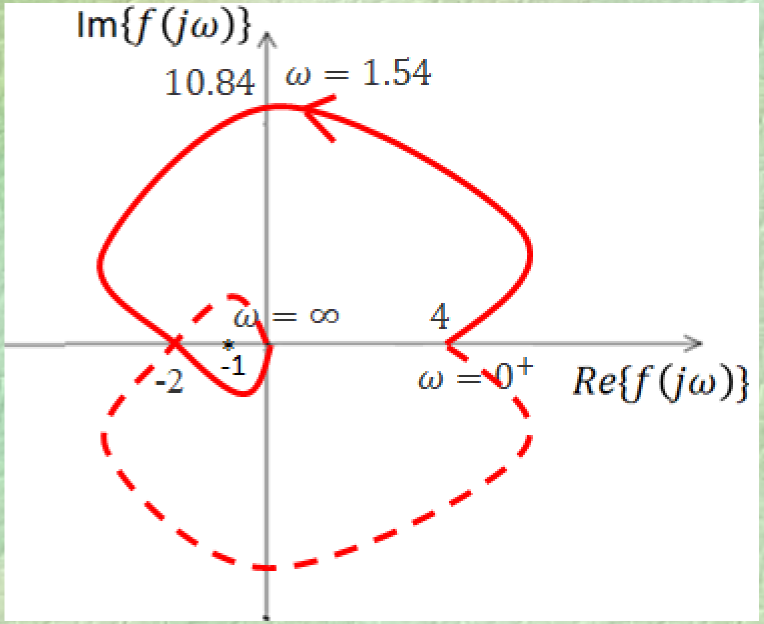


$$Z = N_{-1} + P$$

??

s^3	1	0
s^2	3	10
s	$-\frac{10}{3}$	0
s^0	10	

P=2

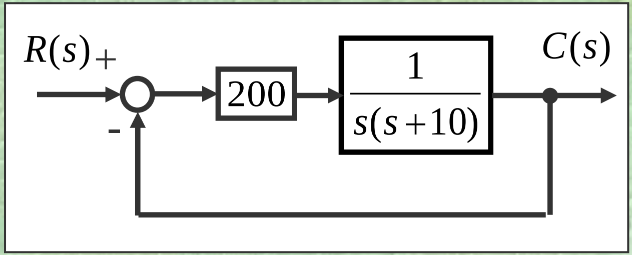
$$Z = N_{-1} + 2$$

$$Z = -2 + 2 = 0$$

Closed-loop system is stable.

Exercises

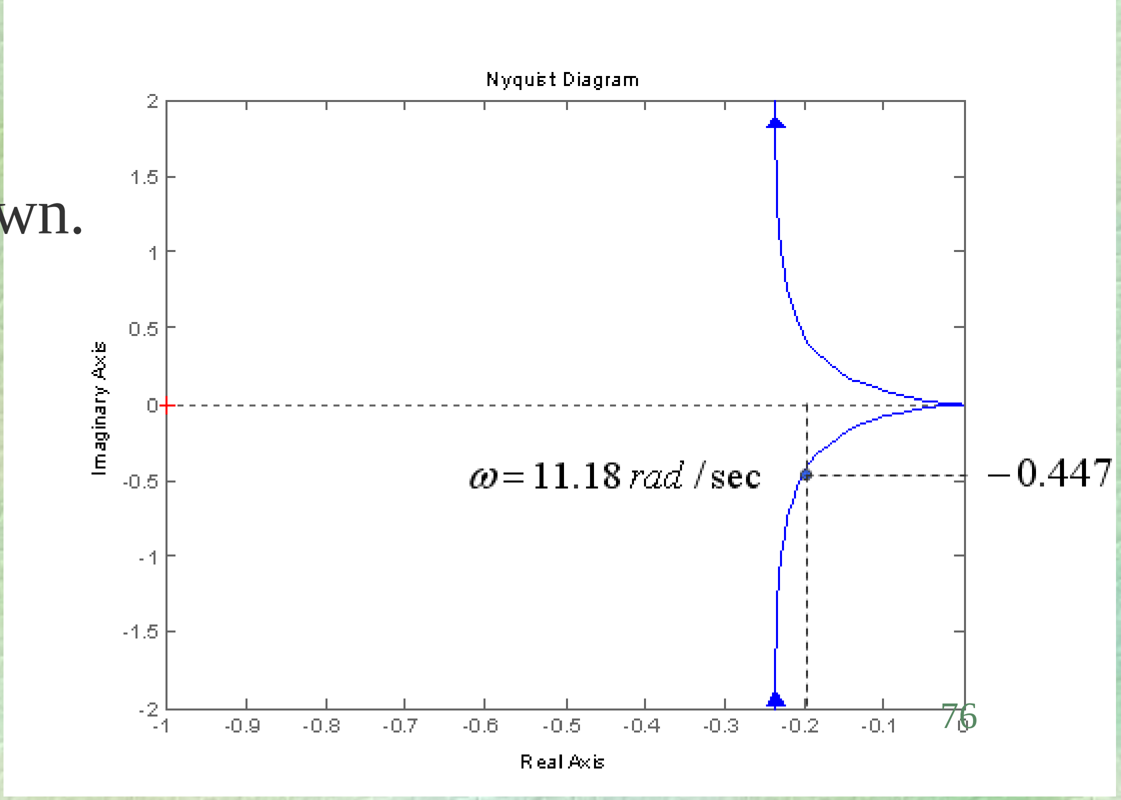
Exercise 1: Derive the gain crossover frequency, phase crossover frequency, GM and PM of following system by use of Bode plot.



$$Answer: \omega_c = 12.5\ ,\ \omega_{180} = \infty, GM = \infty\ and\ \varphi_m = 38^\circ$$

Exercise 2: The polar plot of an openloop system with negativeunit feedback is shown.

a) Find the open loop
b) transfer function.
c) Find the closed loop
d) transfer function.



$$answer\ a: \frac{150}{s(s+25)}\quad b: \frac{150}{s^2+25s+150}$$

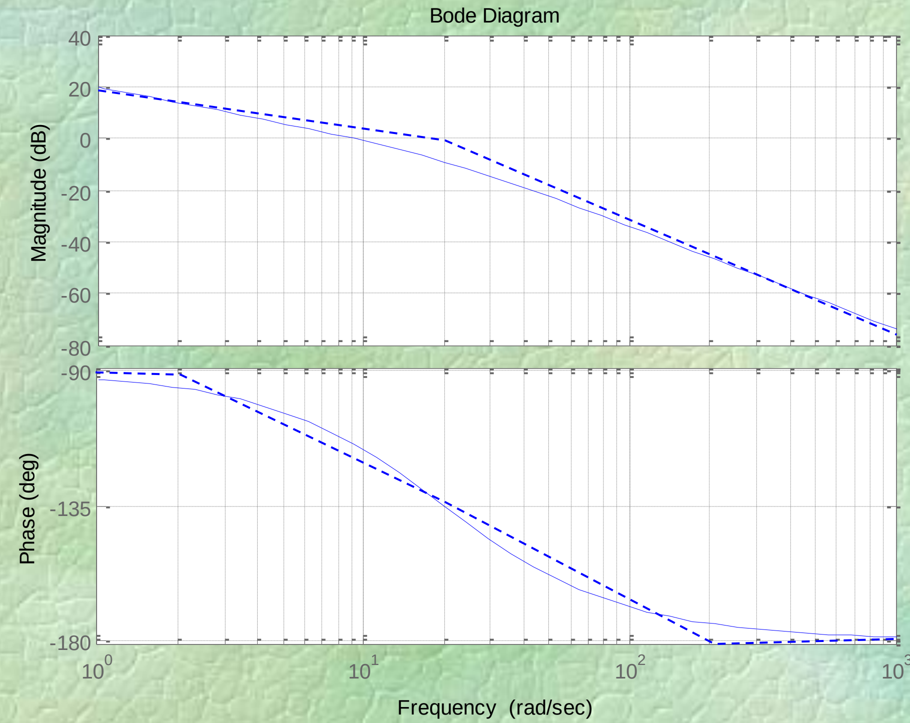
Exercises

Exercise 3: Bode plot of an open loop system with negative unit feedback is shown.

a) Find the open loop transfer function.

b) Find the closed loop transfer function.

$$answer\ a:\frac{200}{s(s+20)}\quad b:\frac{200}{s^2+20s+200}$$

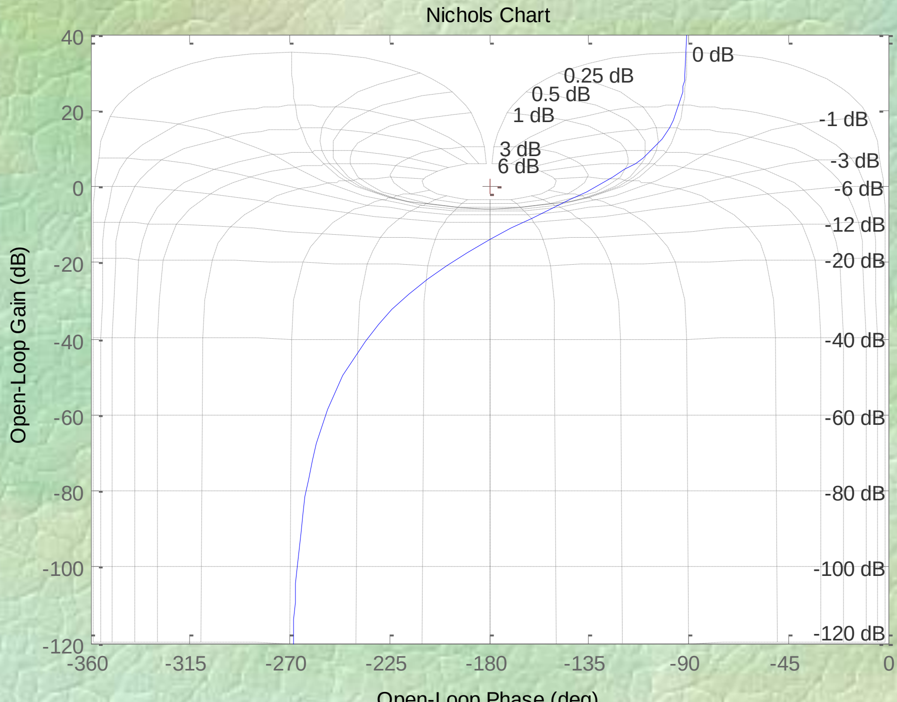


Exercises

Exercise 4: The Nichols chart of an open loop system with negative unit feedback is shown.

a) Find the GM and PM.

b) Find M_P.



$$answer\ a: GM = 14\,db, PM = 45^\circ \quad b: M_p = 1.8\,db$$

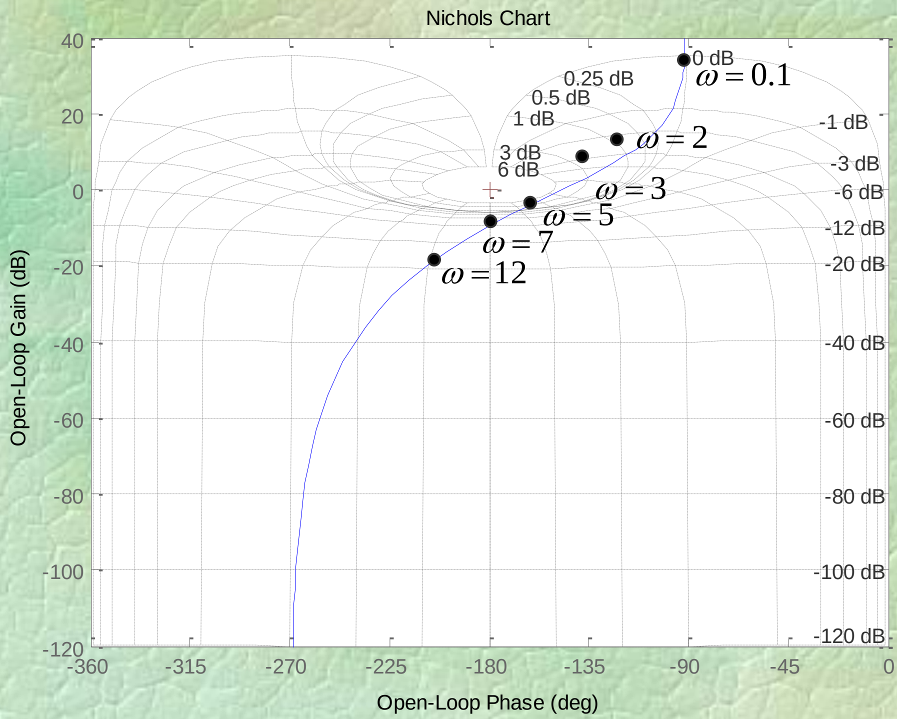
Exercises

Exercise 5: The Nichols chart of a open loop system with negative unit feedback is shown.

a) Find the error constants

b) Find the GM and PM and gain crossover frequency and phase crossover frequency.

c) Find M_P, open loop bandwidth and closed loop bandwidth.



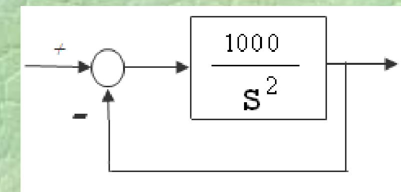
$answer\ a: k_p = \infty, k_v = 5, k_a = 0$

$b: GM = 10\,db, PM = 32^\circ, \omega_c = 3.75\,rad/\sec, \omega_{180} = 7\,rad/\sec$

$c: M_p = 5.3\,db, BW_{openloop} = 4.7\,rad/\sec, BW_{closedloop} = 6.3\,rad/\sec$

Exercises

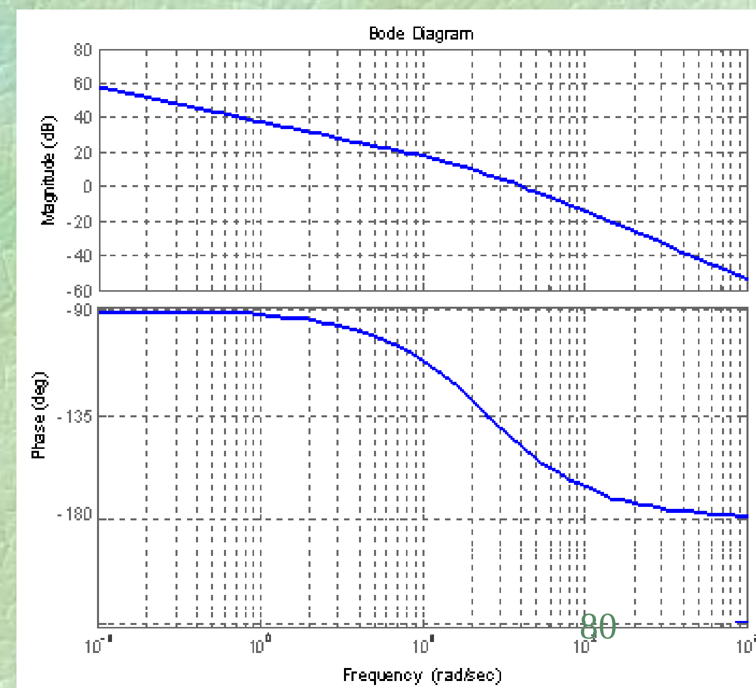
Exercise 6: Draw Nichols chart of following system (Final exam).



Exercise 7: Draw gain-phase plot of a minimum phase type one system with no zero and three poles and GM=2 db and PM=45° (Final exam).

Exercise 8: Bode plot of a minimum phase system is: (Final exam).

a- Derive phase and gain crossover Frequency, Gm and PM.
b- Determine the nonzero error constant.
c- If 0.01 sec delay added inside the feedback loop, derive new Bode plot in the same figure.
d- Derive phase and gain crossover Frequency, Gm and PM of new system.



Exercises

Exercise 9: Nichols chart of a system is given, determine

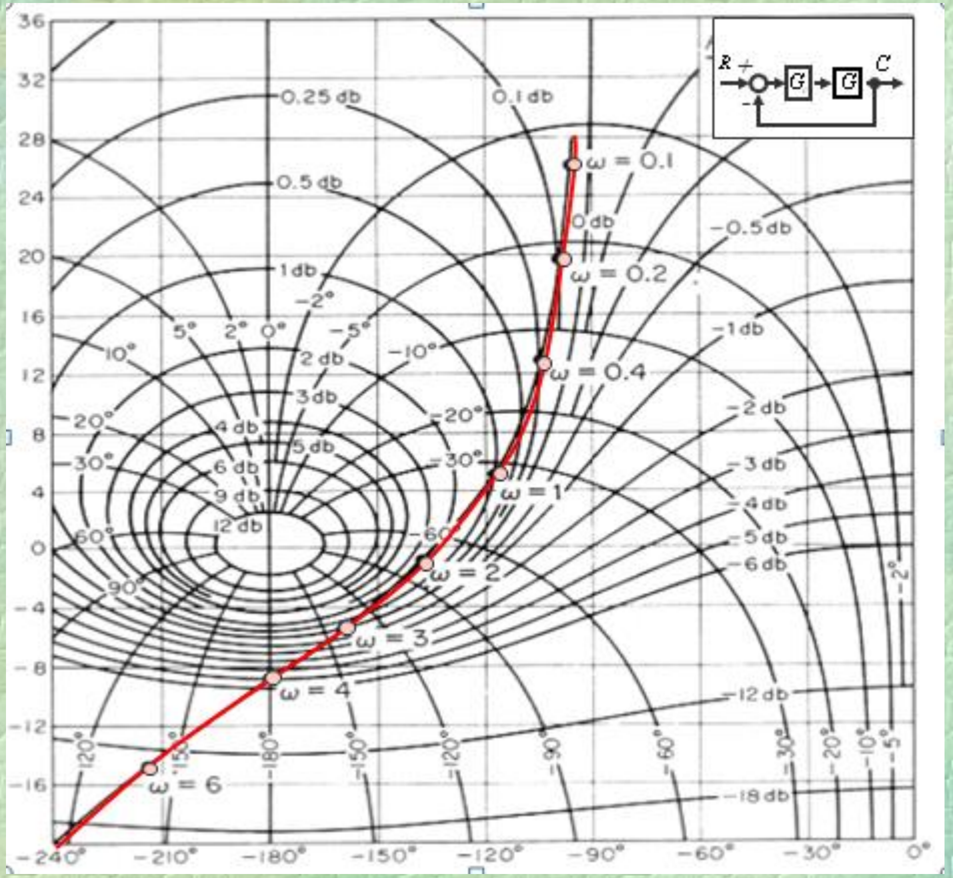
a- Gain and phase cross over frequency.

b- GM and PM.

c- Open loop and closed loop BW.

d- Type of system.

e- All error constant.



Dr. Ali Karimpour Aug 2024