
Electric Machinery III

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Ferdowsi University of Mashhad, Iran

References for Electric Machinery III

Text: A. E. Fitzgerald, Charles Kingsley, Jr And Stephen D. Umans, *Electric Machinery*. Mc Graw Hill, 7th Edition

Reference 1: P.C. Sen, *Principles of Electric Machines and Power Electronics*. Wiley, 3rd Edition

Reference 2: P.S. Bimbhra, *Electrical Machinery*. Khanna Publication

Electrical Machine III Syllabus

1. Introduction

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2. Introduction to Rotating Machines

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3. Generation of Sinusoidal Voltage in a Synchronous Generator

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4. Synchronous-Machine Analyses

- Synchronous-Machine Inductances; Equivalent Circuits
- Steady-State Power-Angle Characteristic
- **Steady-State Operating Characteristic**
- Open- and Short-Circuit Characteristics
- Effects of Salient Poles; Introduction to Direct and Quadrature-Axis Theory
- Steady-State Power-Angle Characteristic of Salient-Pole Machines
- Permanent Magnet Synchronous Machine
- Transient Behavior of Synchronous Machine.

Steady-State Operating Characteristic

1- Synchronous-generator compounding curves (I_f versus I_a for constant pf.)

The curve showing the field current required to maintain **rated terminal voltage** as the **constant-power-factor** load is varied.

2- Synchronous-generator V curves (I_a versus I_f for constant P)

The curve showing the relation between armature current and field current at a **constant terminal voltage** and with a **constant real power**

3- Capability curves (Q-P curve)

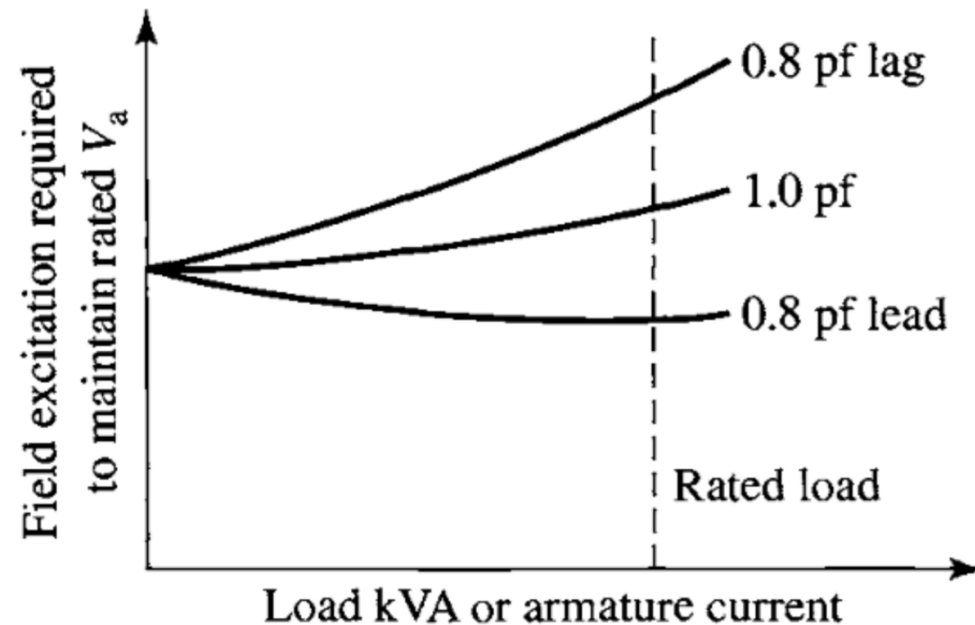
They give the maximum reactive-power loadings corresponding to various real power loadings with operation at **rated terminal voltage**.

Steady-State Operating Characteristic-Compounding curves

Lecture #4-2

(I_f versus I_a for constant pf.)

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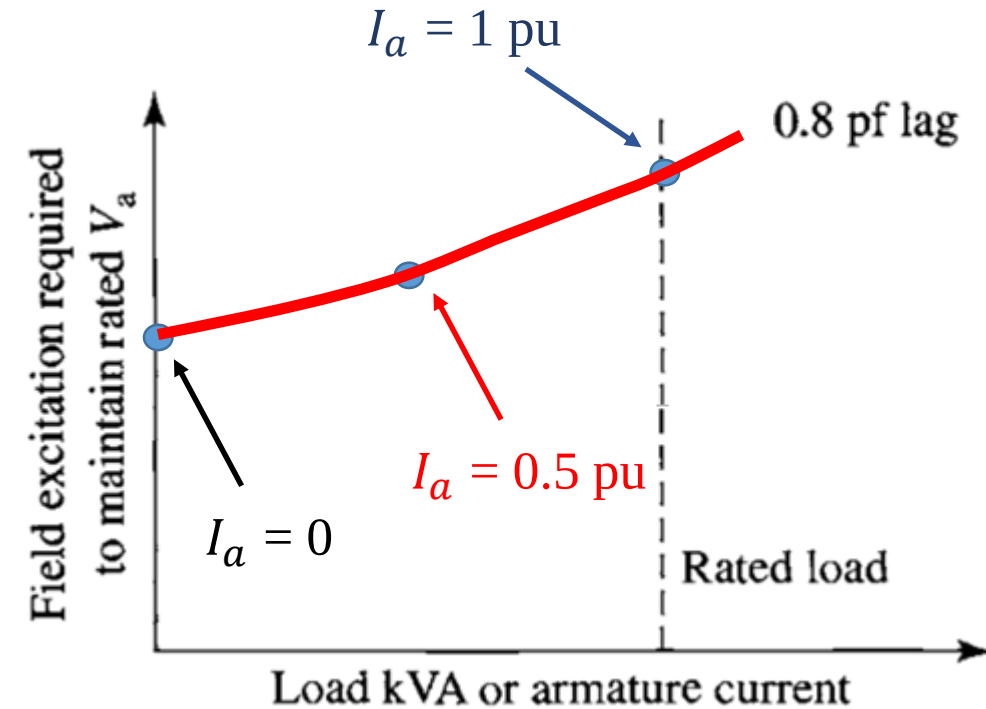
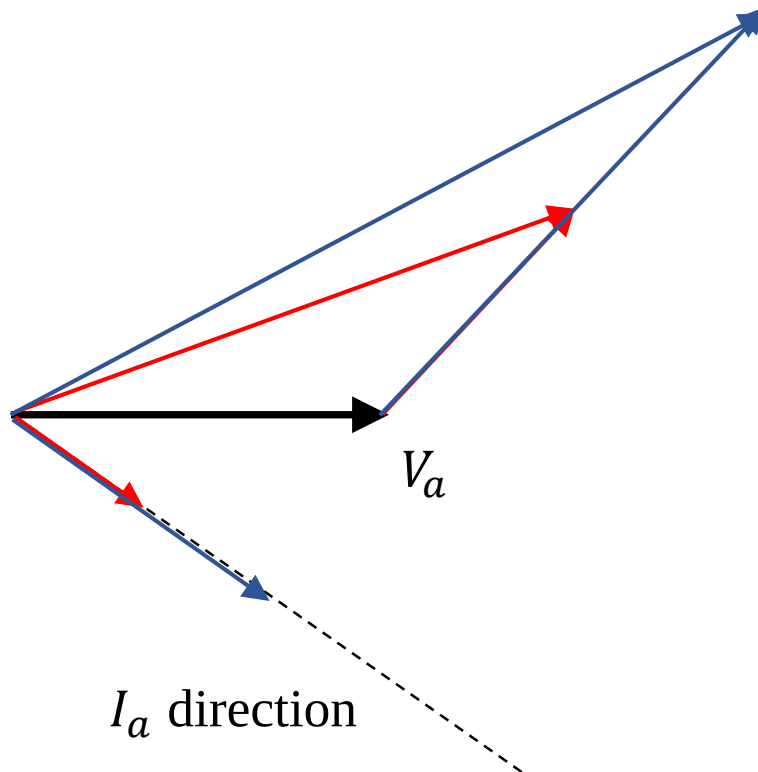


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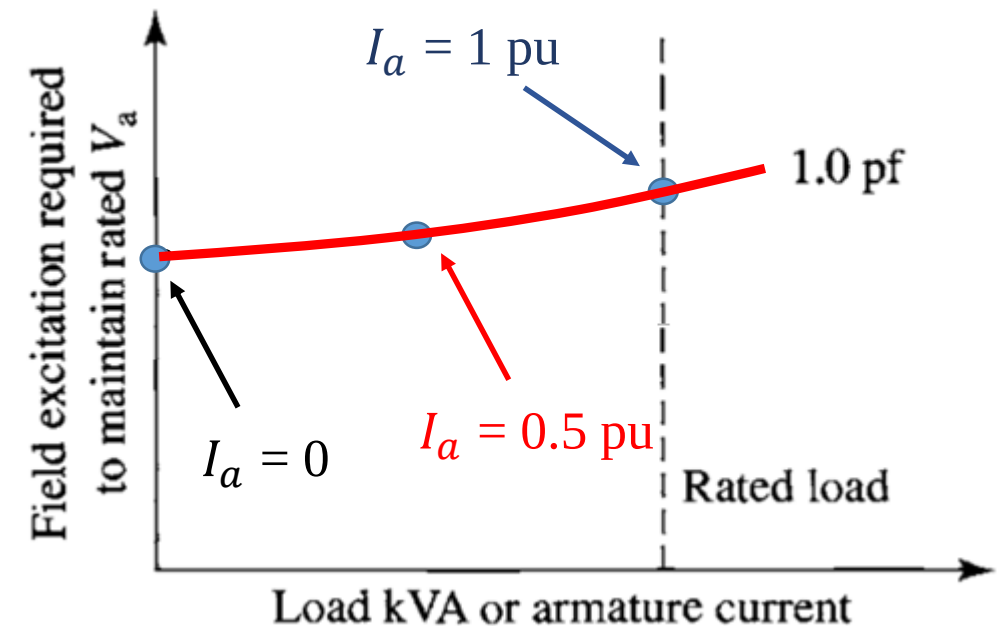
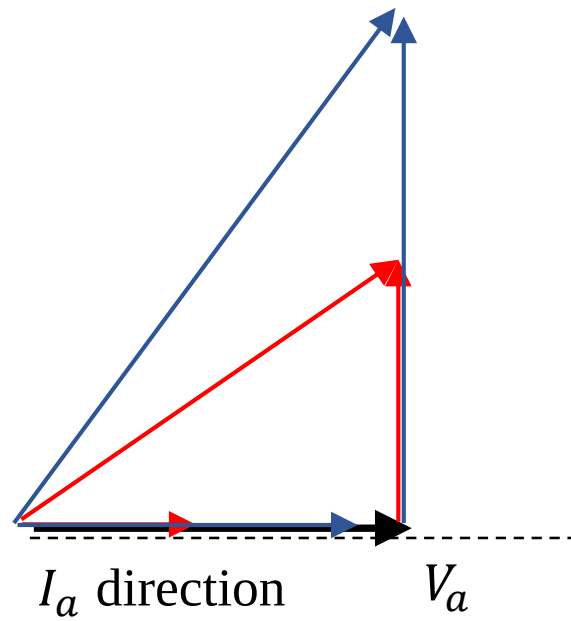


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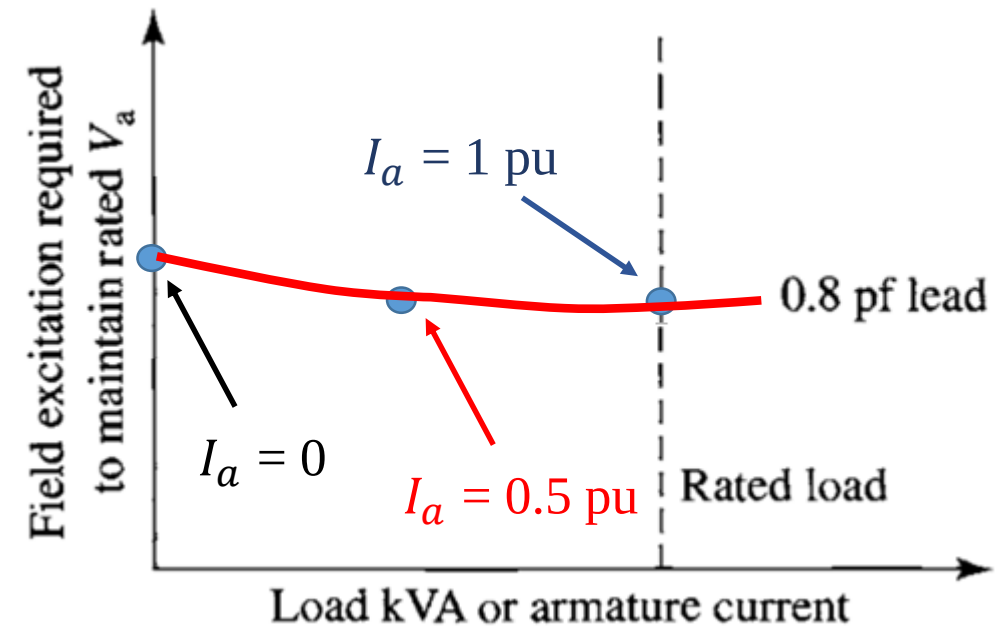
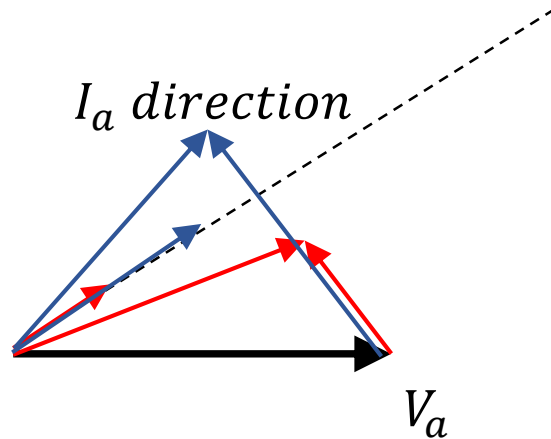
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Steady-State Operating Characteristic-Compounding curves Lecture #4-2

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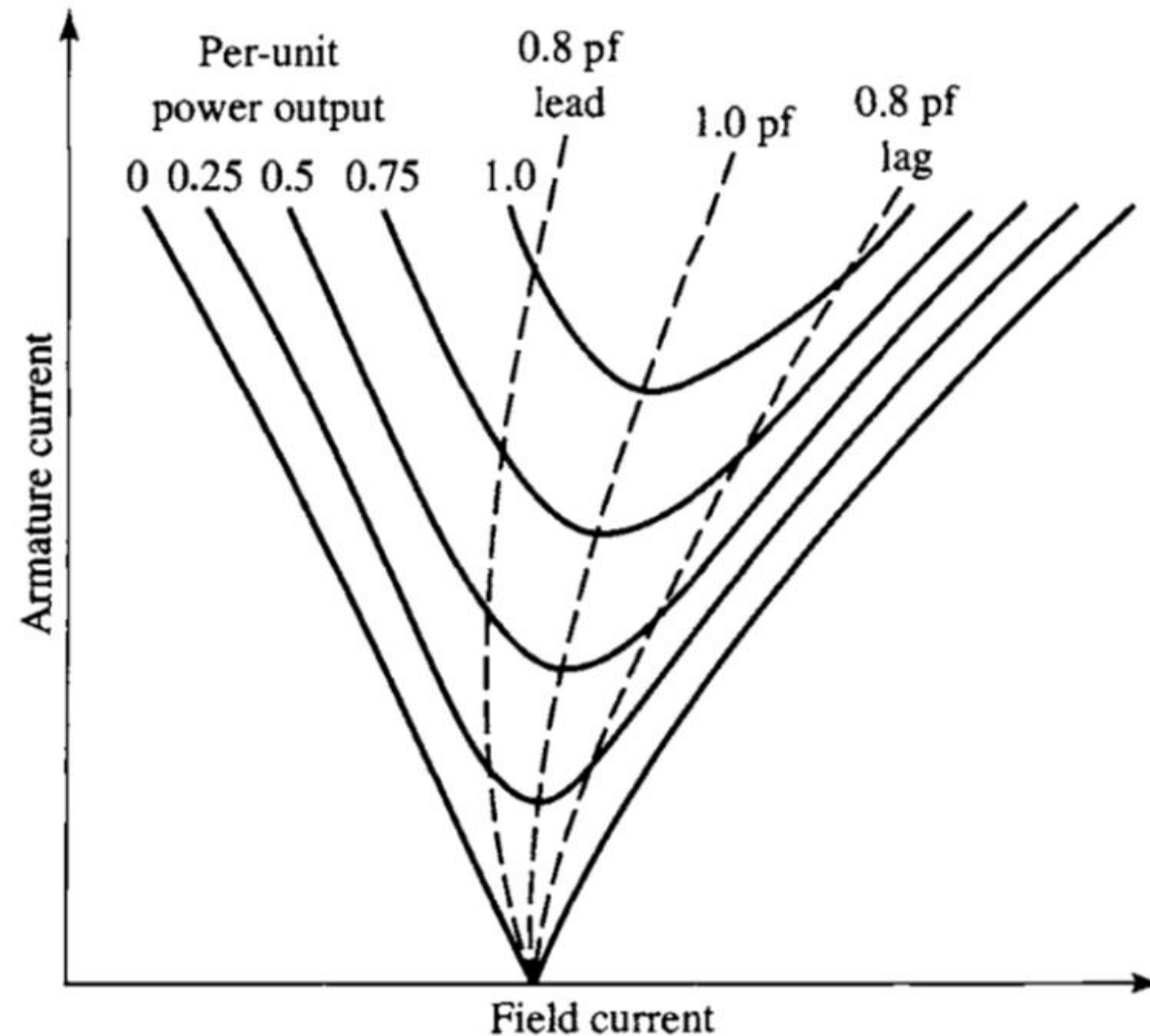


Steady-State Operating Characteristic-V curves

Lecture #4-2

(I_a versus I_f for constant active power)

The curve showing the relation between I_a and I_f at a **constant V_a** and with **constant P** .

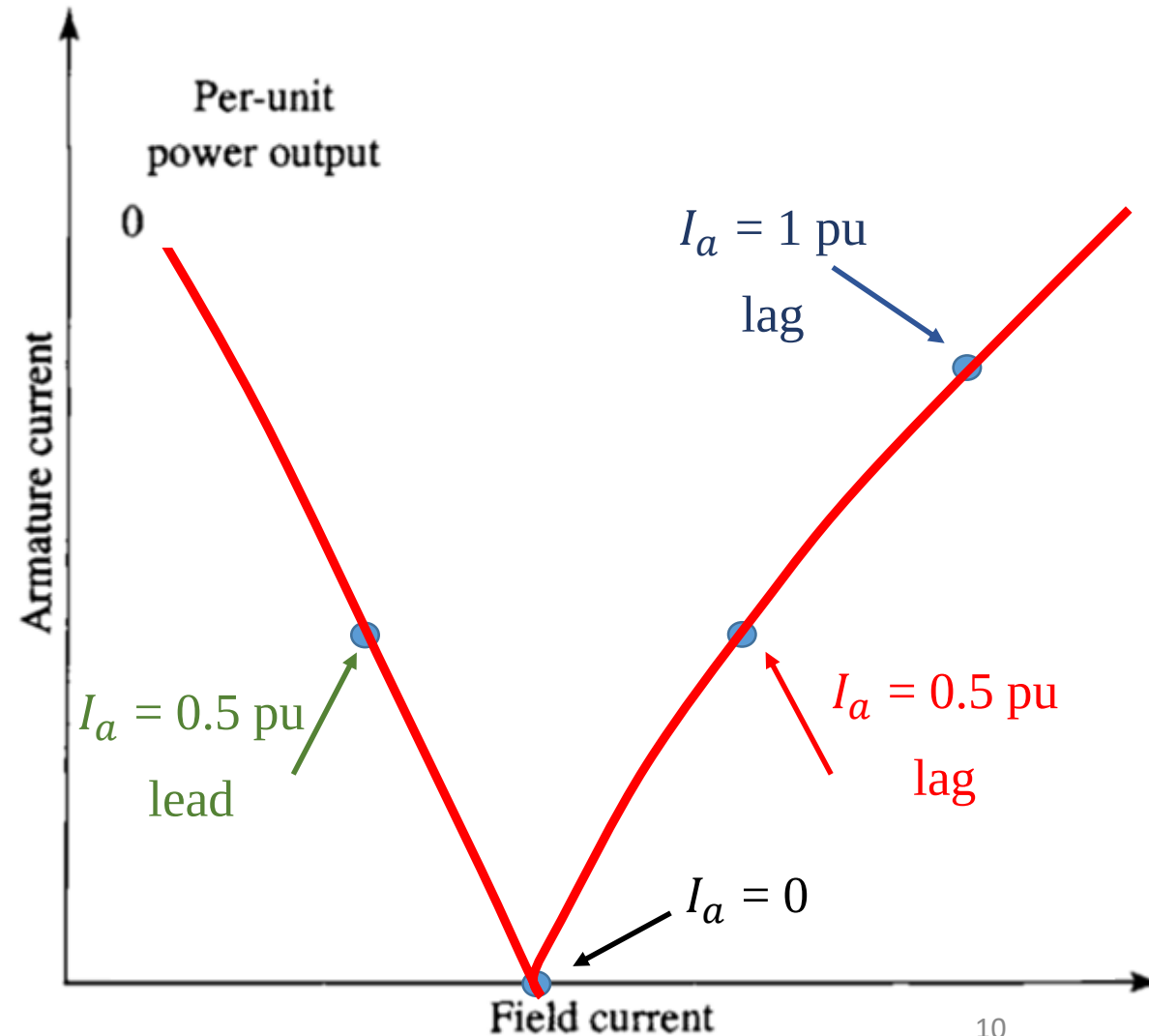
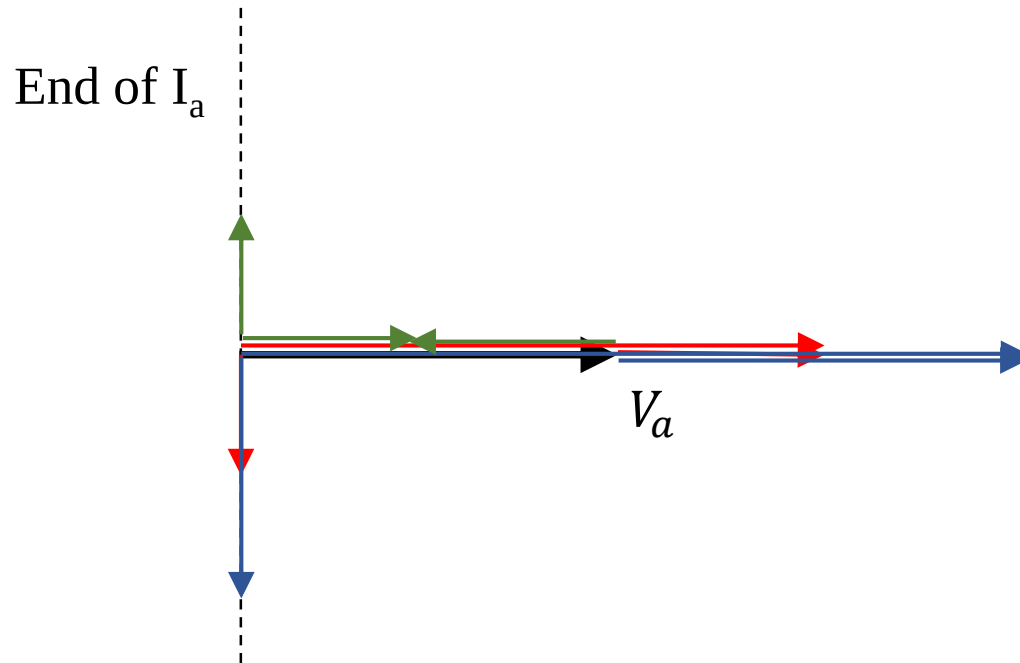


Steady-State Operating Characteristic-V curves

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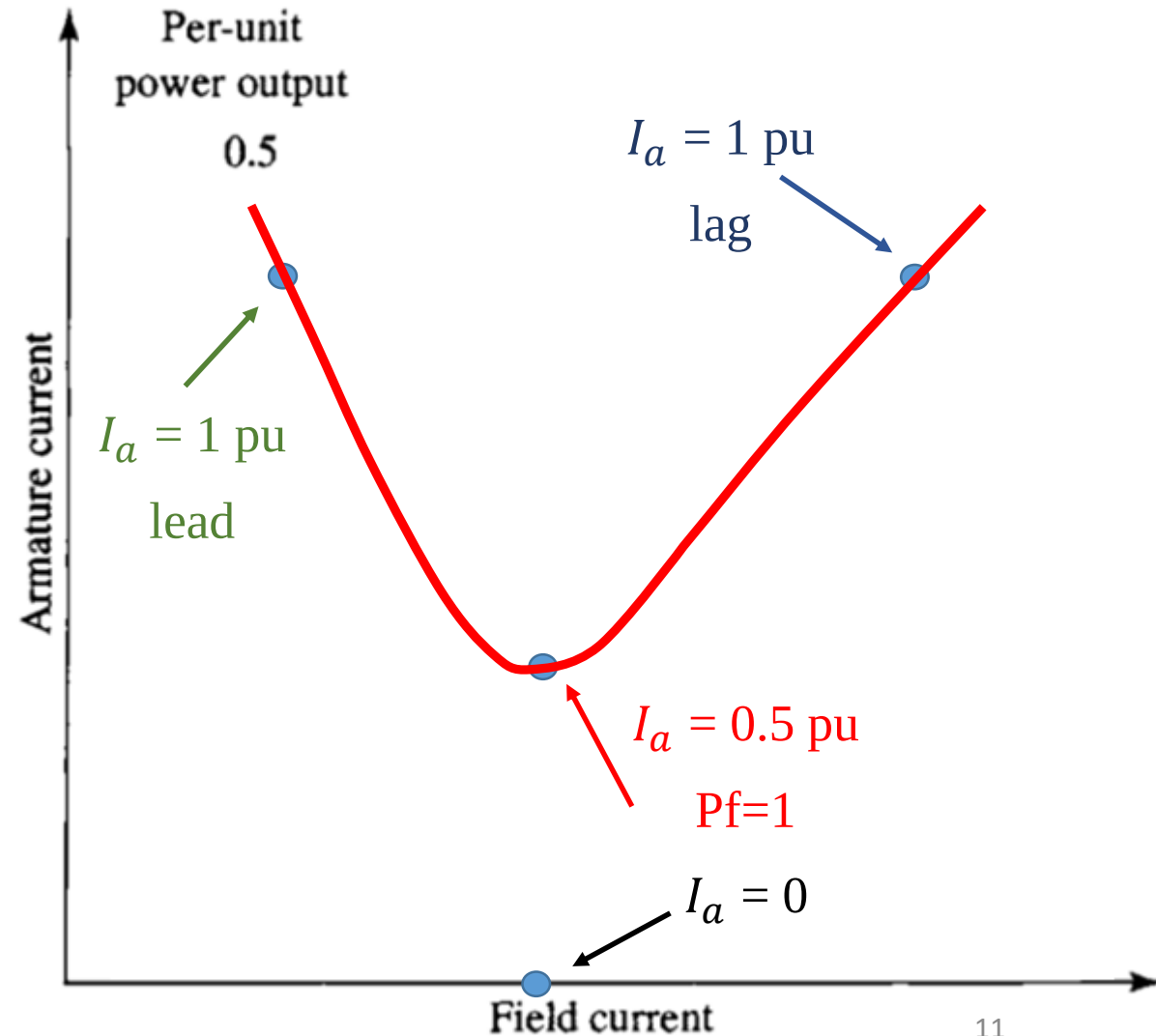
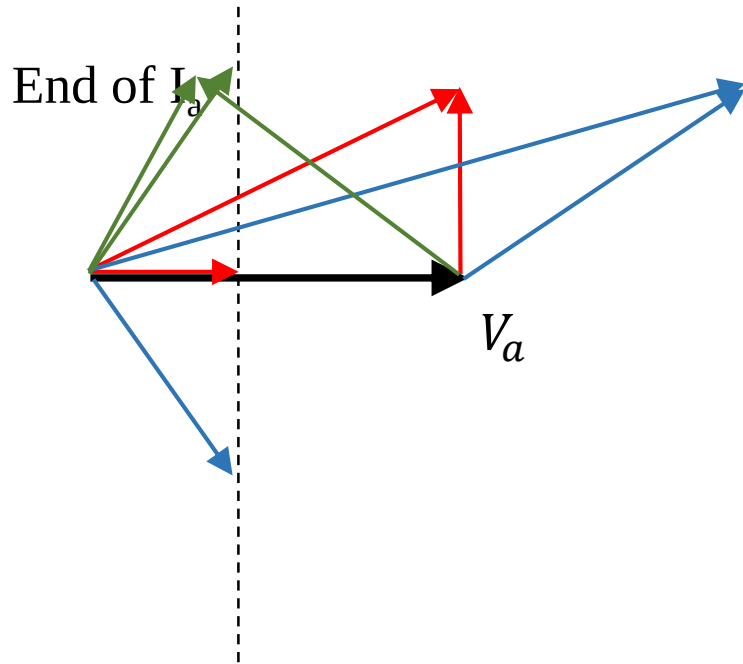


Steady-State Operating Characteristic-V curves

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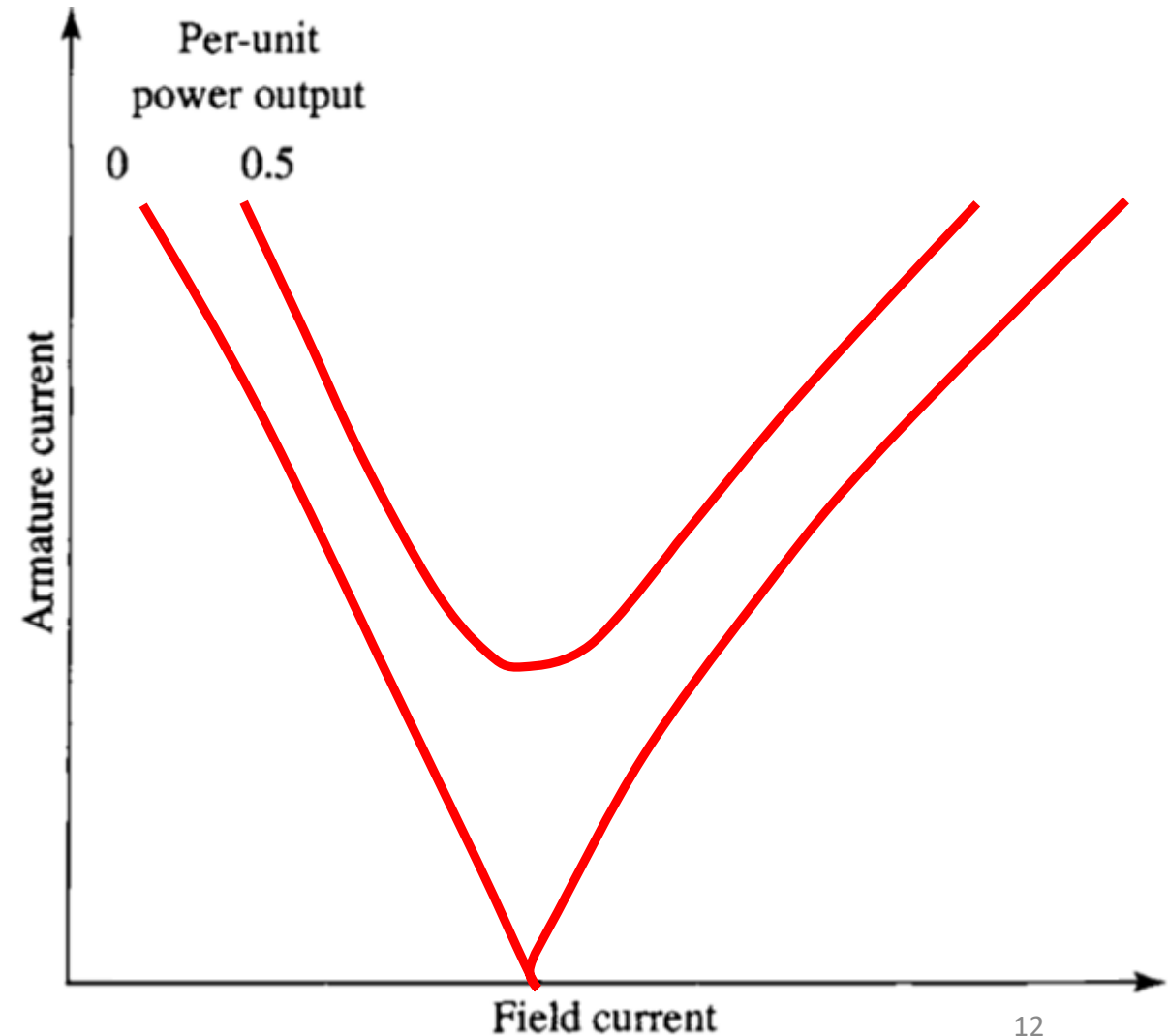


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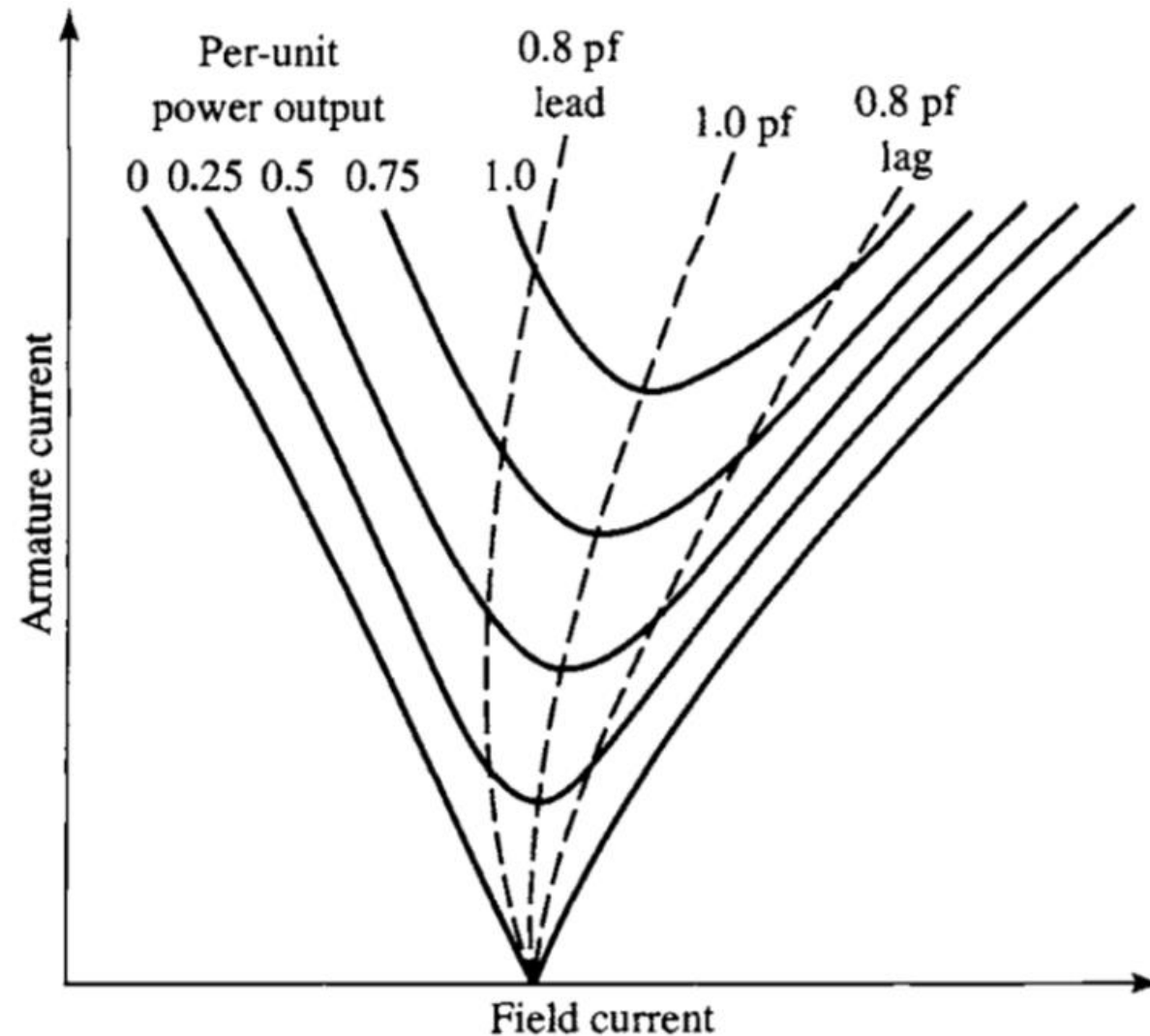


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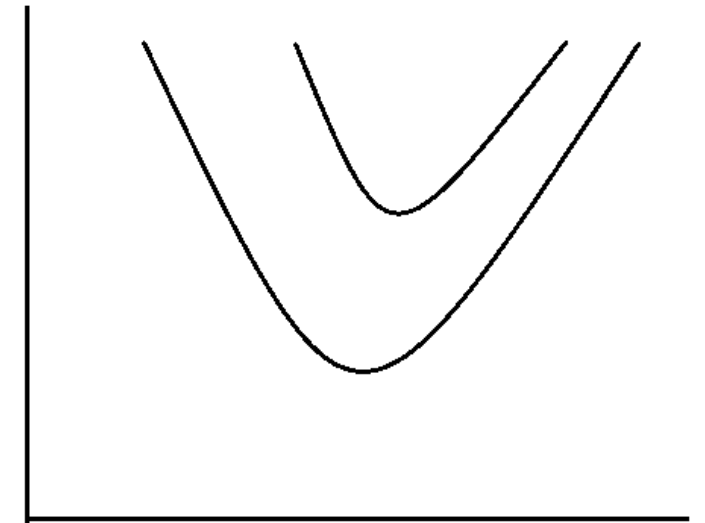


Exercise 1: The following figure shows the V curve of a synchronous generator for two power levels: 22.5 kW and 45 kW, with AFNL = 40 and AFSC = 40. What is the minimum armature current in the V curve corresponding to 22.5 kW?

- (a) AFNL (b) 118 A (c) 59 A (d) AFSC

Exercise 2: The following figure shows the VVV curve of a synchronous generator for two power levels: 22.5 kW and 45 kW, with AFNL = 40 and AFSC = 40. What is the field current corresponding to the minimum armature current in the VVV curve for 45 kW?

- (a) AFNL (b) 118.2 A (c) 56.6 A (d) AFSC



Exercise 3: A synchronous generator is operating at its rated load. With a very small increase in the field current, the armature current slightly decreases. Which option is correct?

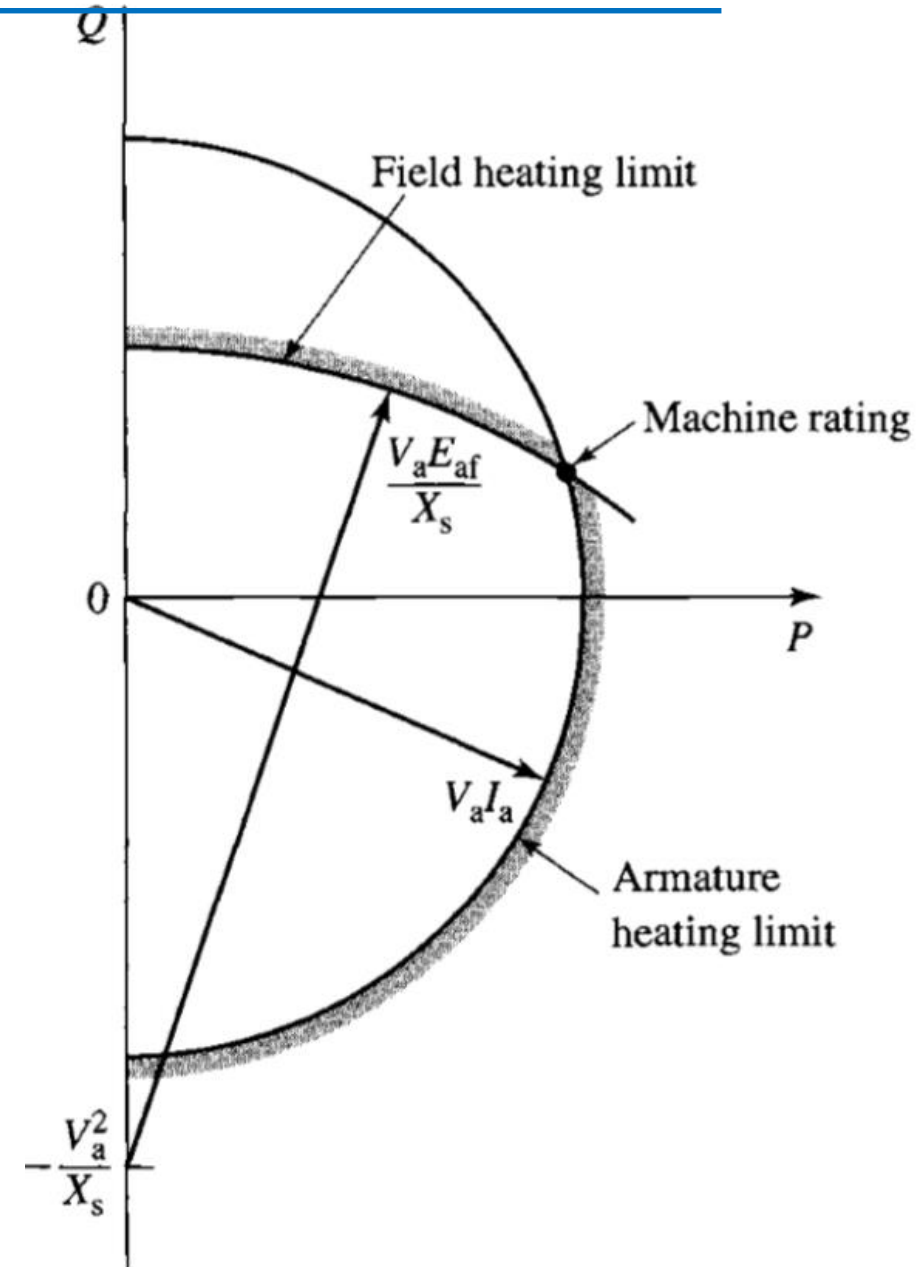
- (a) The generator has an inductive role in the network.
- (b) Increasing the field current cannot cause a decrease in the armature current.
- (c) The generator has a capacitive role in the network.
- (d) None of the above.

Exercise 4: Consider a 45 kVA, 220 V generator with $AFNL = AFSC = 40$. A point on the compounding curve for unity power factor corresponds to (0, 40). Which of the following shows another point on this curve?

- (a) (118, 86.6)
- (b) (59, 86.6)
- (c) (118, 56.6)
- (d) (59, 56.6)

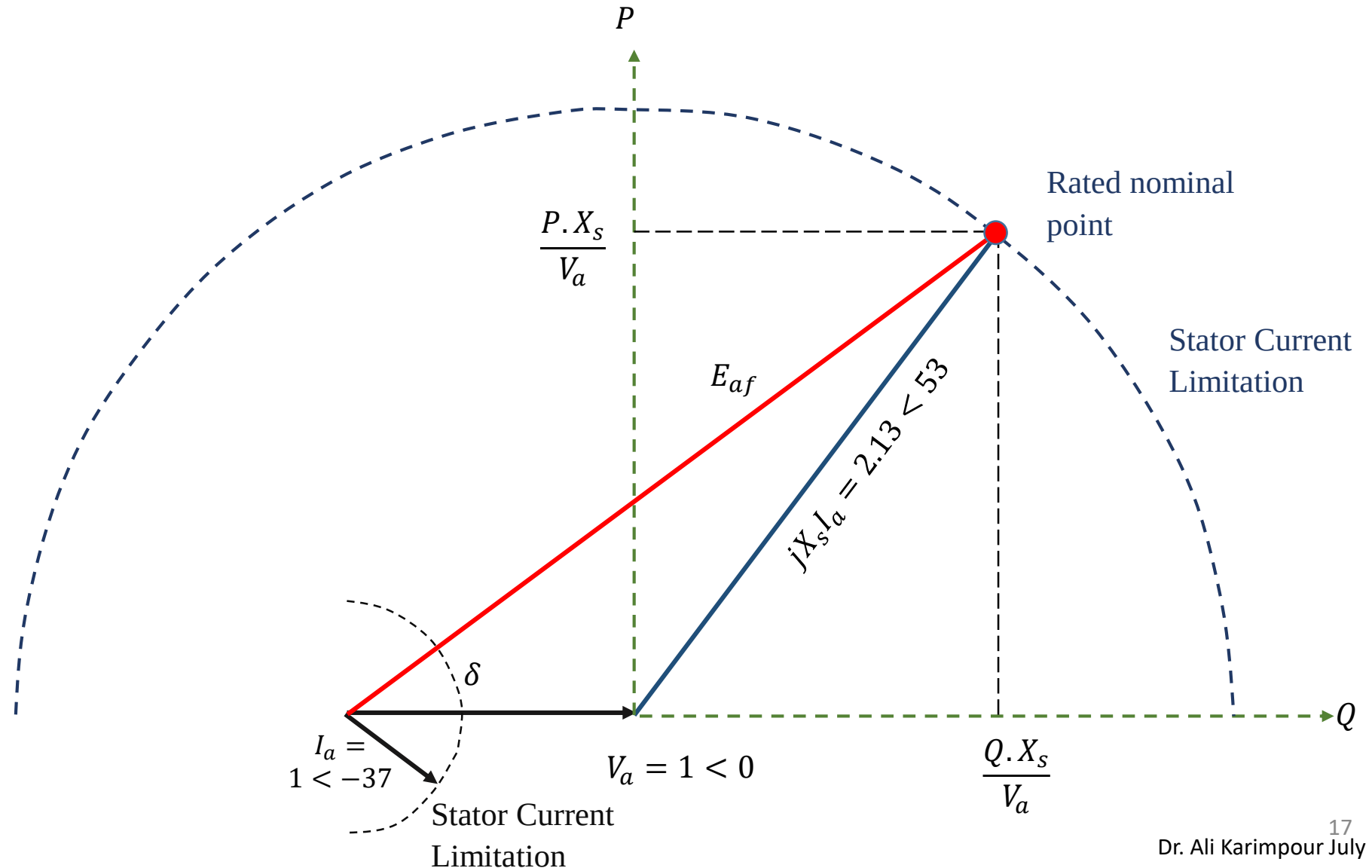
3- Capability curves (Q-P curve)

It gives the maximum reactive-power loadings corresponding to various real power loadings with operation at rated terminal voltage.



Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2

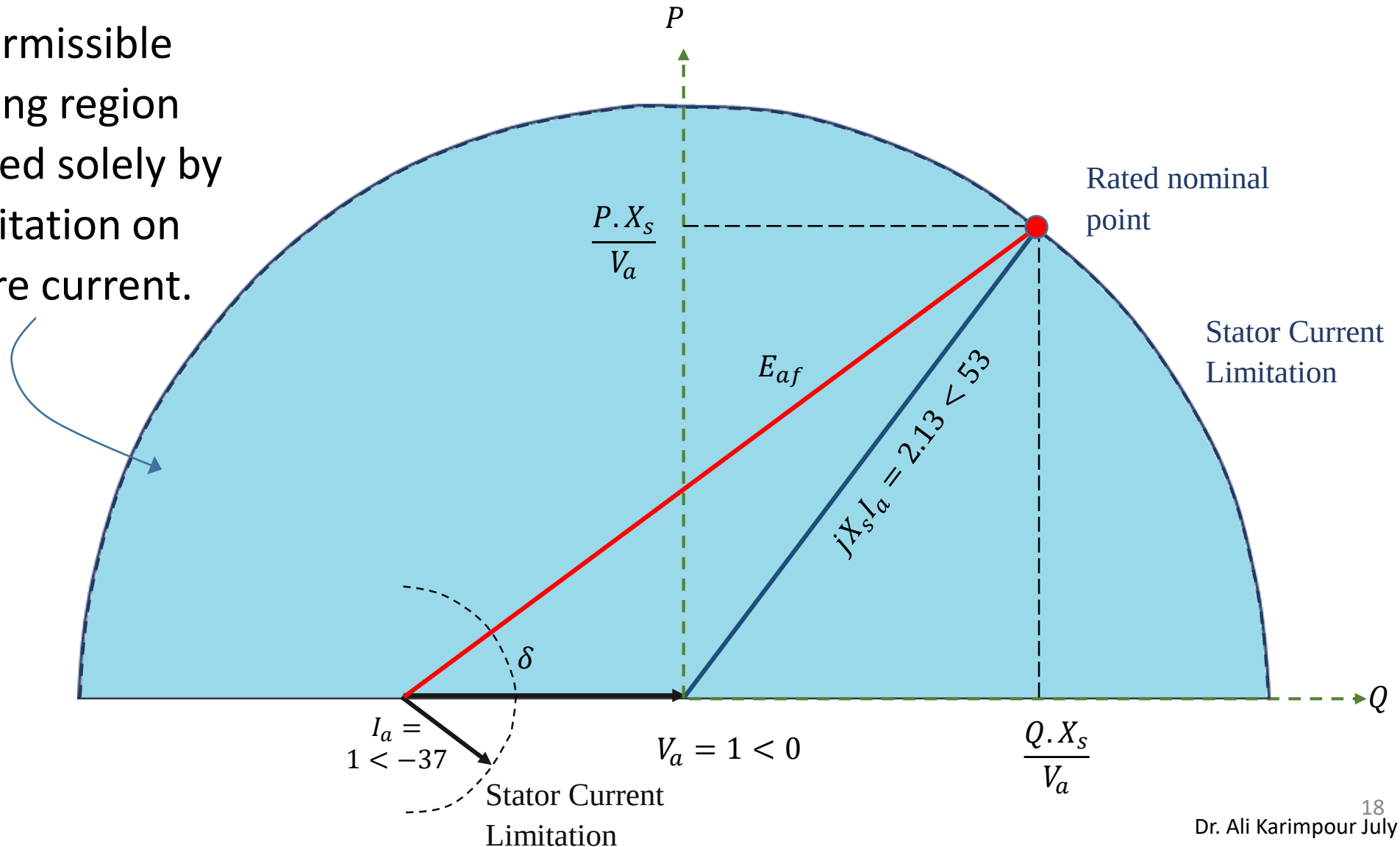
Ansaldo generator: S=200-MVA , 0.8 power factor lag, SCR=0.47, V=15750 V



Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2

Ansaldo generator: S=200-MVA , 0.8 power factor lag, SCR=0.47, V=15750 V

The permissible operating region determined solely by the limitation on armature current.

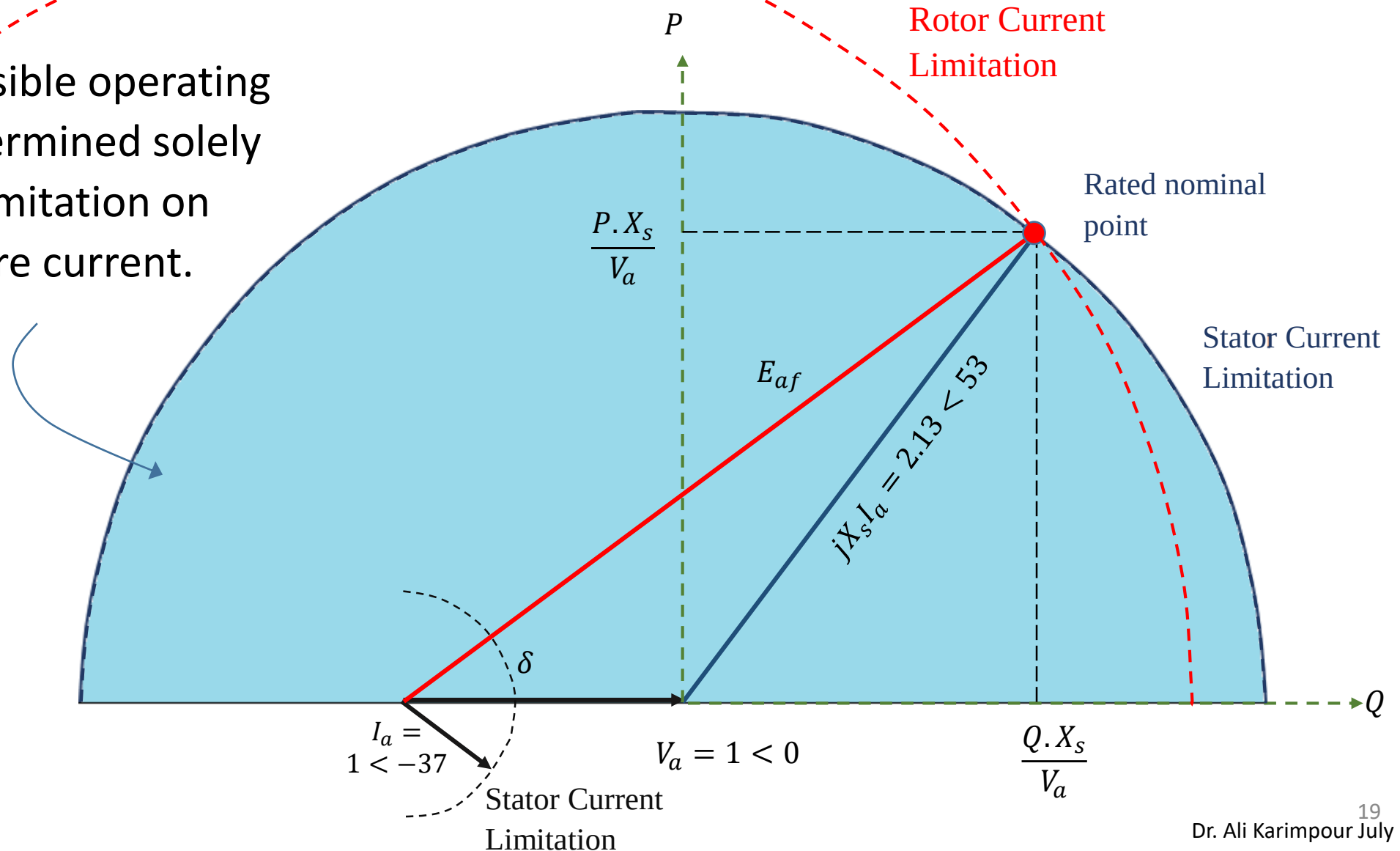


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

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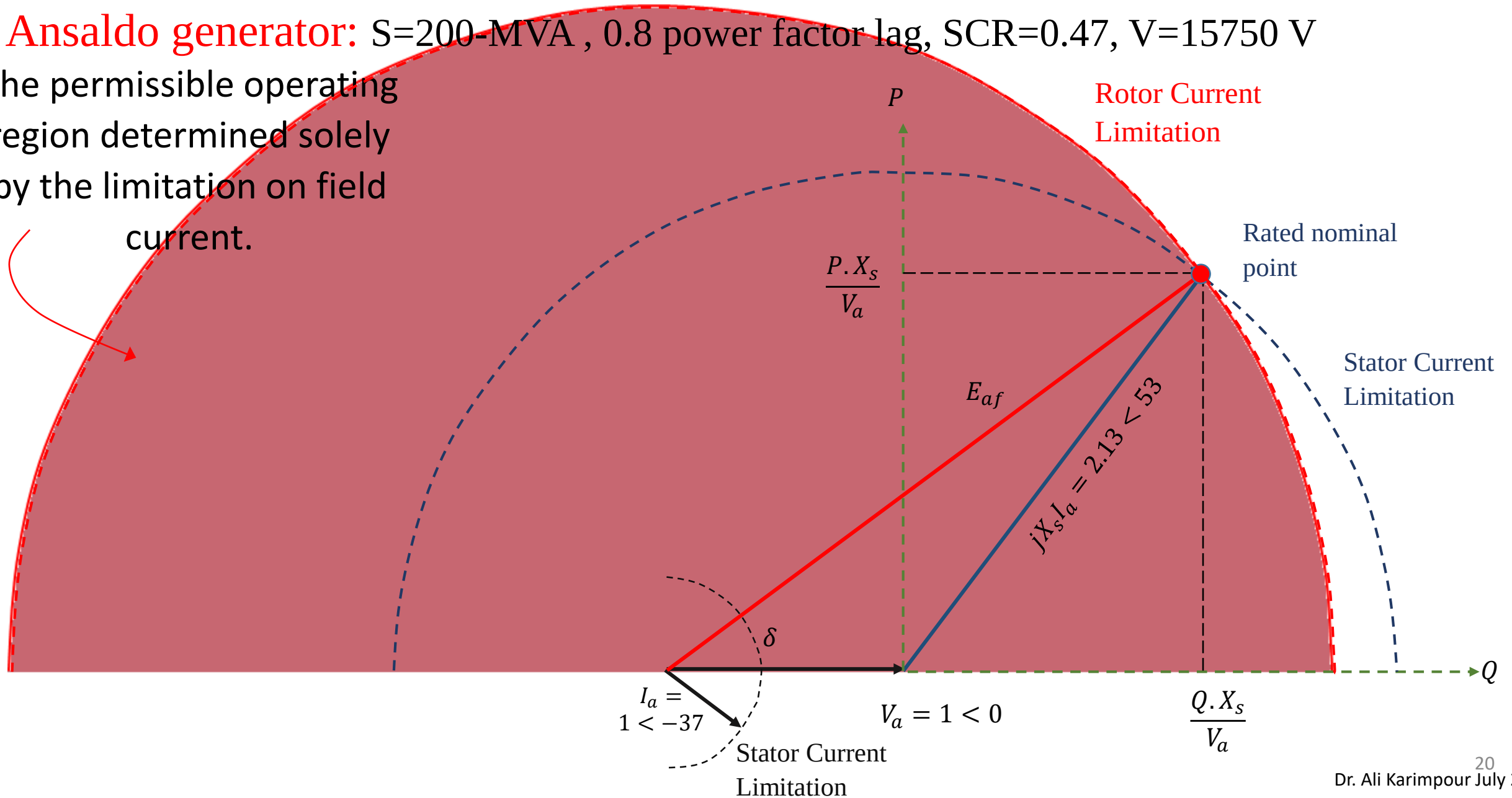


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

Ansaldo generator: $S=200$ -MVA , 0.8 power factor lag, $SCR=0.47$, $V=15750$ V

The permissible operating region determined solely by the limitation on field current.

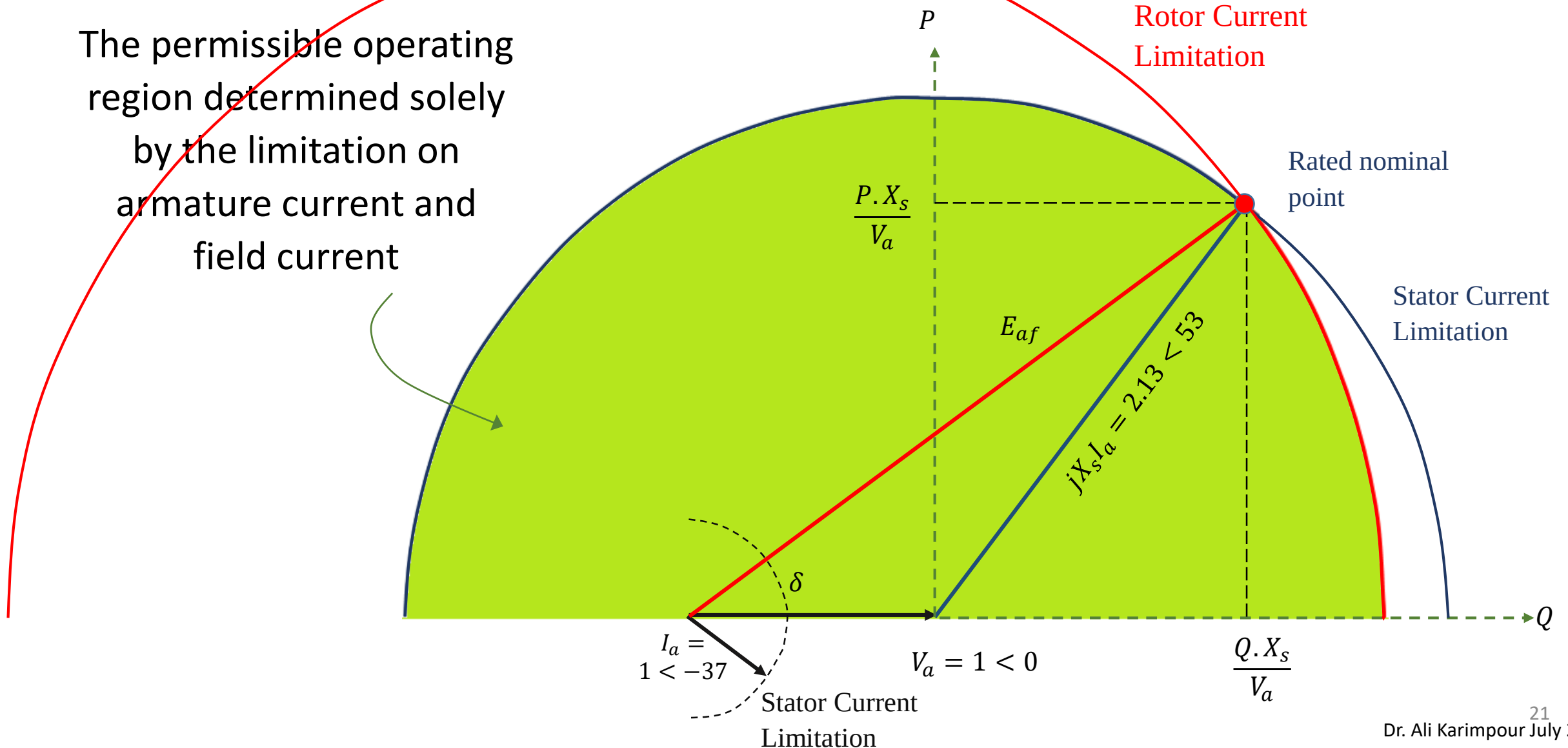


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

Ansaldo generator: $S=200$ MVA , 0.8 power factor lag, $SCR=0.47$, $V=15750$ V

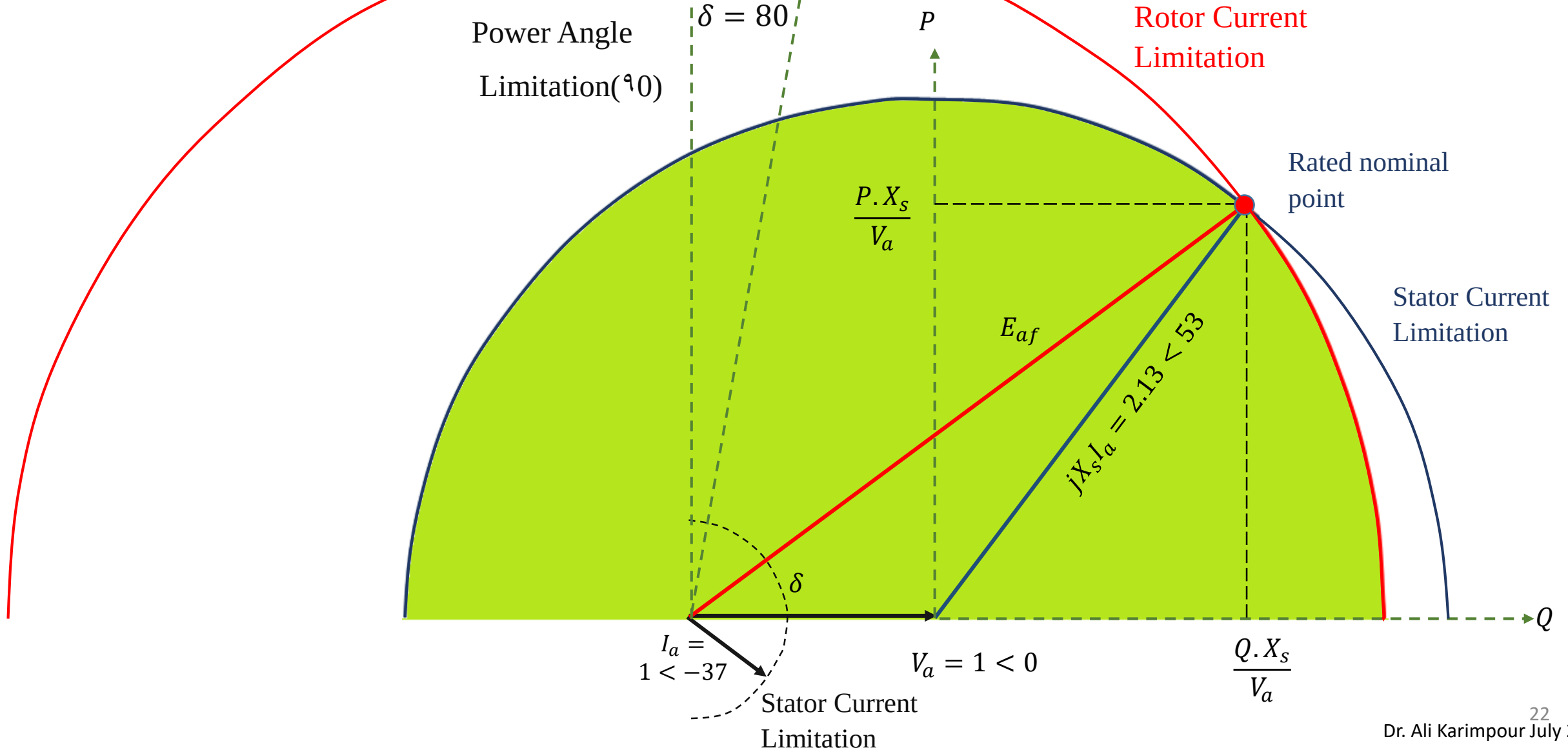
The permissible operating region determined solely by the limitation on armature current and field current



Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

Ansaldo generator: $S=200$ MVA, 0.8 power factor lag, $SCR=0.47$, $V=15750$ V

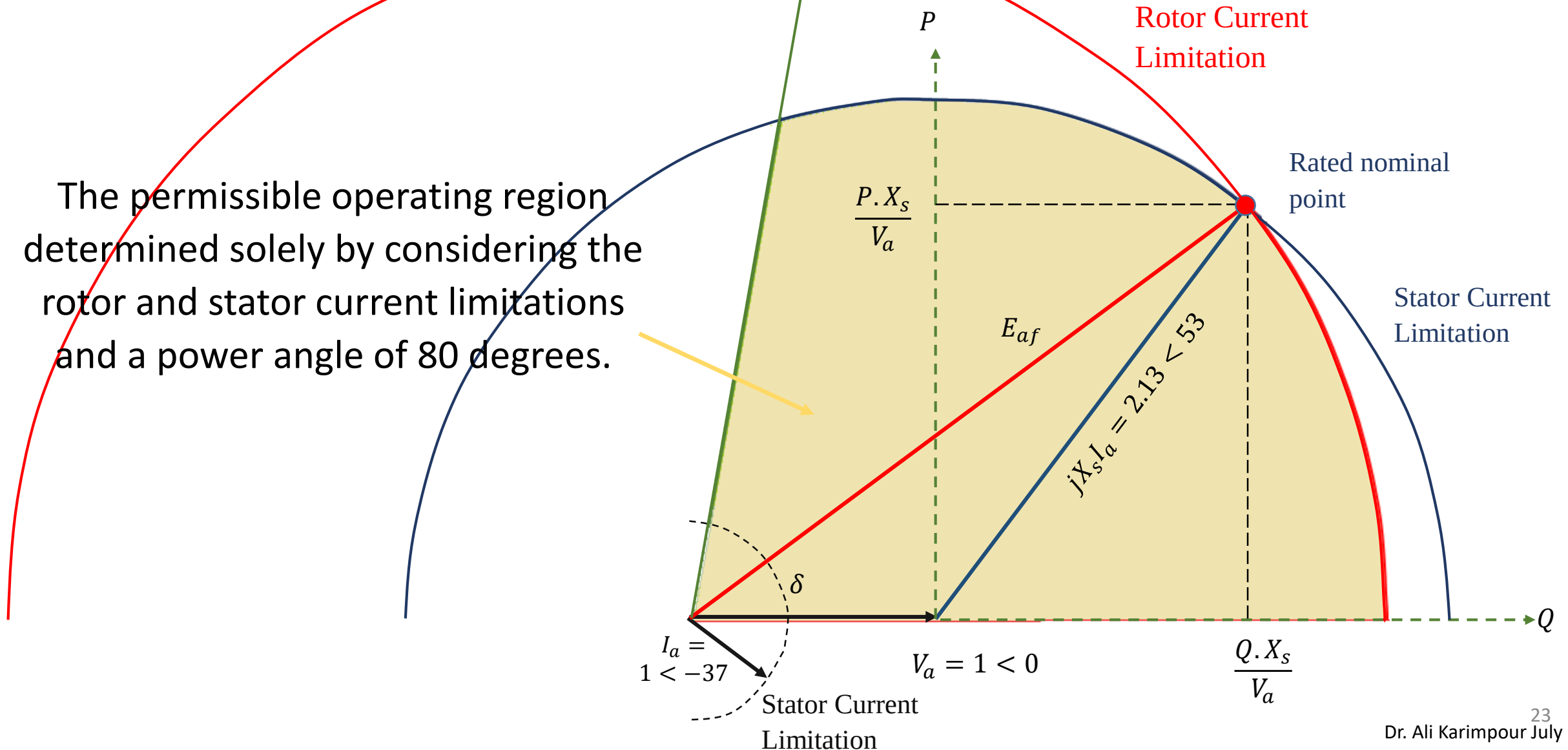


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

Ansaldo generator: $S=200$ MVA , 0.8 power factor lag, $SCR=0.47$, $V=15750$ V

The permissible operating region determined solely by considering the rotor and stator current limitations and a power angle of 80 degrees.



Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

Capability Curve for Ansaldo

Ansaldo generator:

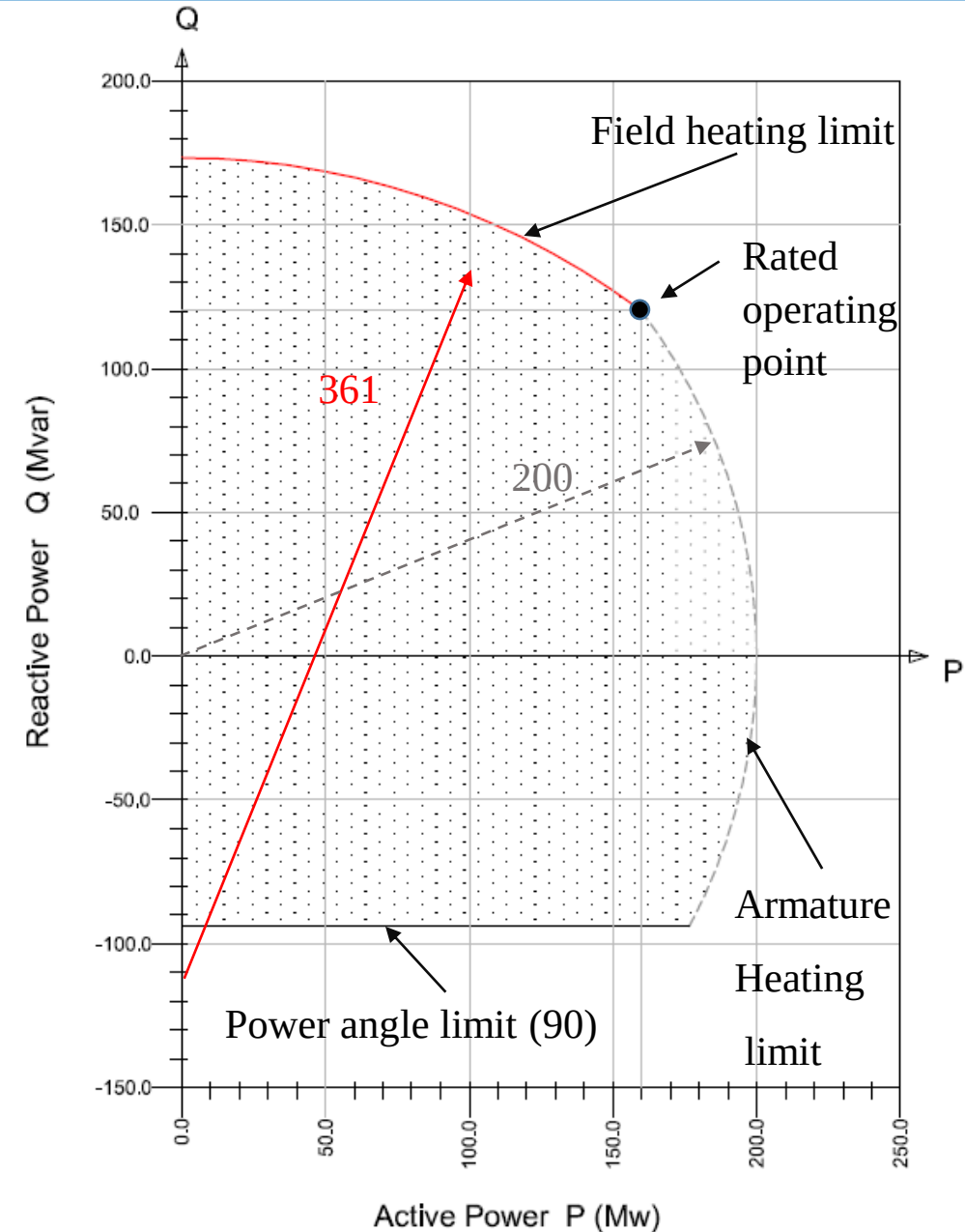
$S=200\text{-MVA}$, 0.8 power factor lag,

$\text{SCR}=0.47$, $V=15750\text{ V}$

$$P_{\text{rated}} = S \cos \varphi = 160\text{ MW}$$

$$Q_{\text{rated}} = S \sin \varphi = 120\text{ MVAR}$$

synchronous generator capability curve.



Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2

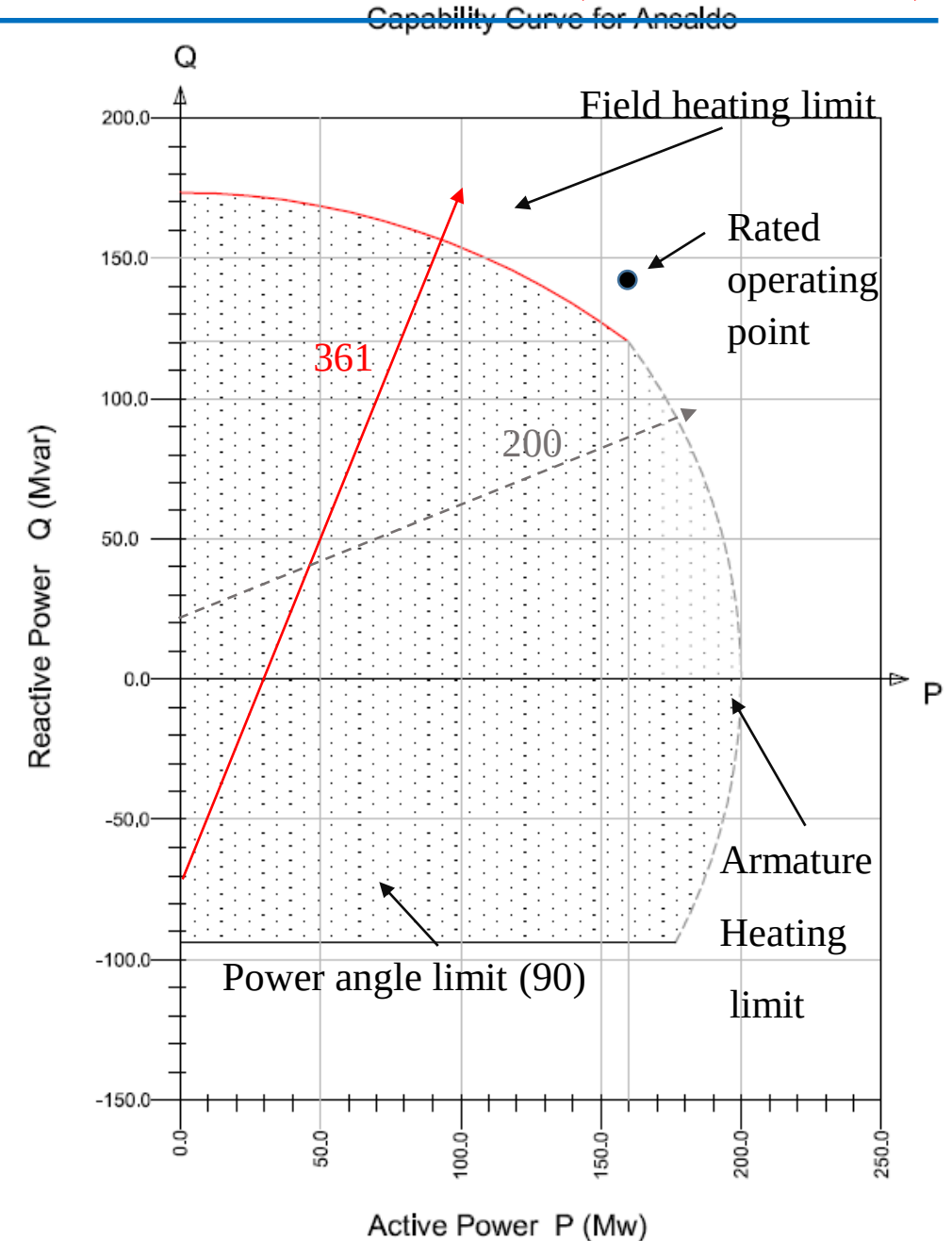
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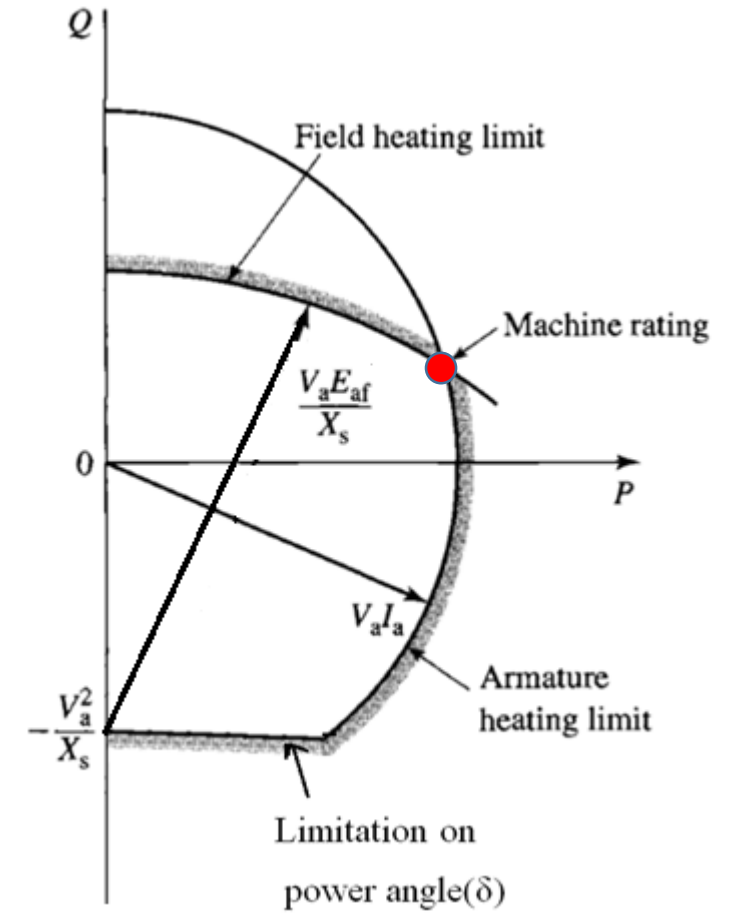
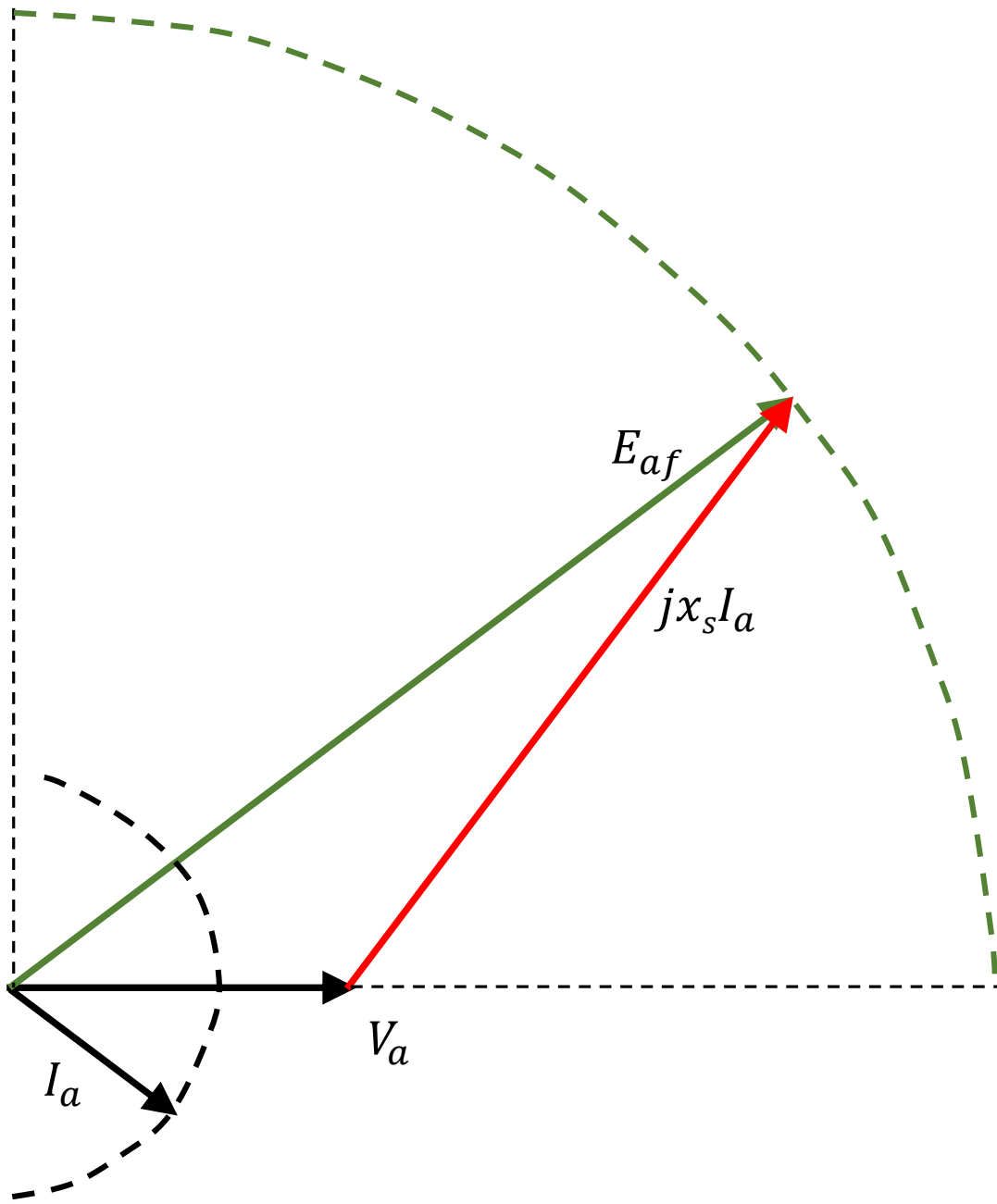
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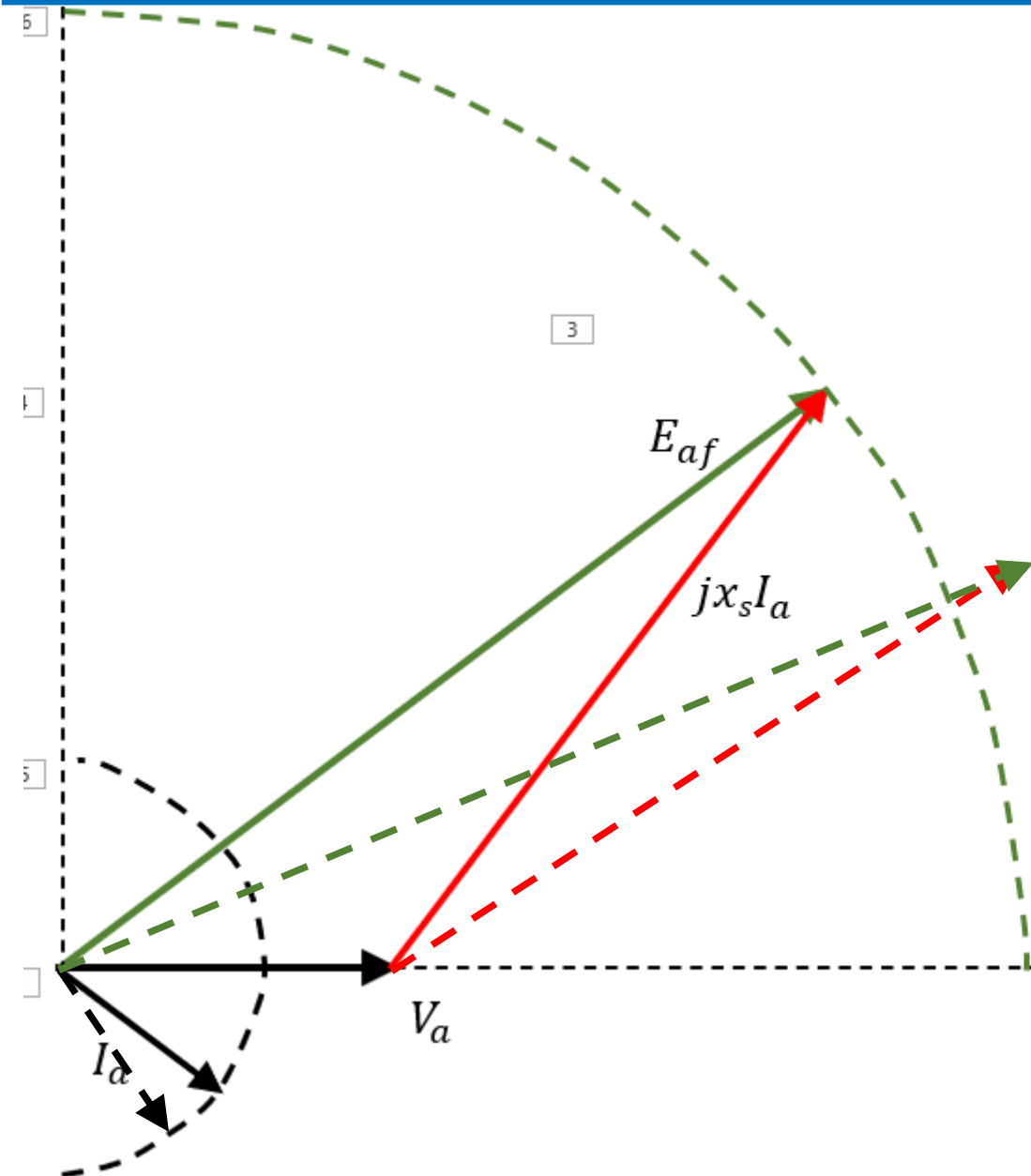


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

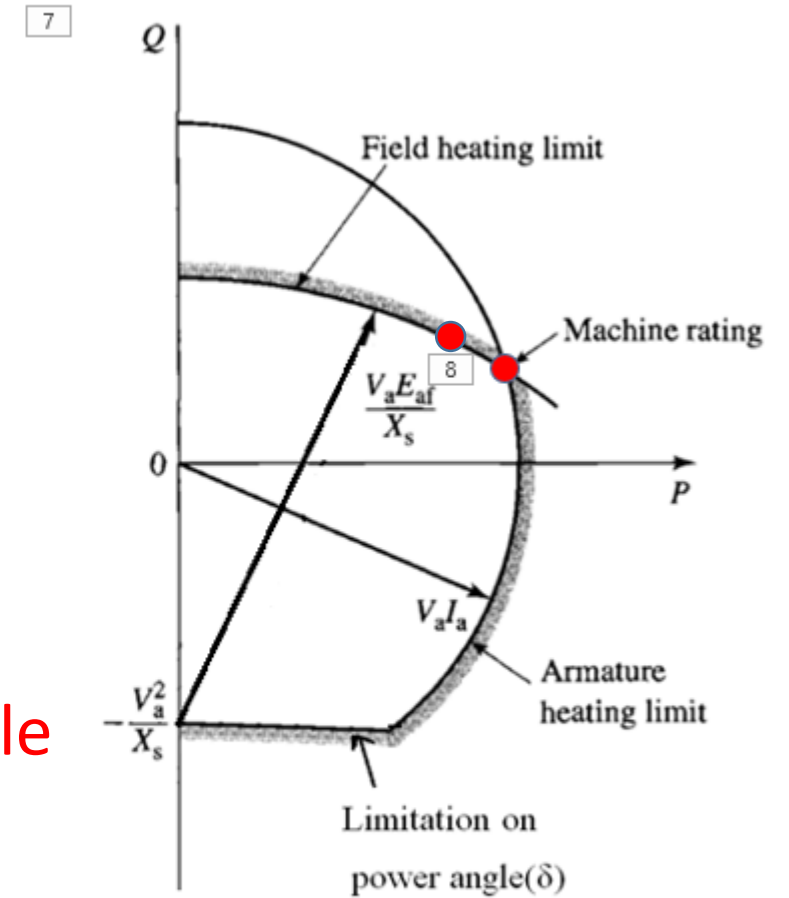
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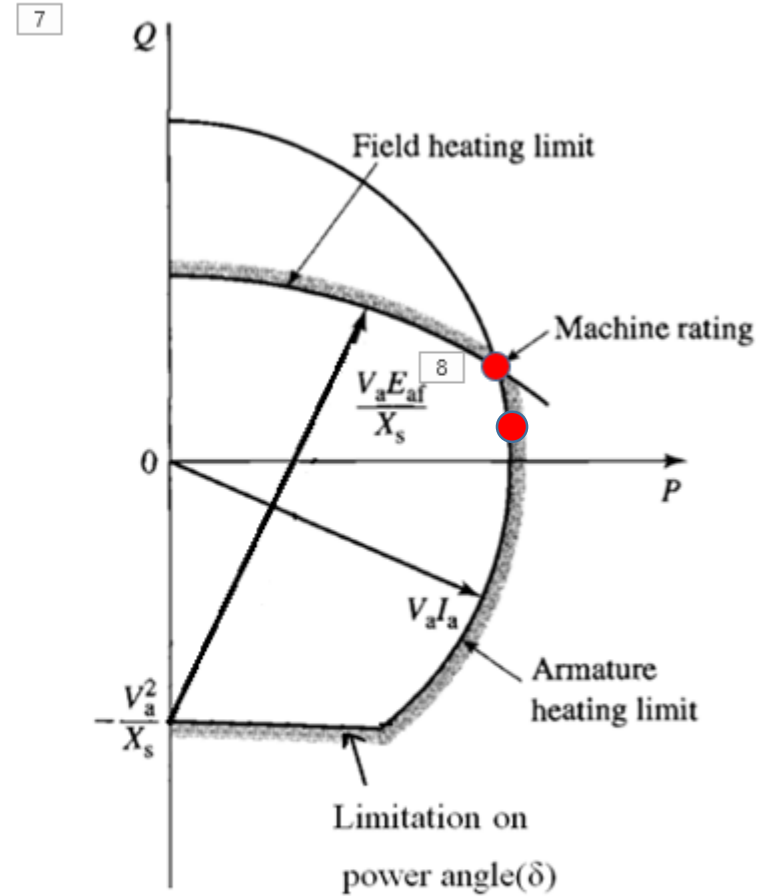
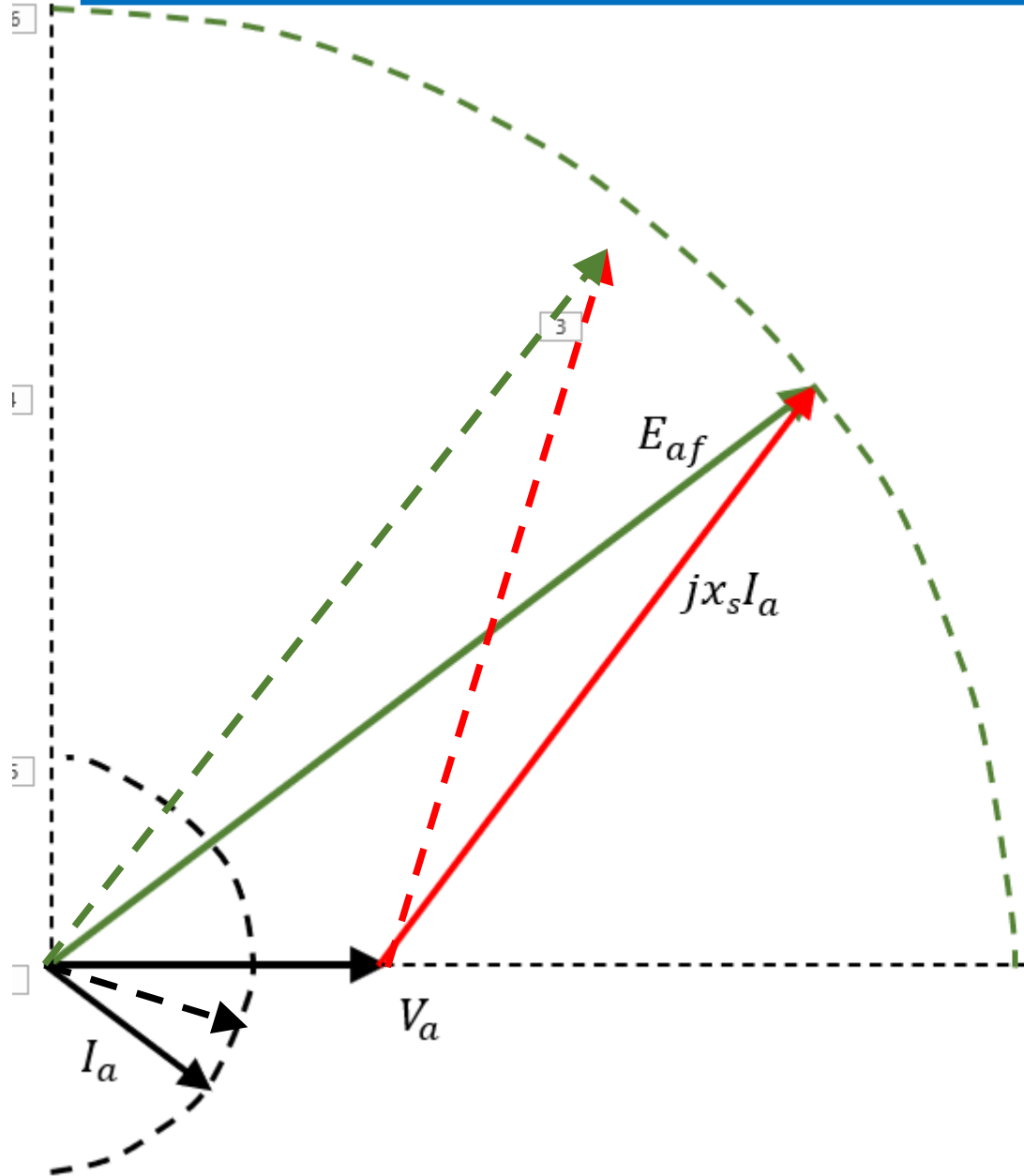
Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2



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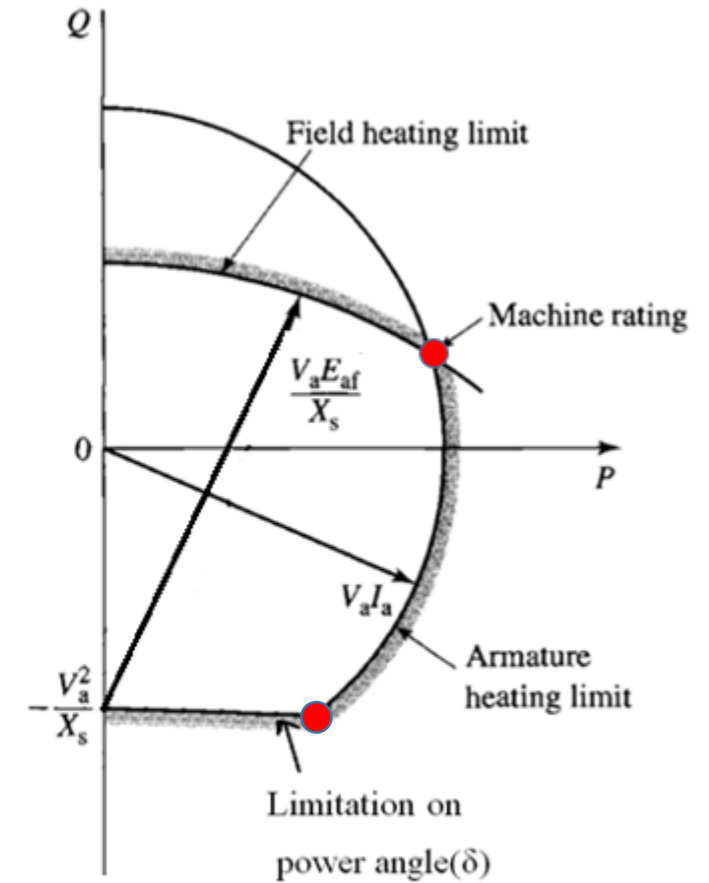
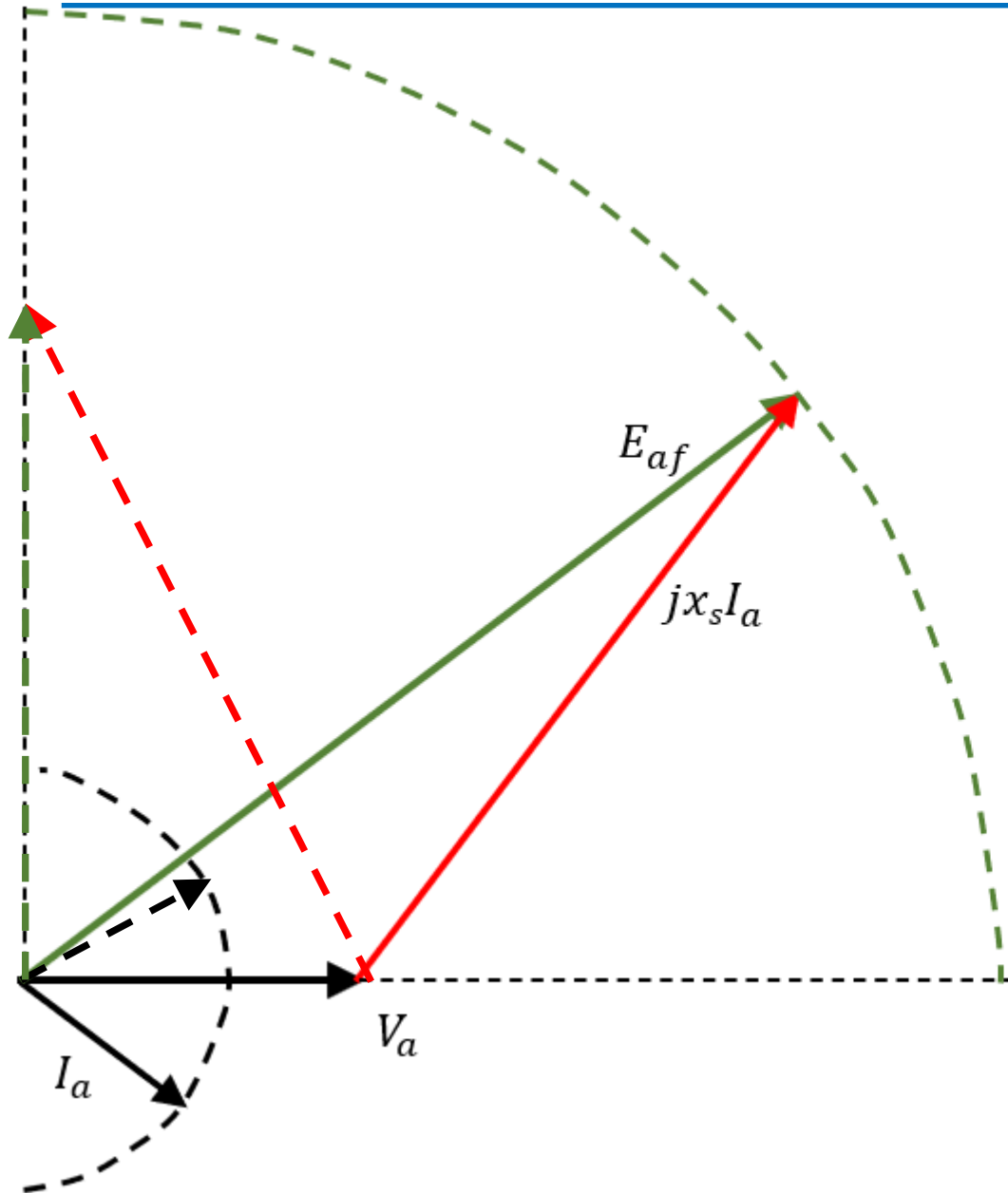


Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2



Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Lecture #4-2



Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2

Let

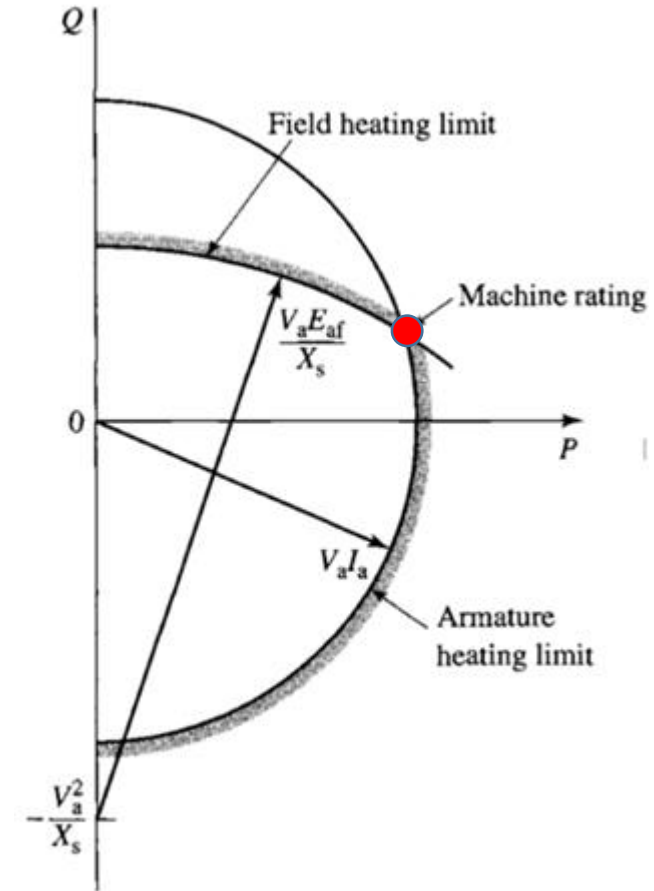
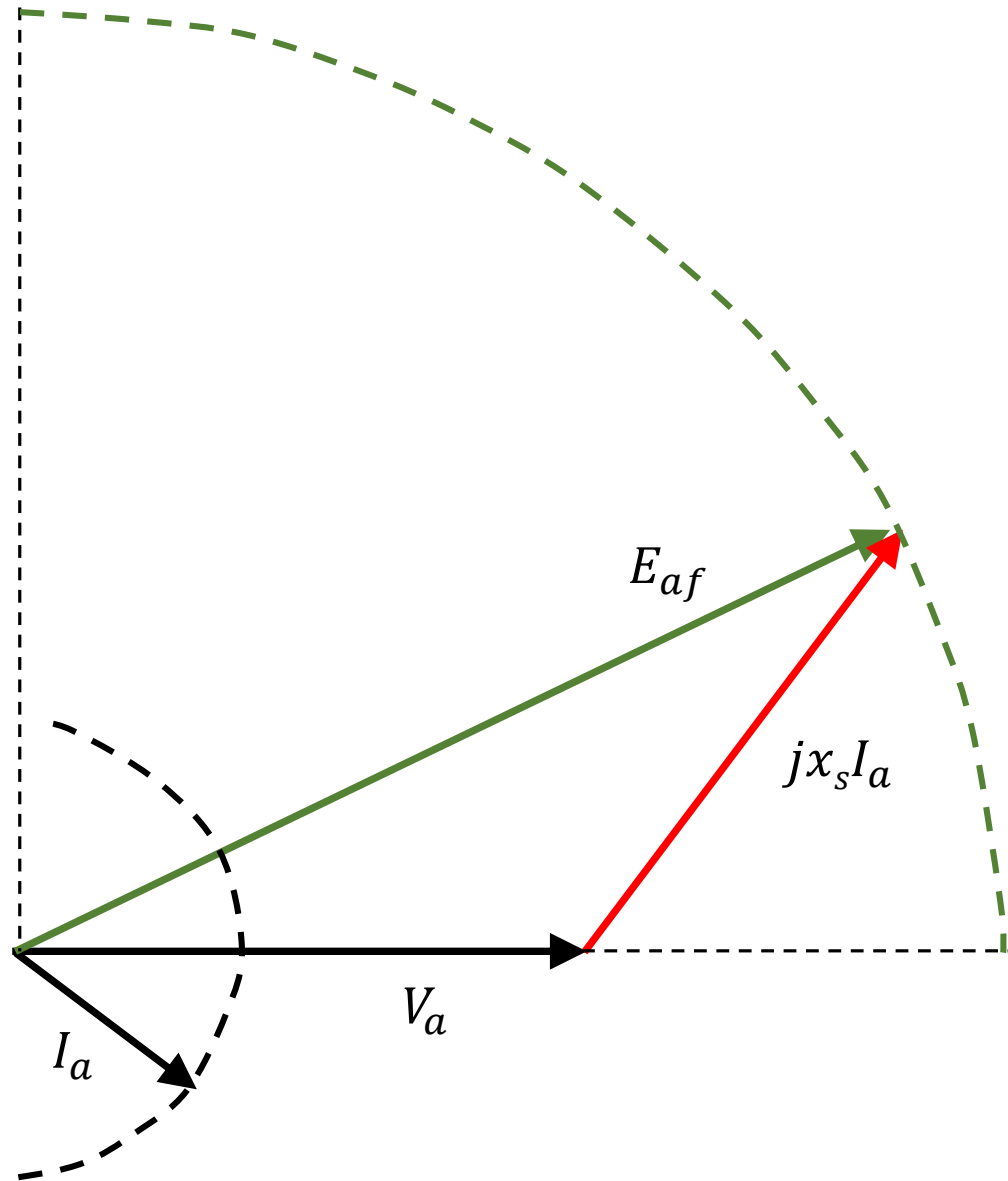
S=165-MVA , V=13-kV, 50-Hz, three-phase, 0.8 power factor lag, $x_s = 0.95 \Omega$, stator resistance negligible.

The generator is operating at rated terminal voltage and its rated load. So:

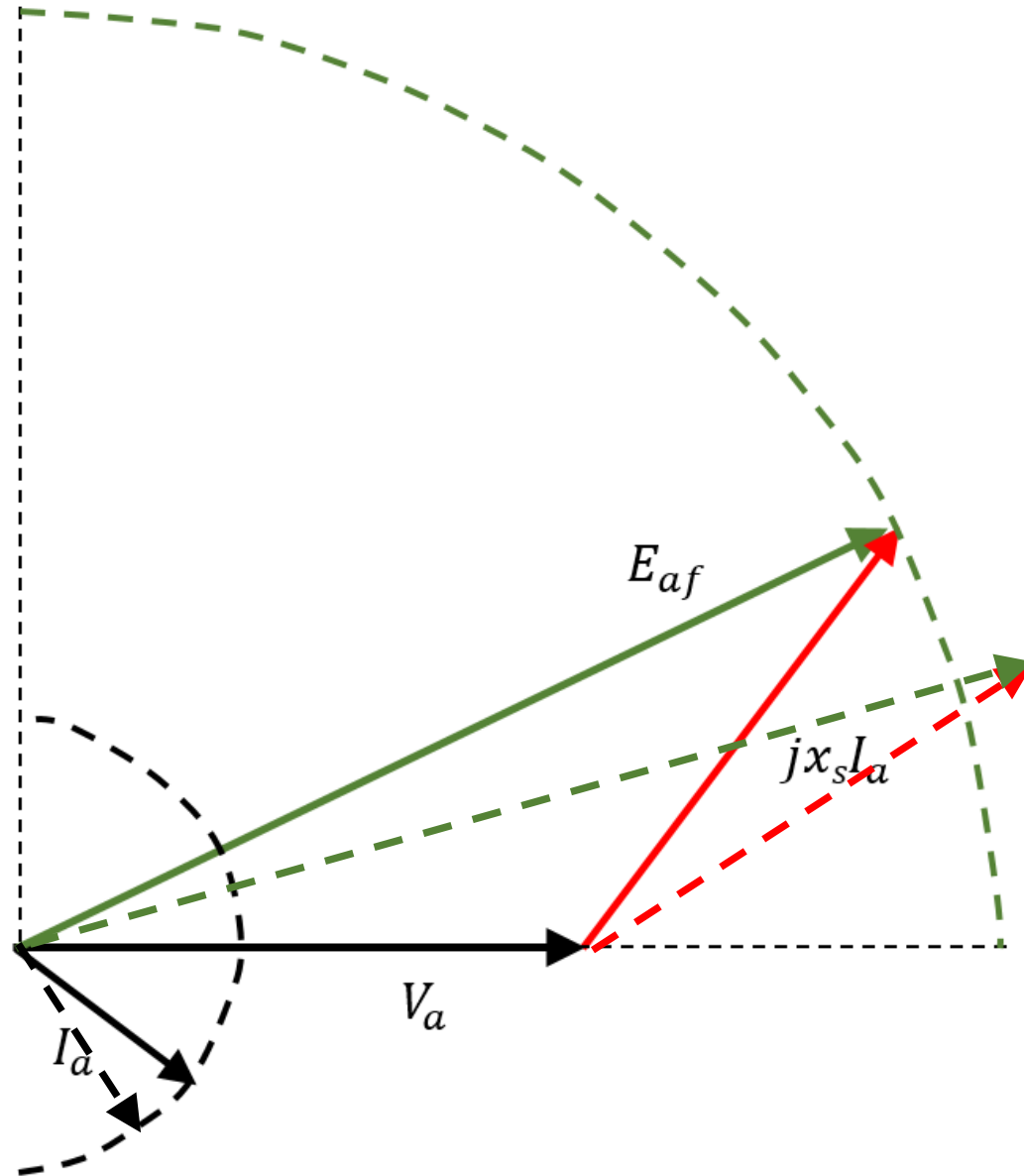
$$V_a = \frac{13000}{\sqrt{3}} = 7506 \angle 0^\circ, \quad I_a = \frac{165000000}{\sqrt{3}13000} \angle -\cos^{-1}(0.8) = 7328 \angle -37^\circ, \quad x_s = 1.5 \Omega,$$

$$E_{af} = V_a + R_a I_a + j x_s I_a = 7506 \angle 0^\circ + 0.95 \angle 90^\circ (7328 \angle -37^\circ)$$

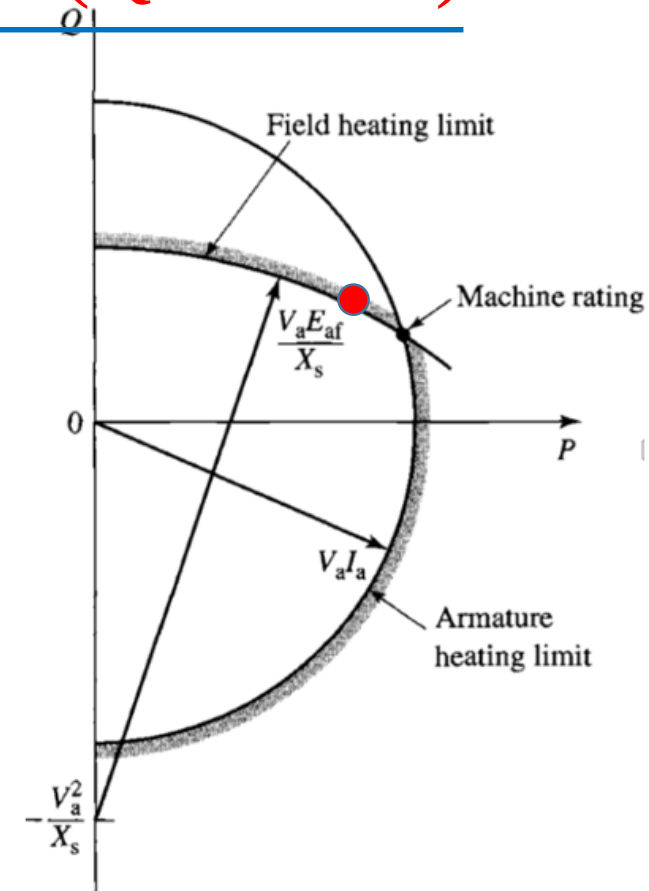
Steady-State Operating Characteristic-Capacity curves (Q-P curve) Lecture #4-2



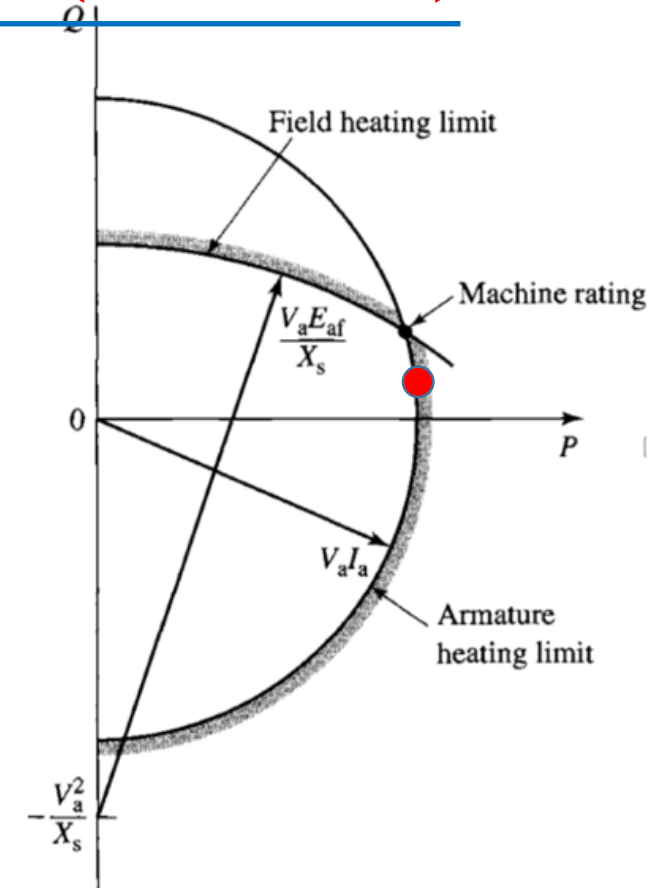
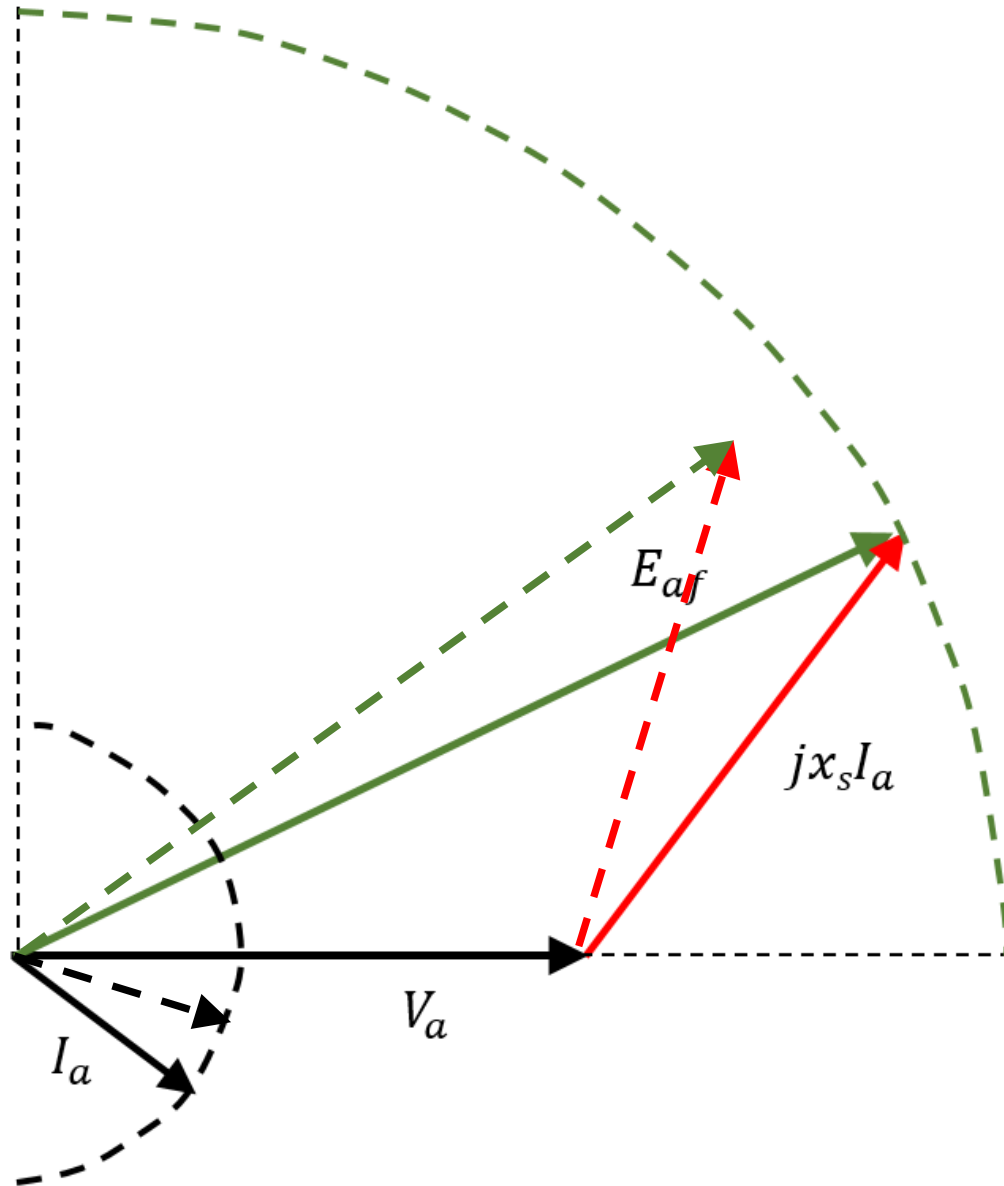
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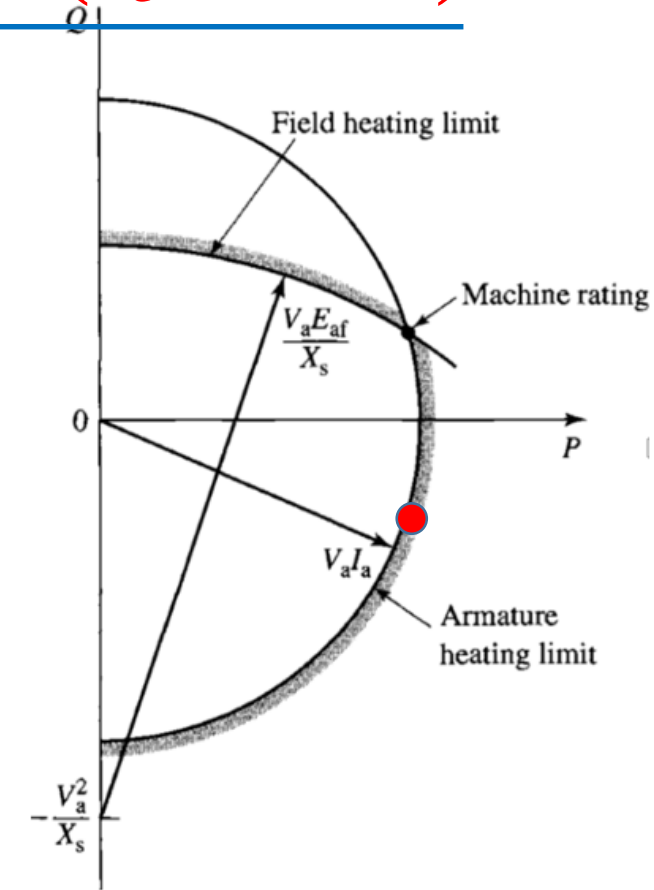
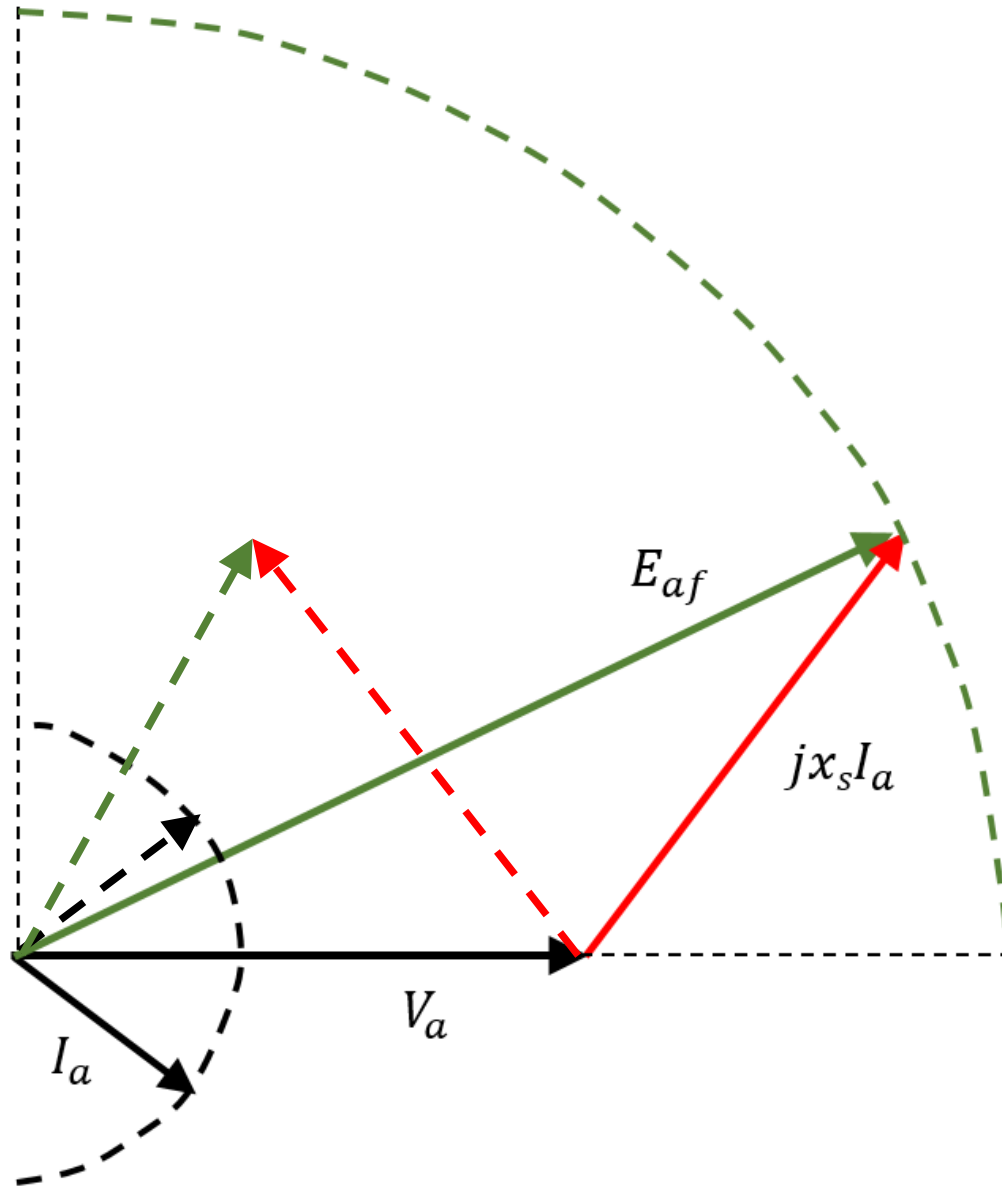
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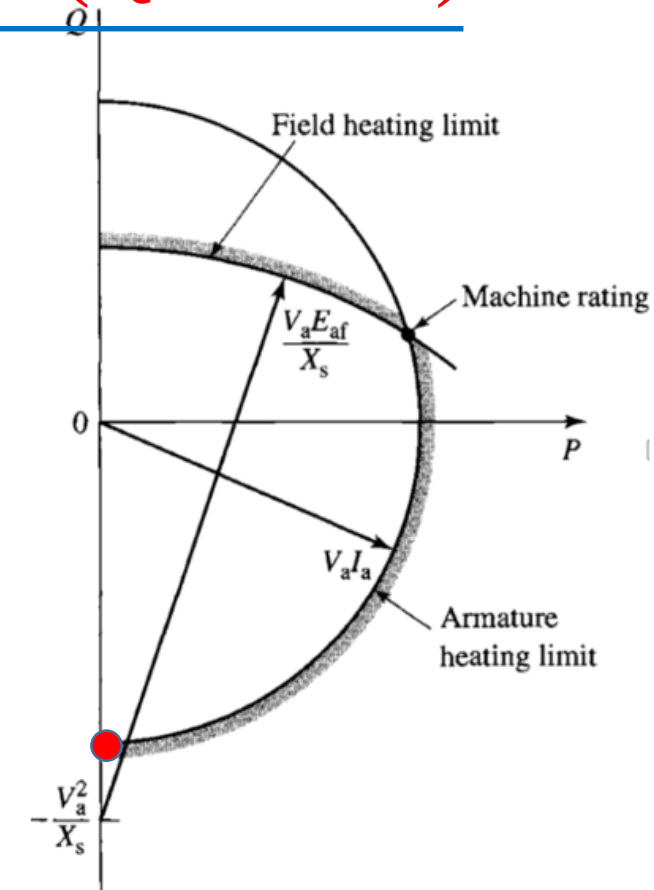
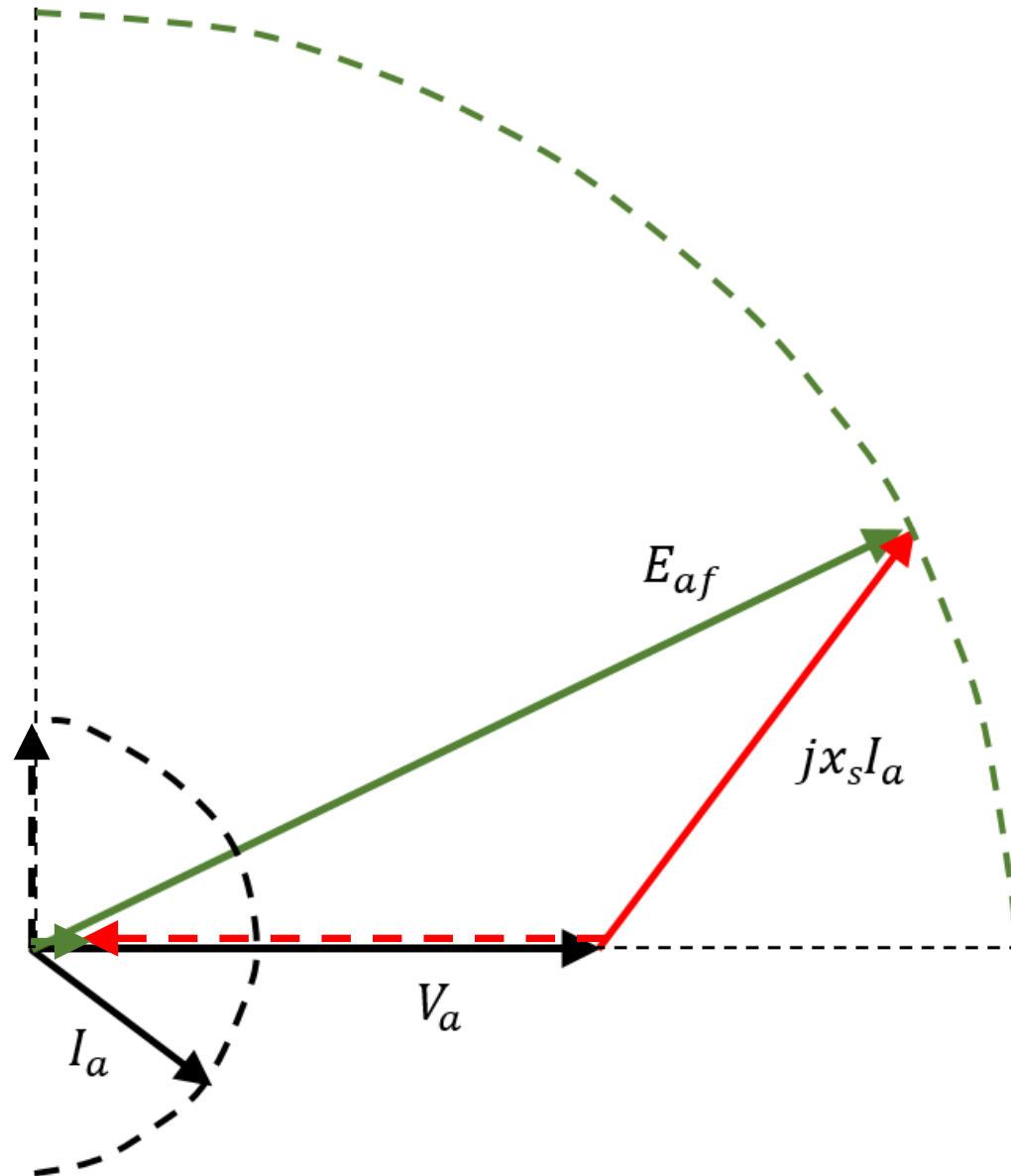
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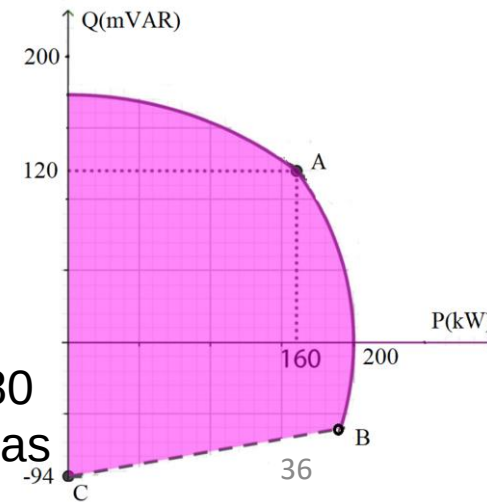


Steady-State Operating Characteristic-Capacity curves (Q-P curve)

Exercise 5: Consider a 200 MVA, 15750 V three-phase Ansaldo generator with negligible armature resistance and a synchronous reactance of 2.64 ohms per phase, connected to an infinite bus with a rated voltage. The generator operates at its rated load with a power factor of 0.8 lagging.

Draw Figures 1 through 5 as follows:

- Figure 1** - On the P-Q plane, shade the area where the generator current is less than or equal to the rated value in blue. (Note that in this case, we are talking about the generator, and consider neither the power angle (δ) limitation nor the permissible field current.)
- Figure 2** - Overlay Figure 2 on Figure 1. On the P-Q plane, shade the area where the field current is less than or equal to the rated value in red. (Note that in this case, we are talking about the generator, and consider neither the power angle (δ) limitation nor the permissible armature current.)
- Figure 3** - Overlay Figure 3 on Figure 2. On the P-Q plane, shade the area where the power angle is less than or equal to 90 degrees in green. (Note that in this case, we are talking about the generator, and consider neither the permissible field current nor the permissible armature current.)
- Figure 4** - Overlay Figure 4 on Figure 2. On the P-Q plane, shade the area where the power angle is less than or equal to 80 degrees in green. (Note that in this case, we are talking about the generator, and consider neither the permissible field current nor the permissible armature current.)
- Figure 5** - On the P-Q plane, shade the area where the power angle is less than or equal to 80 degrees, and both armature and field currents are within permissible limits. This curve is known as the load ability curve.



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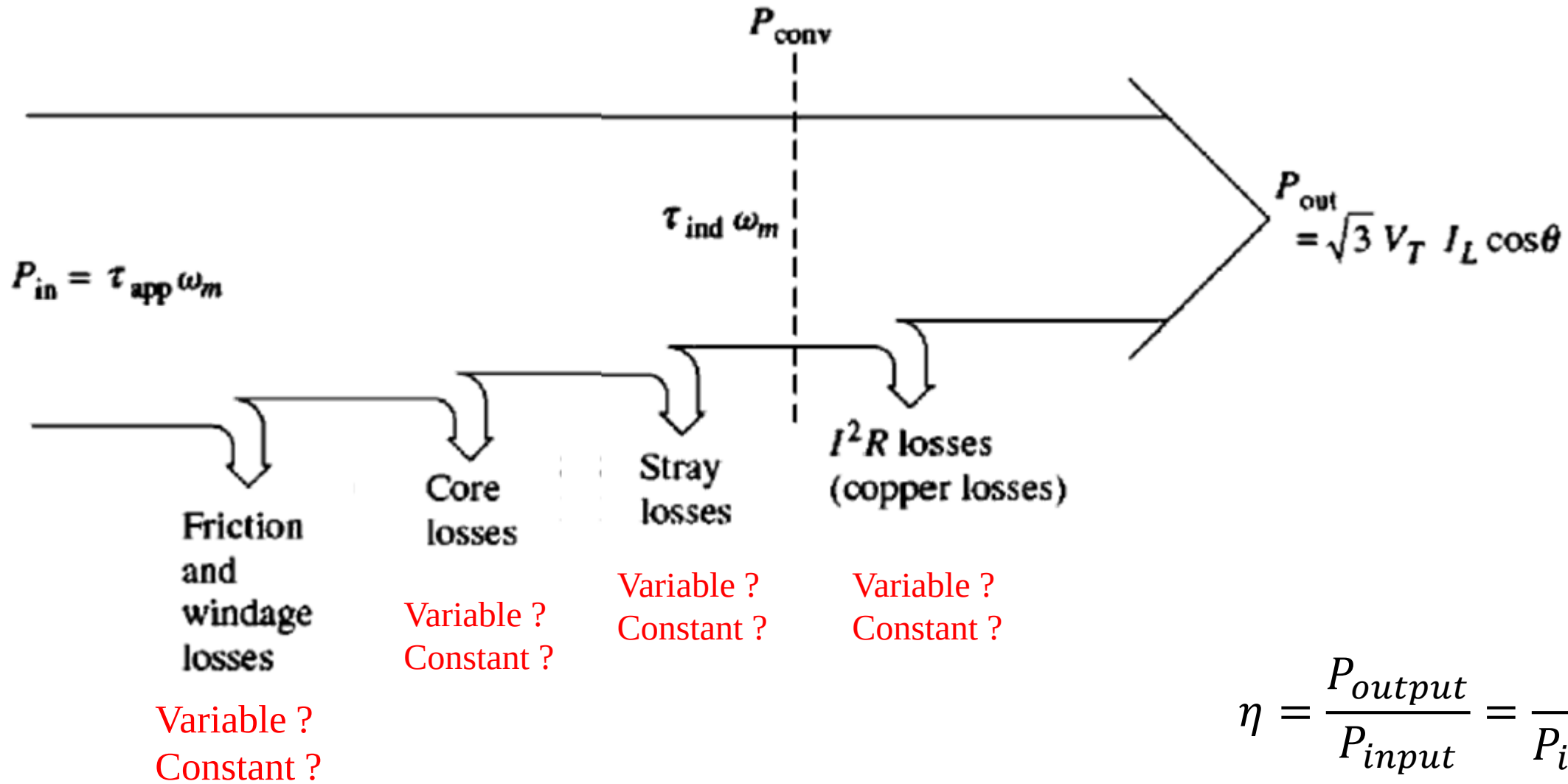
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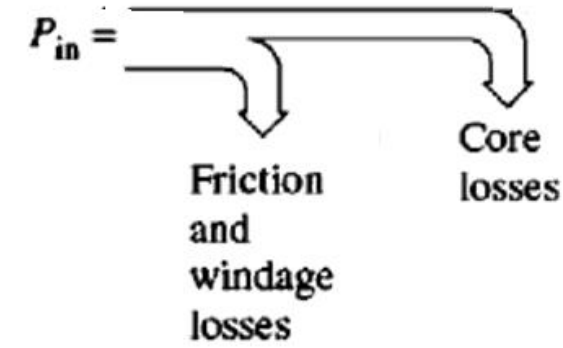
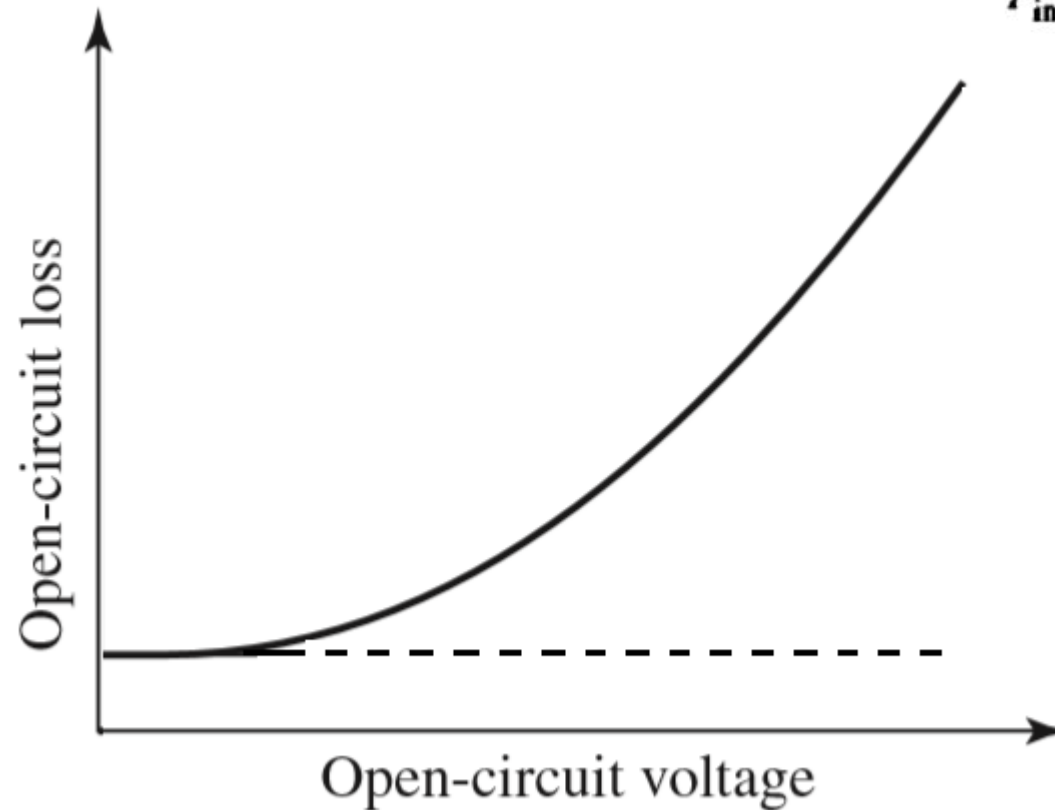
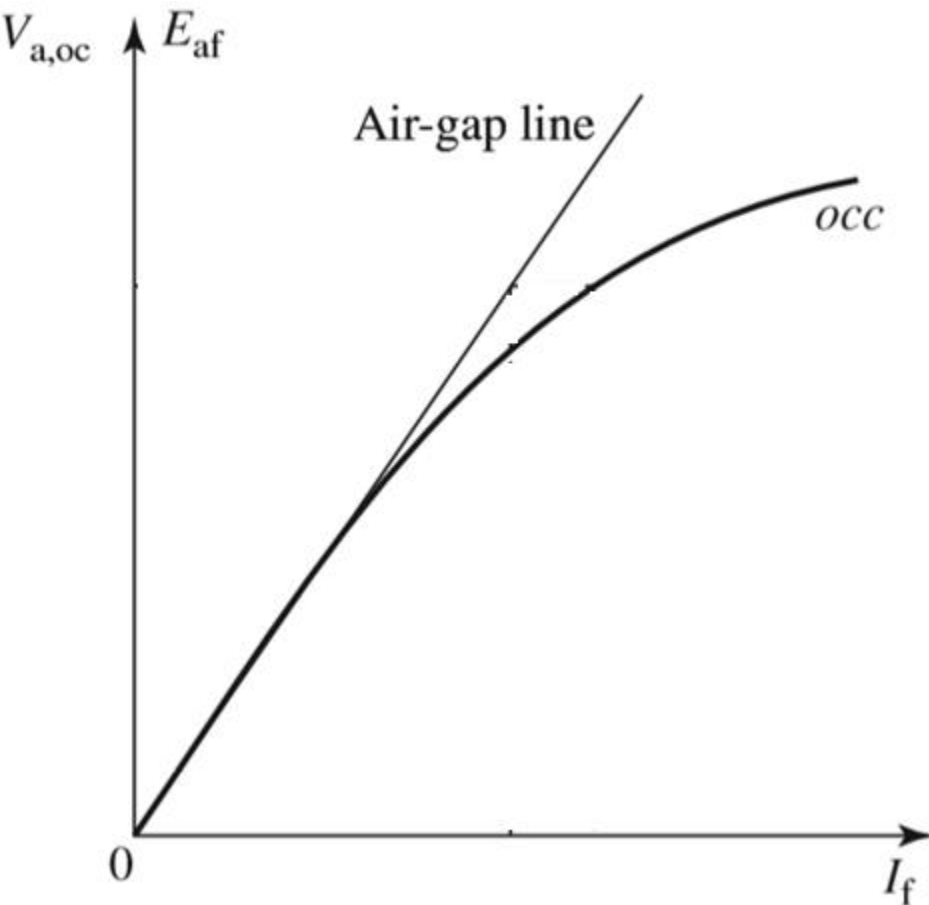
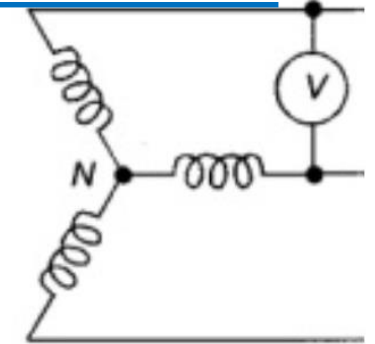
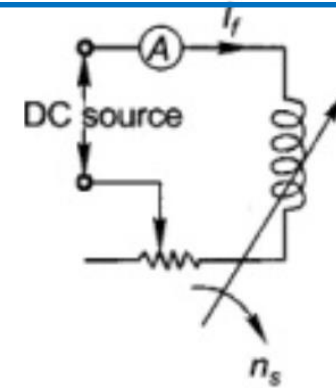
Open- and Short-Circuit Characteristics



$$\eta = \frac{P_{output}}{P_{input}} = \frac{P_{out}}{P_{in} + r_f I_f^2}$$

Open- and Short-Circuit Characteristics

Field current I_f	...	...	...	...
Terminal voltage V_{oc}	...	...	...	...
Input Mechanical Power (Core losses+Friction)	...	...	...	...



Example 1: The following data are taken from the open-circuit test of a 45-kVA, three-phase, Y-connected, 220-V(line-to-line), six-pole, 60-Hz synchronous generator.

Field current, A	0	1.10	2.20	2.84	3.5
Terminal Voltage(L-L), V	0	101	202	220	230
Input Mechanical power(W)	450	480	345	1000	1100

The generator connected to rated voltage and deliver 45 kW. The rotor current in this situation is 3 A and rotor resistance is 124Ω and generator efficiency this situation is 92%.

In this situation:

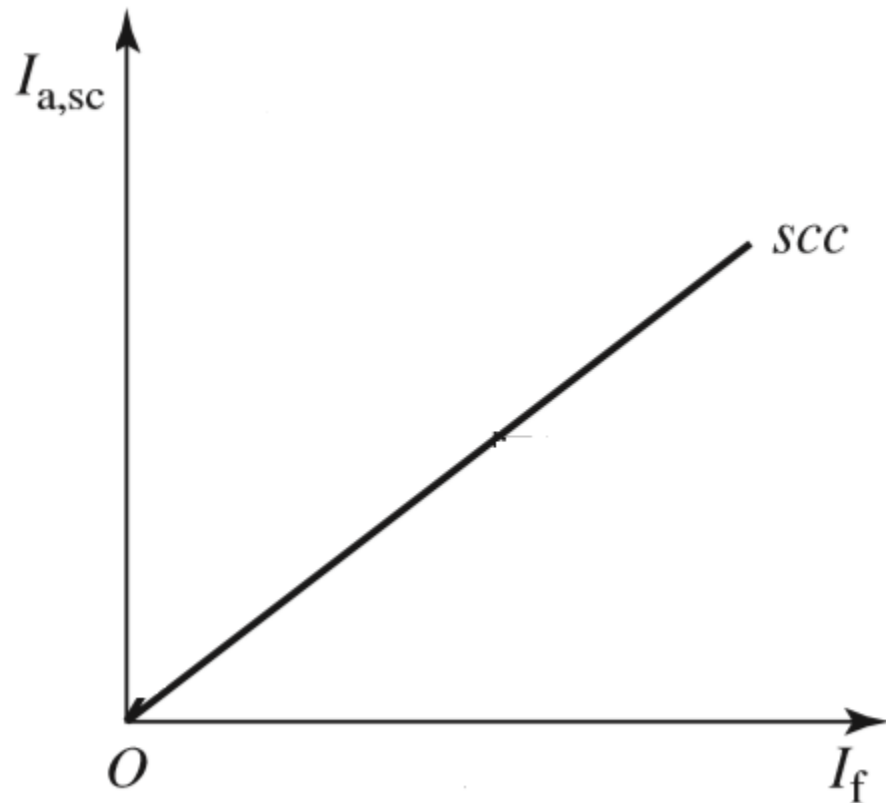
- a- What is the value of Friction and windage losses?
- b- What is the value of core losses?
- c- What is the sum of stray load losses and armature load losses?

Solution a) 450 W b) $1000 - 450 = 550$ W

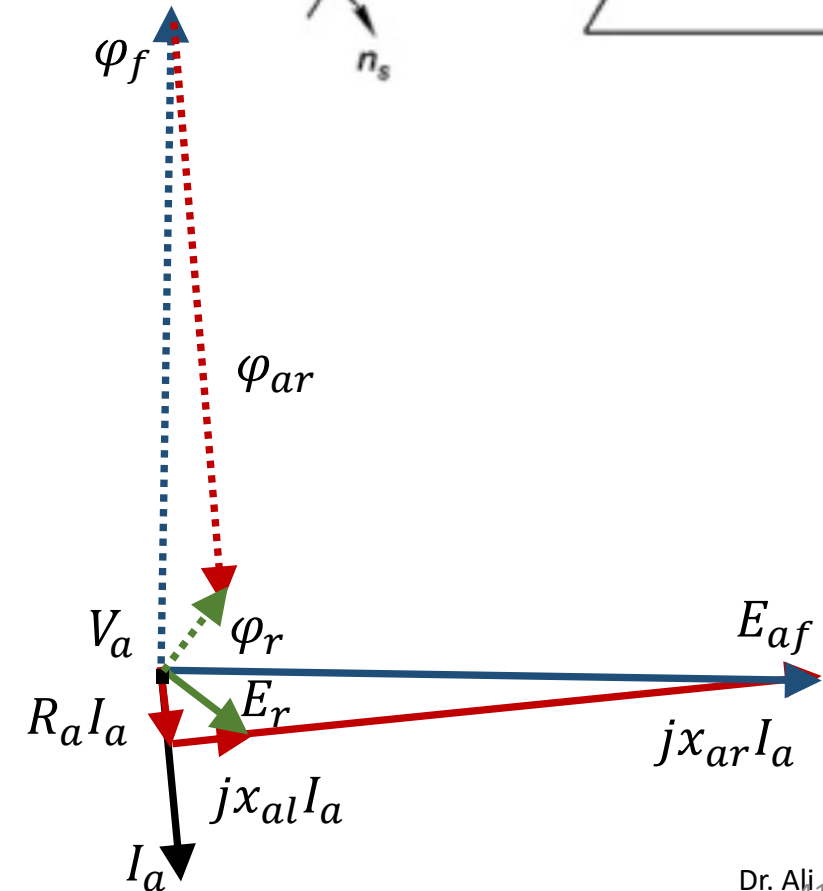
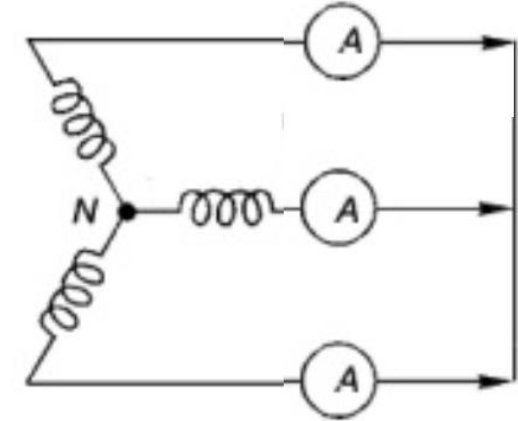
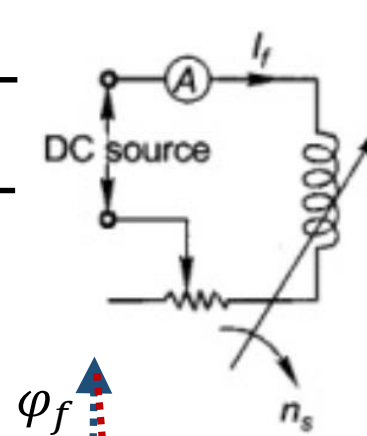
c) $P_{\text{losses}} = 45000 / 0.92 - 45000 = 3913$ $P_{\text{stray}} + P_{\text{custator}} = 3913 - 550 - 450 - 124 * 3^2 = 1797$ W

Open- and Short-Circuit Characteristics

Field current I_f	...	...	...	...
Stator Current I_{sc}	...	...	...	...
Input Mechanical Power (Short-circuit load losses+ Friction)	...	...	...	...

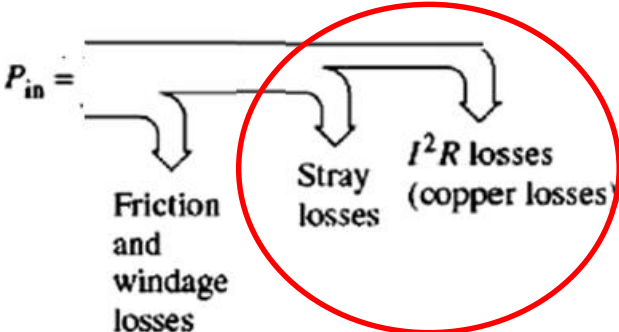
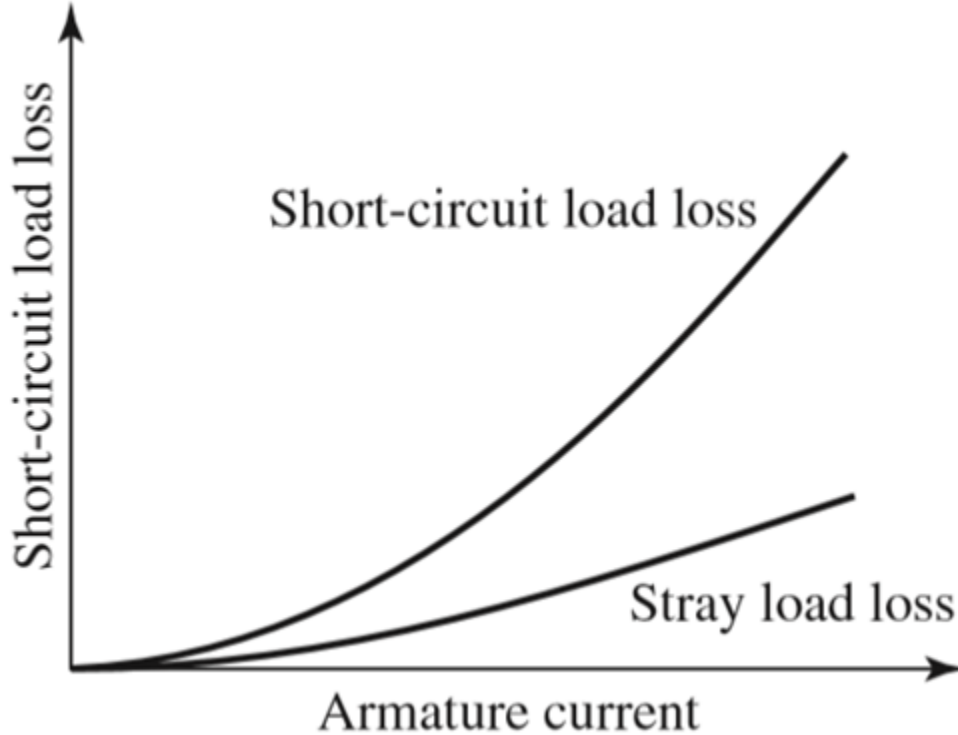
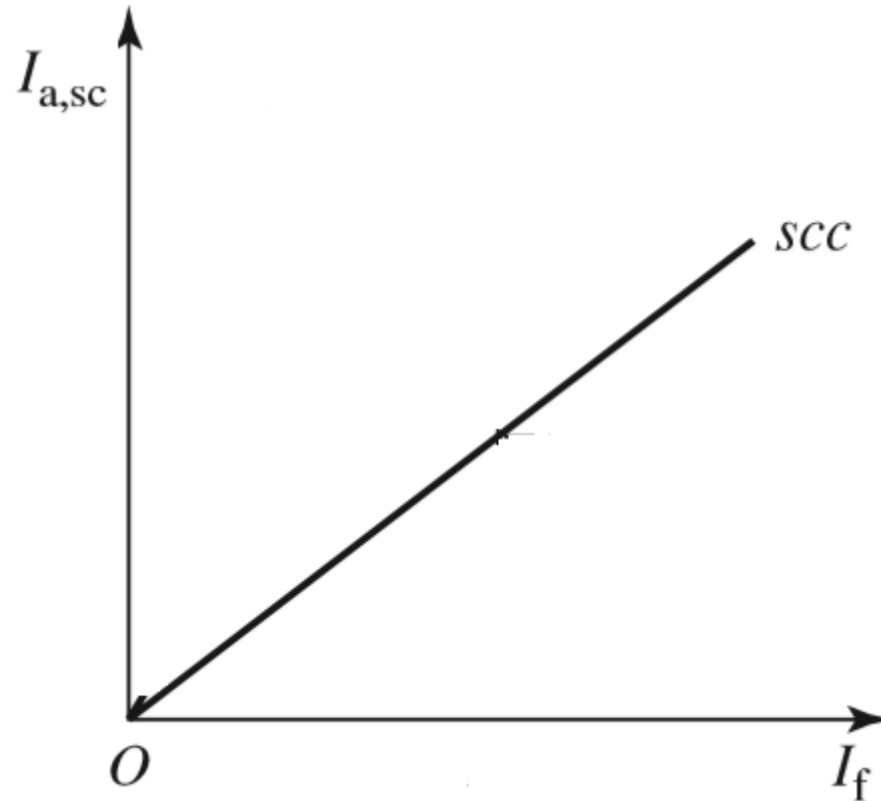
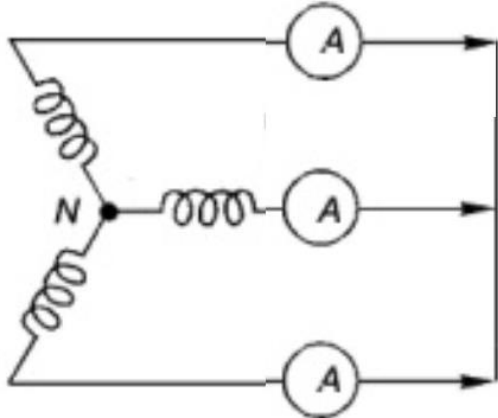
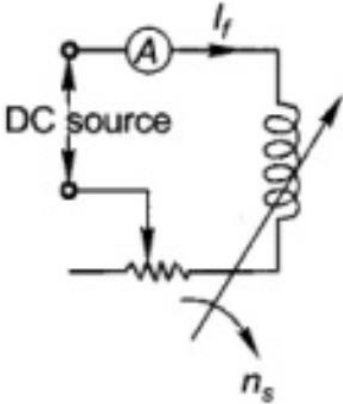


Is it linear??!!



Open- and Short-Circuit Characteristics

Field current I_f	...	...	...	...
Stator Current I_{sc}	...	...	...	...
Input Power (Short-circuit load losses+ Friction)	...	...	...	...



Short-Circuit Load Loss

$$R_{aeff} = \frac{\text{Short cicuit load losses}}{3I_a^2}$$

Example 2 : The following data are taken from the open-short-circuit tests of a 45-kVA, three-phase, Y-connected, 220-V(line-to-line), six-pole, 60-Hz synchronous generator.

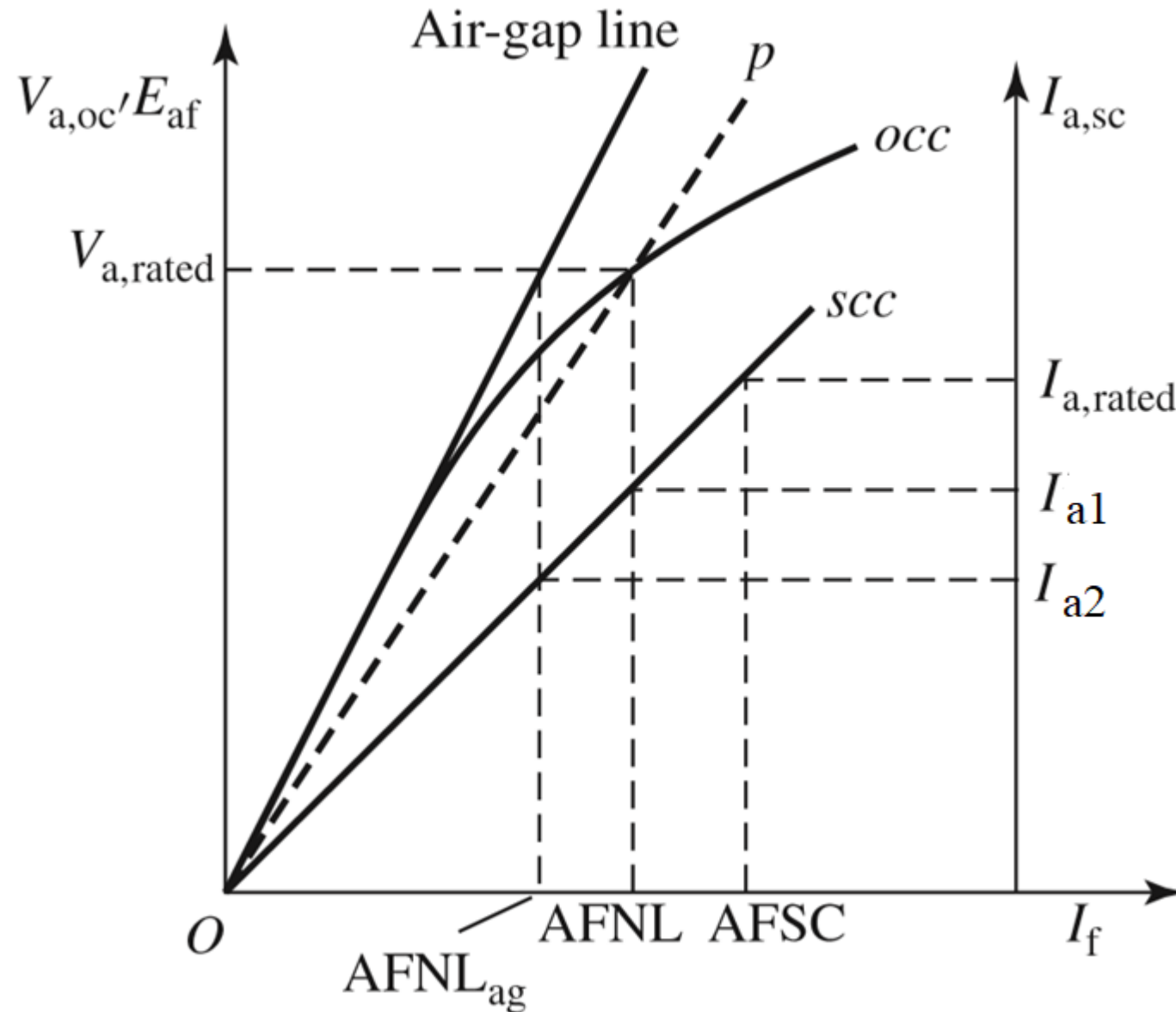
Field current, A	0	1.10	2.20	2.84	3.5
Terminal Voltage(L-L), V	0	101	202	220	230
Input Mechanical power(W)	450	480	345	1000	1100
Field current, A	0	2.20	2.84		
Armature current, A	0	118	152		
Input Mechanical power(W)	450	2250	3430		

The generator connected to rated voltage and deliver its rated power at unity power factor. The rotor current in this situation is 3 A and rotor resistance is 124Ω . Derive the generator efficiency in this situation and R_{aeff} .

Solution: $P_{\text{friction and windage}} = 450 \text{ W}$ $P_{\text{core}} = 1000 - 450 = 550 \text{ W}$ $P_{\text{stray+custator}} = 2250 - 450 = 1800 \text{ W}$

$$P_{\text{curotor}} = 124 * 3^2 = 1116 \text{ W} \quad \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \% \frac{45000}{45000 + 450 + 550 + 1800 + 1116} \times 100 = \%92 \quad R_{aeff} = \frac{1800/3}{43118^2}$$

Open- and Short-Circuit Characteristics



AFNL (Amperes Field No Load),

AFNL_{ag}

AFSC (Amperes Field Short Circuit)

$$X_{s,u} = \frac{V_a(\text{air gap})}{\sqrt{3}I_a(\text{corresponding to } V_a)} \quad \Omega$$

$$X_s = \frac{V_{a,\text{rated}}}{\sqrt{3}I_{a1}} \quad \Omega$$

$$X_s = \frac{1}{I_{a1}/I_{a,\text{rated}}} = \frac{AFSC}{AFNL} \text{ per unit}$$

$$SCR = \frac{AFNL}{AFSC} \quad X_s = \frac{1}{SCR} \text{ p.u.}$$

Example 3: The following data are taken from the open- and short-circuit characteristics of a 45-kVA, three-phase, Y-connected, 220-V(line-to-line), six-pole, 60-Hz synchronous machine. From the open-circuit characteristic:

Field current=2.84A Line-to-line voltage=220V

and from the air-gap line:

Field current=2.20A Line-to-line voltage=202V

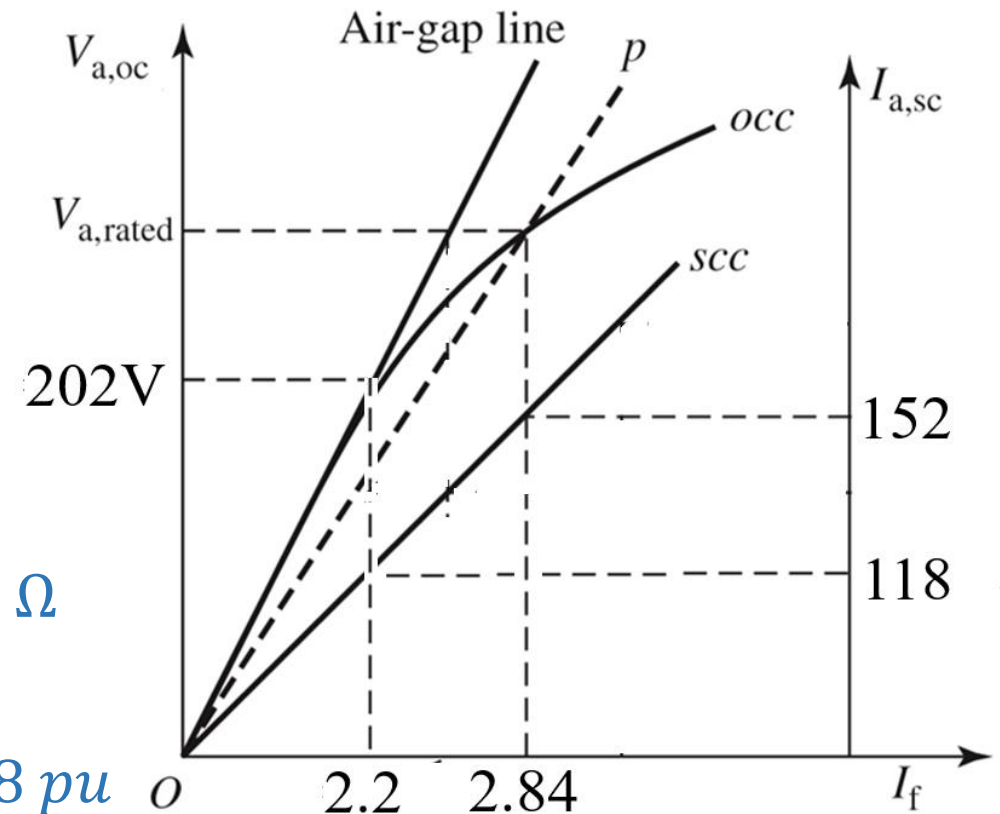
From the short-circuit characteristic:

Field current, A	2.20	2.84
Armature current, A	118	152

Compute the unsaturated value of the synchronous reactance and its saturated value at rated voltage, both in ohms per phase and in per unit.

$$X_{s,u} = \frac{V_a(\text{air gap})}{I_a(\text{corresponding to } V_a)} = \frac{202/\sqrt{3}}{118} = 0.988 \, \Omega$$

$$X_{s,u} = \frac{V_a(\text{air gap})}{I_a(\text{corresponding to } V_a)} = \frac{202/220}{1} = 0.918 \, pu$$



Example 3: The following data are taken from the open- and short-circuit characteristics of a 45-kVA, three-phase, Y-connected, 220-V(line-to-line), six-pole, 60-Hz synchronous machine.

From the open-circuit characteristic:

Field current=2.84A Line-to-line voltage=220V

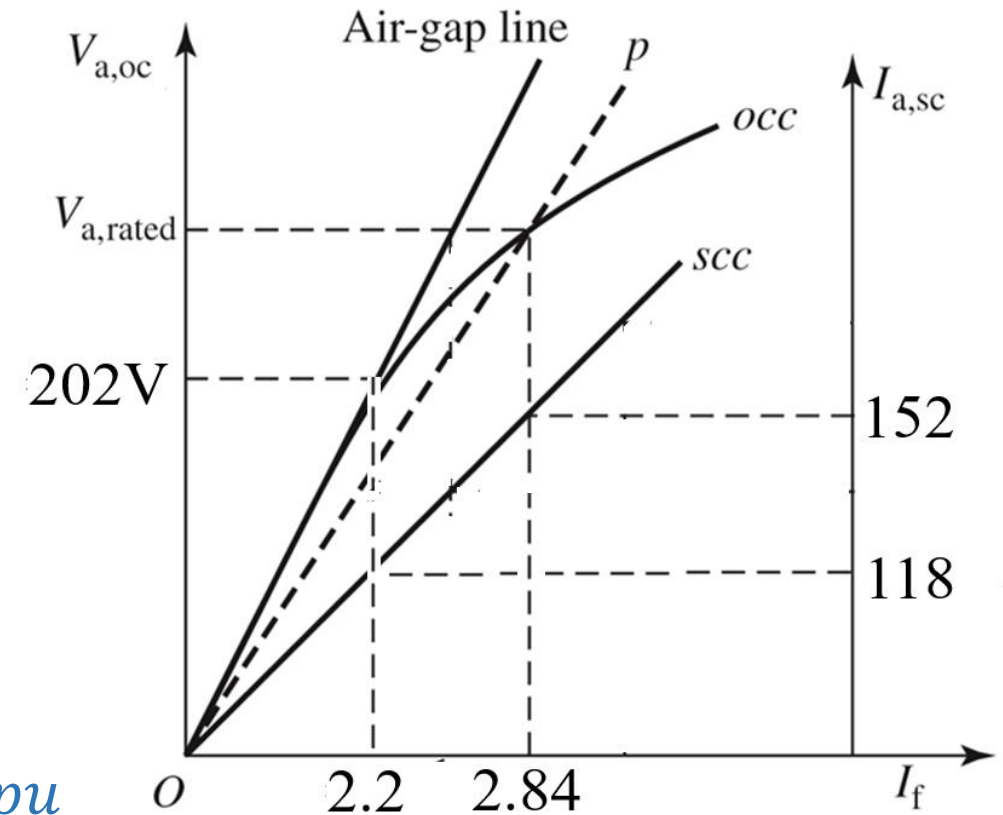
and from the air-gap line:

Field current=2.20A Line-to-line voltage=202V

From the short-circuit characteristic:

Field current, A	2.20	2.84
Armature current, A	118	152

Compute the unsaturated value of the synchronous reactance and its saturated value at rated voltage, both in ohms per phase and in per unit.



$$X_s = \frac{V_{a,rated}}{I_{a1}} = \frac{220/\sqrt{3}}{152} = 0.836 \Omega$$

$$SCR = \frac{AFNL}{AFSC} = \frac{2.84}{2.2} = 1.291 \quad X_s = \frac{1}{SCR} = 0.775 \text{ pu}$$

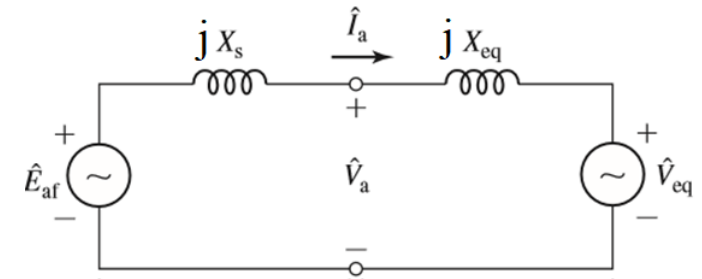
Example 4: Consider a synchronous condenser connected to a power system bus which can be represented by a Thevenin-equivalent voltage V_{eq} (13.75 kV) and series reactance X_{eq} (0.02 p.u.). The synchronous condenser is rated at 75 MVA, 13.8 kV with a synchronous reactance of 0.95 per unit and AFNL = 830 A. Calculate the generator field current required to raise the generator terminal voltage, and thus the local system voltage, to 13.8kV and the reactive power supplied under that operating condition.

$$v_a = v_{eq} + jx_{eq}I_a \quad v_{eq} = \frac{13.75}{13.8} = 0.9964 \text{ pu}$$

$$I_a = \frac{1 \angle 0 - 0.9964 \angle 0}{j0.02} = -j0.181$$

$$E_{af} = 1 \angle 0 + j0.95(-j0.181) = 1.172 \angle 0 \text{ pu}$$

$$I_f = 1.172 * 830 = 973 \text{ A} \quad Q = 0.181 \text{ pu} = 13.6 \text{ MVAR}$$



Exercise 6: A 25-MVA, 11.5 kV synchronous machine is operating as a synchronous condenser, as discussed in Appendix D (section D.4.1). The generator short-circuit ratio is 1.68 and the field current at rated voltage, no load is 420 A. Assume the generator to be connected directly to an 11.5 kV source.

- a. What is the saturated synchronous reactance of the generator in per unit and in ohms per phase?

The generator field current is adjusted to 150 A.

- b. Draw a phasor diagram, indicating the terminal voltage, internal voltage, and armature current.
- c. Calculate the armature current magnitude (per unit and amperes) and its relative phase angle with respect to the terminal voltage.
- d. Under these conditions, does the synchronous condenser appear inductive or capacitive to the 11.5 kV system?
- e. Repeat parts (b) through (d) for a field current of 700 A.

Electrical Machine III Syllabus

1. Introduction

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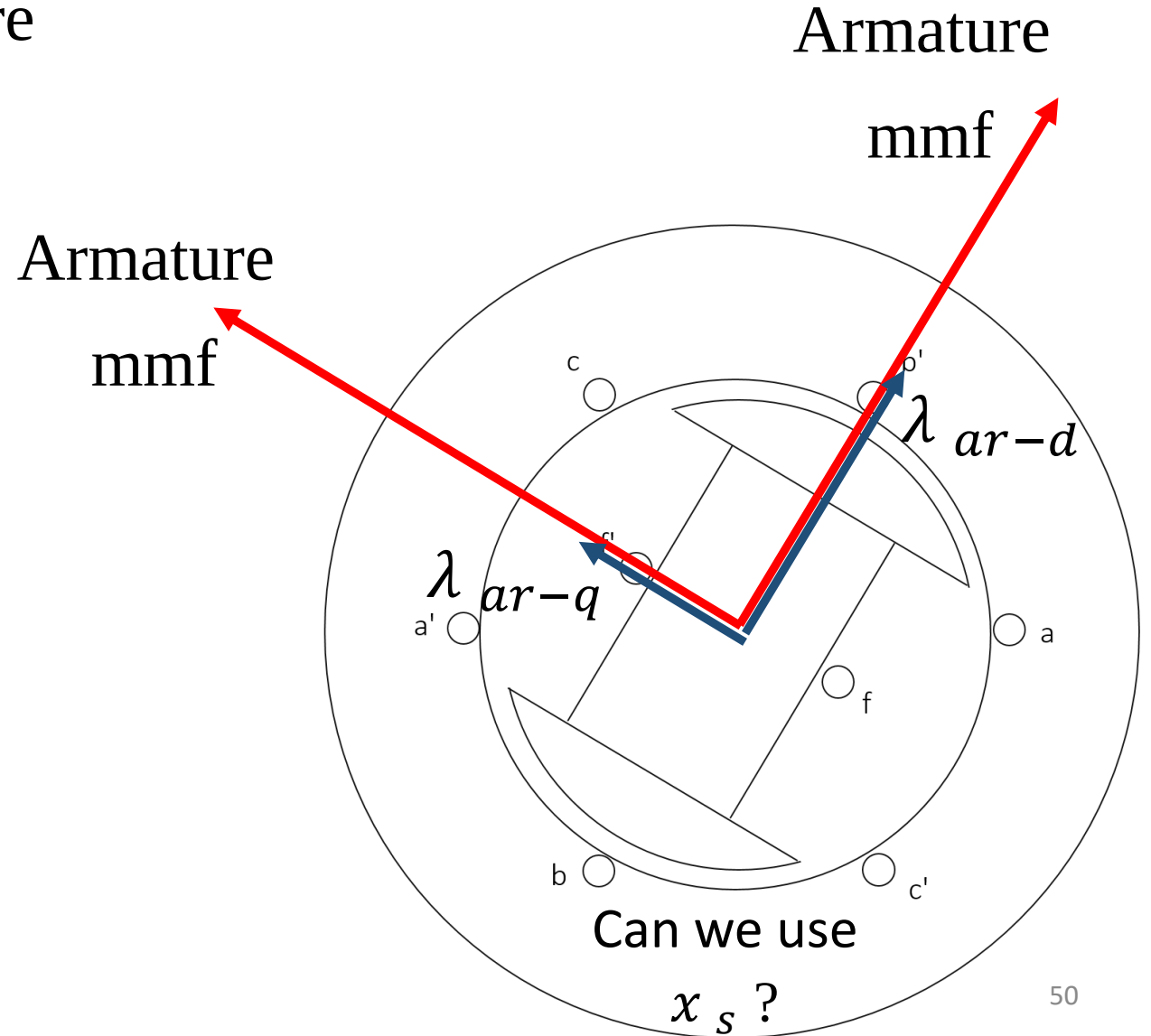
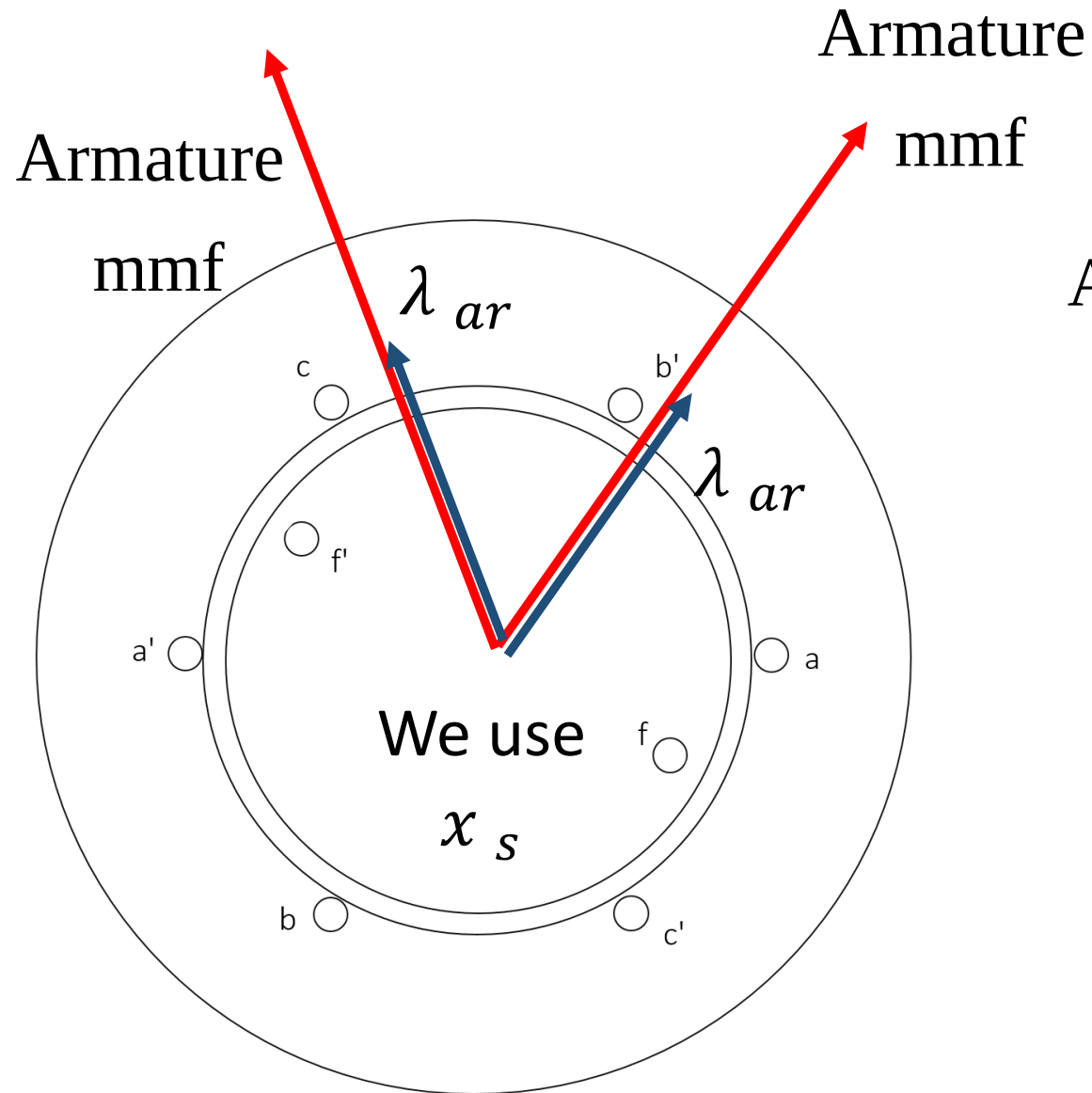
3. Generation of Sinusoidal Voltage in a Synchronous Generator

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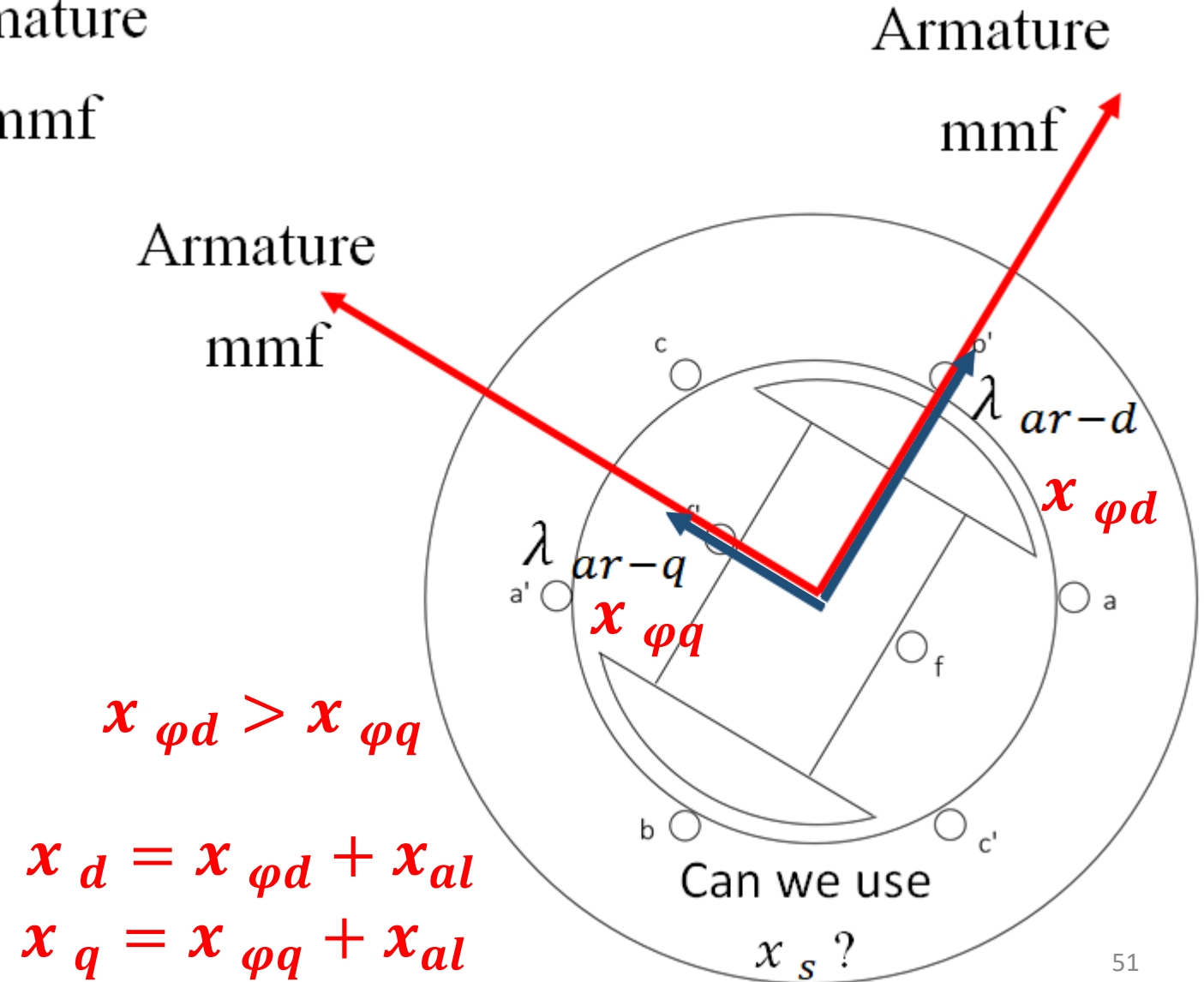
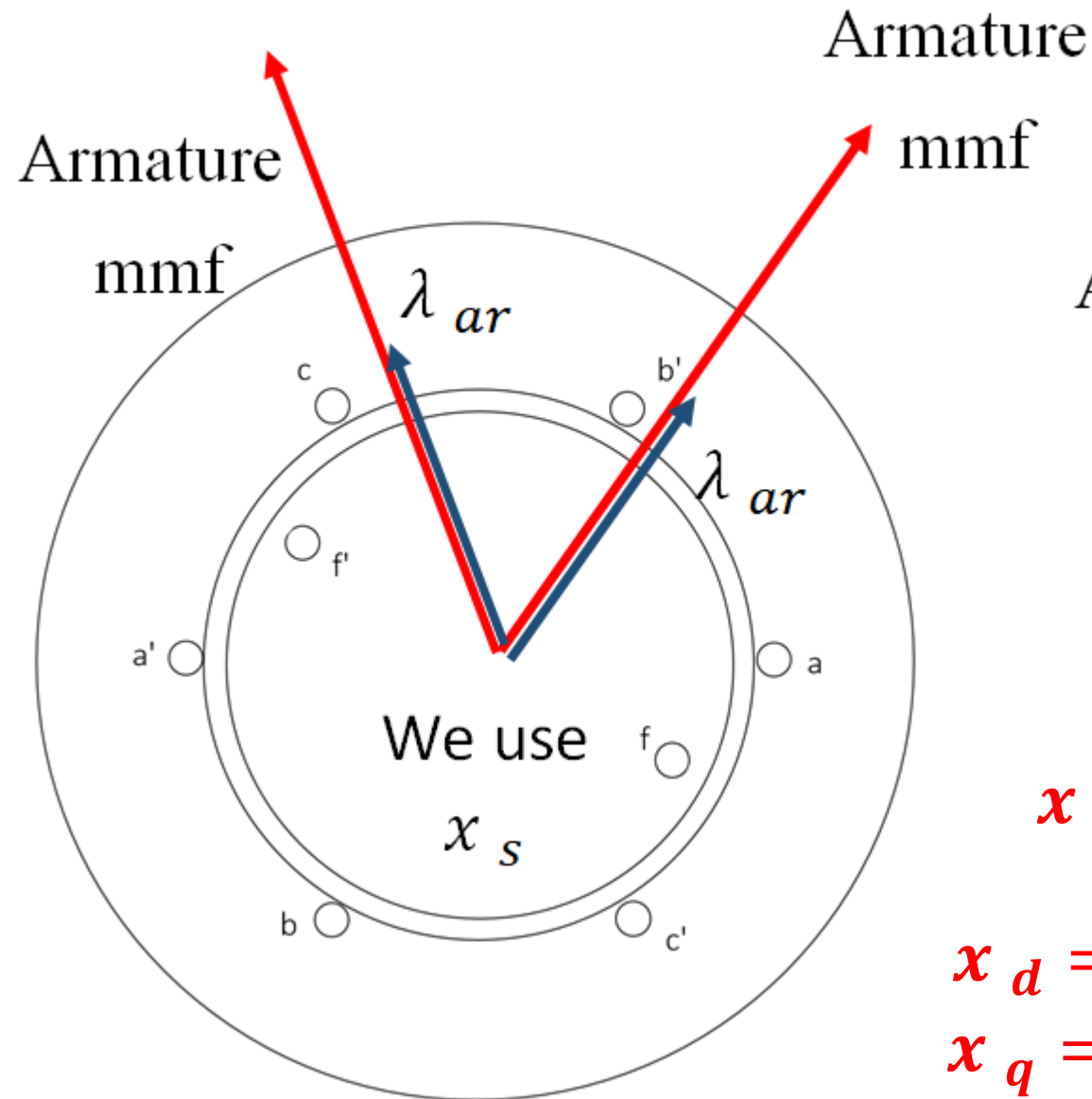
4. Synchronous-Machine Analyses

- Synchronous-Machine Inductances; Equivalent Circuits
- Steady-State Power-Angle Characteristic
- Steady-State Operating Characteristic
- Open- and Short-Circuit Characteristics
- Effects of Salient Poles; Introduction to Direct and Quadrature-Axis Theory
- Steady-State Power-Angle Characteristic of Salient-Pole Machines
- Permanent Magnet Synchronous Machine
- Transient Behavior of Synchronous Machine.

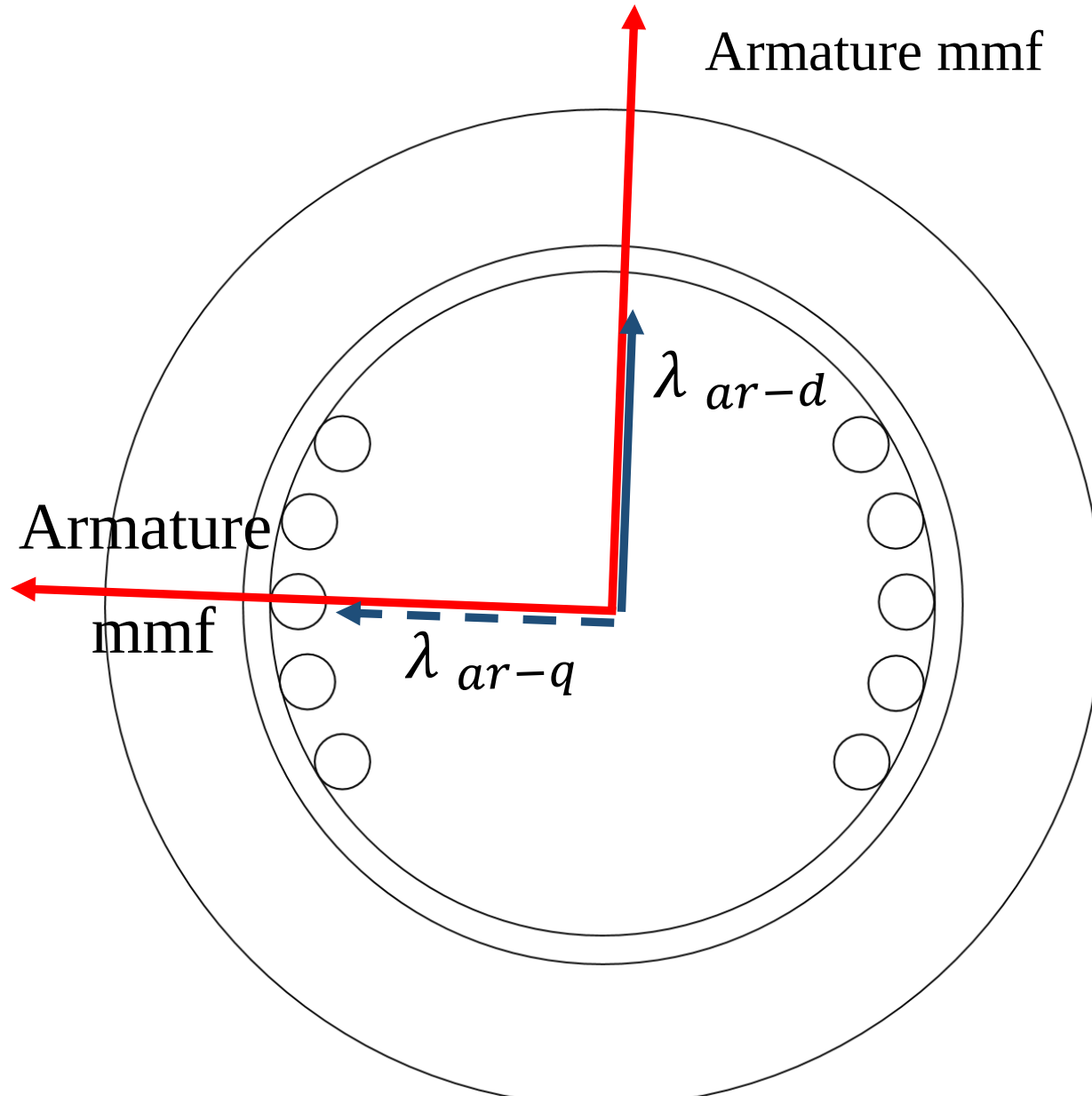
Is the armature reaction different in salient-pole and cylindrical-pole machines?



Is the armature reaction different in salient-pole and cylindrical-pole machines?



Is the direct axis reactance different from quadrature axis reactance?

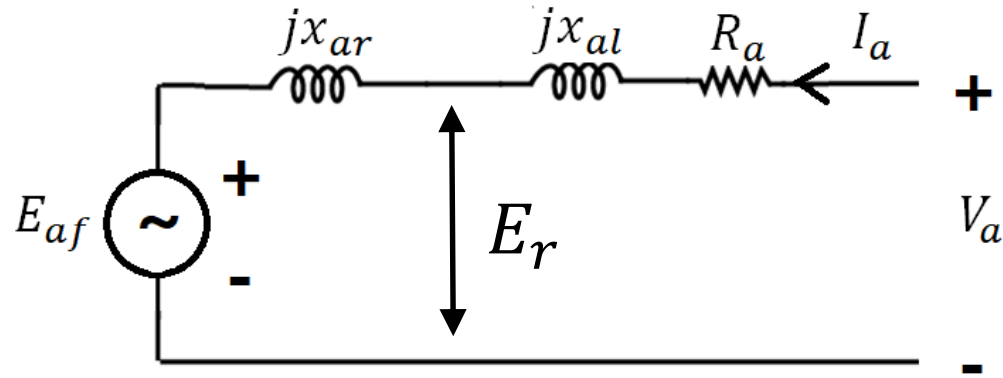


$$x_d = x_q = x_s ??$$

$$x_d \approx x_q \approx x_s$$

But really: $x_d > x_q$

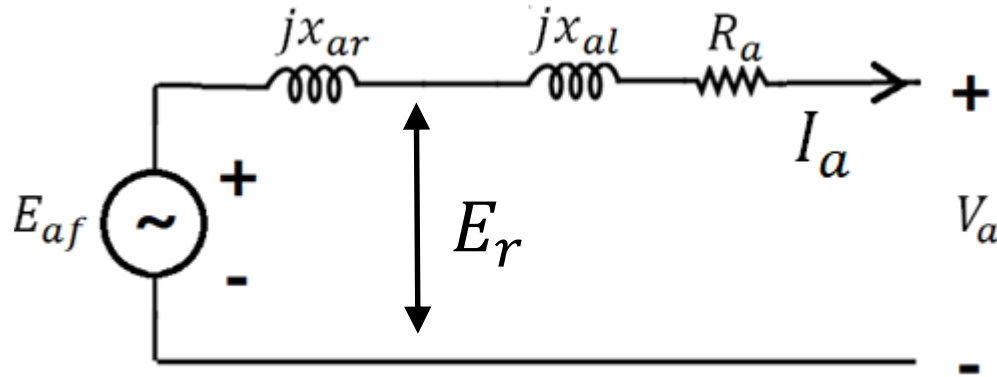
Equivalent Circuit of Cylindrical-Salient-pole Synchronous Machine



Motor Configuration:

$$V_a = R_a I_a + jx_s I_a + E_{af}$$

$$V_a = R_a I_a + jx_d I_d + jx_q I_q + E_{af}$$



Generator Configuration:

$$E_{af} = V_a + R_a I_a + jx_s I_a$$

$$E_{af} = V_a + R_a I_a + jx_d I_d + jx_q I_q$$

Electrical Machine III Syllabus

1. Introduction

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3. Generation of Sinusoidal Voltage in a Synchronous Generator

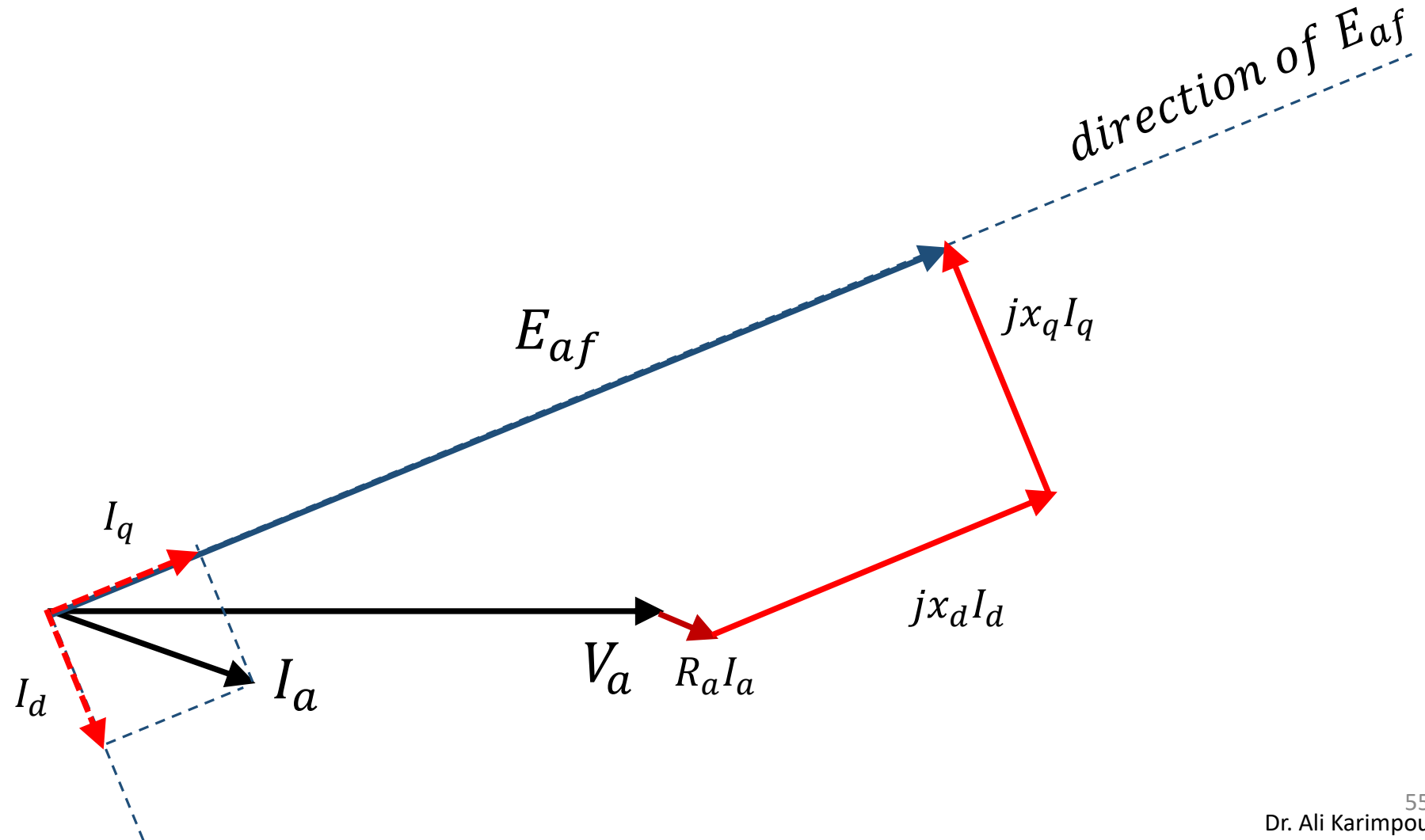
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- Steady-State Operating Characteristic
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- **Steady-State Power-Angle Characteristic of Salient-Pole Machines**
- Permanent Magnet Synchronous Machine
- Transient Behavior of Synchronous Machine.

Generator Configuration:

$$E_{af} = V_a + R_a I_a + jx_d I_d + jx_q I_q$$



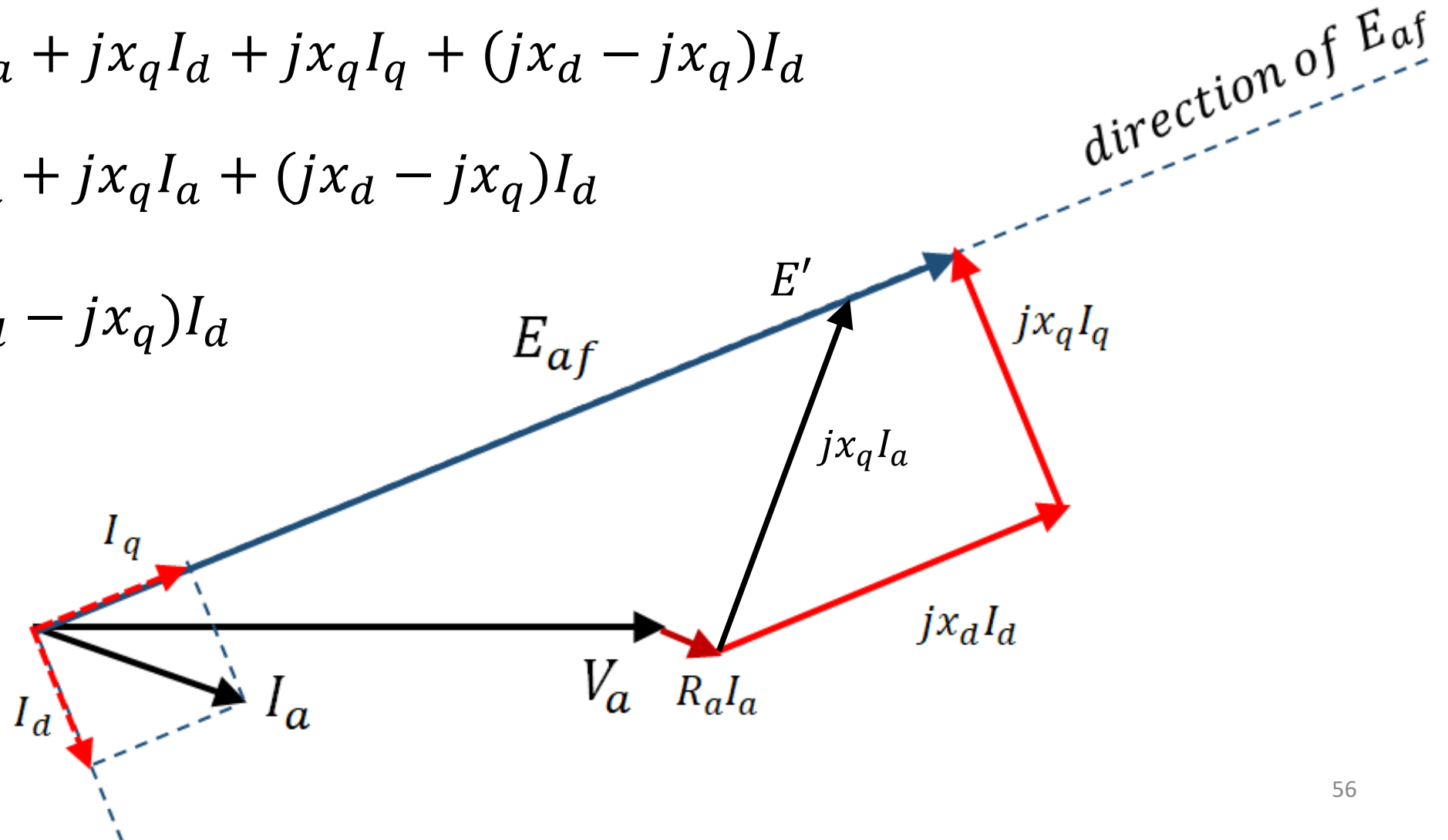
Generator Configuration:

$$E_{af} = V_a + R_a I_a + jx_d I_d + jx_q I_q$$

$$E_{af} = V_a + R_a I_a + jx_q I_d + jx_q I_q + (jx_d - jx_q) I_d$$

$$E_{af} = V_a + R_a I_a + jx_q I_a + (jx_d - jx_q) I_d$$

$$E_{af} = E' + (jx_d - jx_q) I_d$$



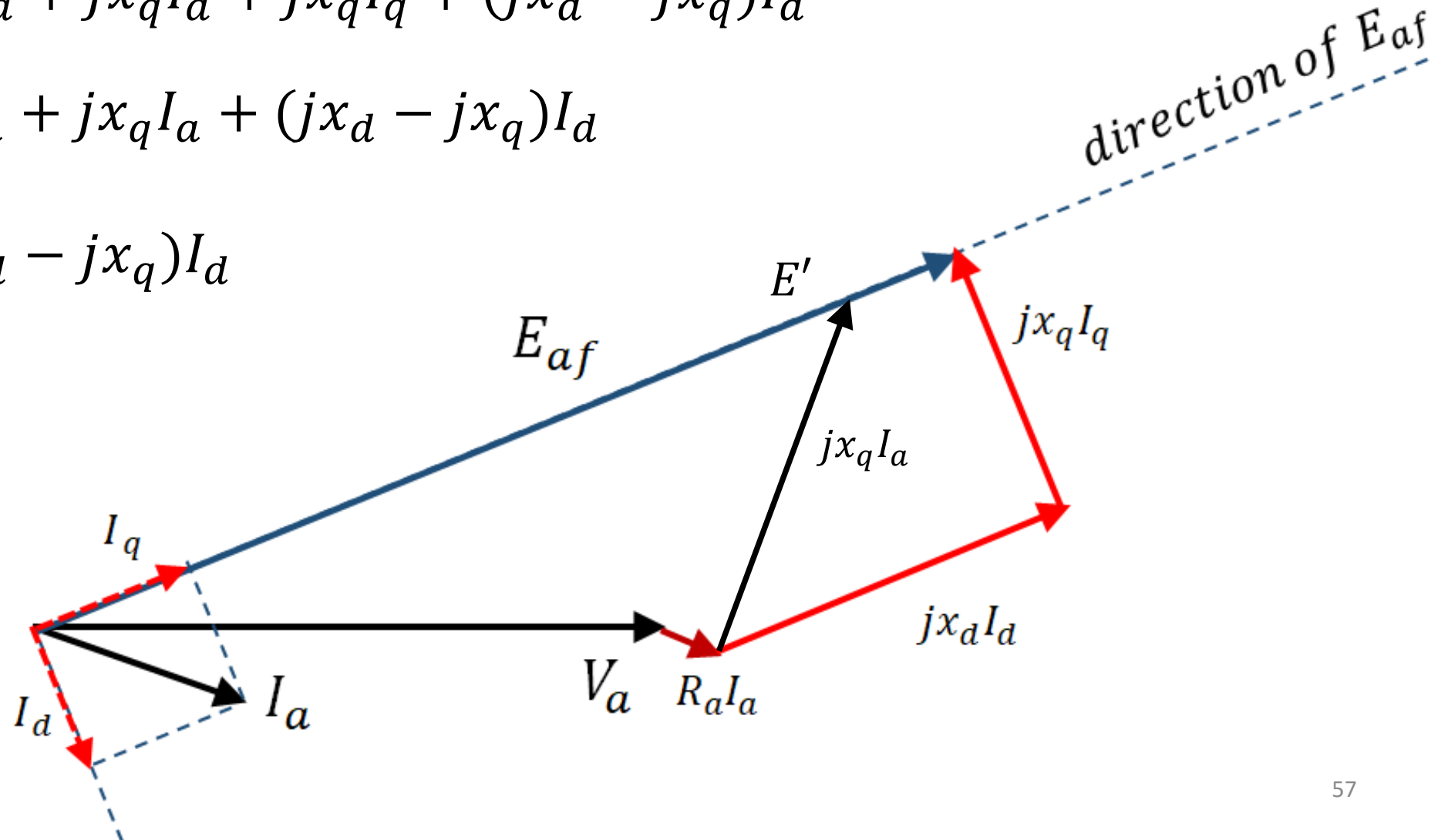
Generator Configuration:

$$E_{af} = V_a + R_a I_a + jx_d I_d + jx_q I_q$$

$$E_{af} = V_a + R_a I_a + jx_q I_d + jx_q I_q + (jx_d - jx_q) I_d$$

$$E_{af} = V_a + R_a I_a + jx_q I_a + (jx_d - jx_q) I_d$$

$$E_{af} = E' + (jx_d - jx_q) I_d$$



Example 5: The reactances of a salient-pole synchronous generator are 1.00 and 0.60 per unit. Neglect the armature resistance. Compute the generator voltage regulation when the generator delivers its rated kVA at 0.8 lagging power factor and rated terminal voltage.

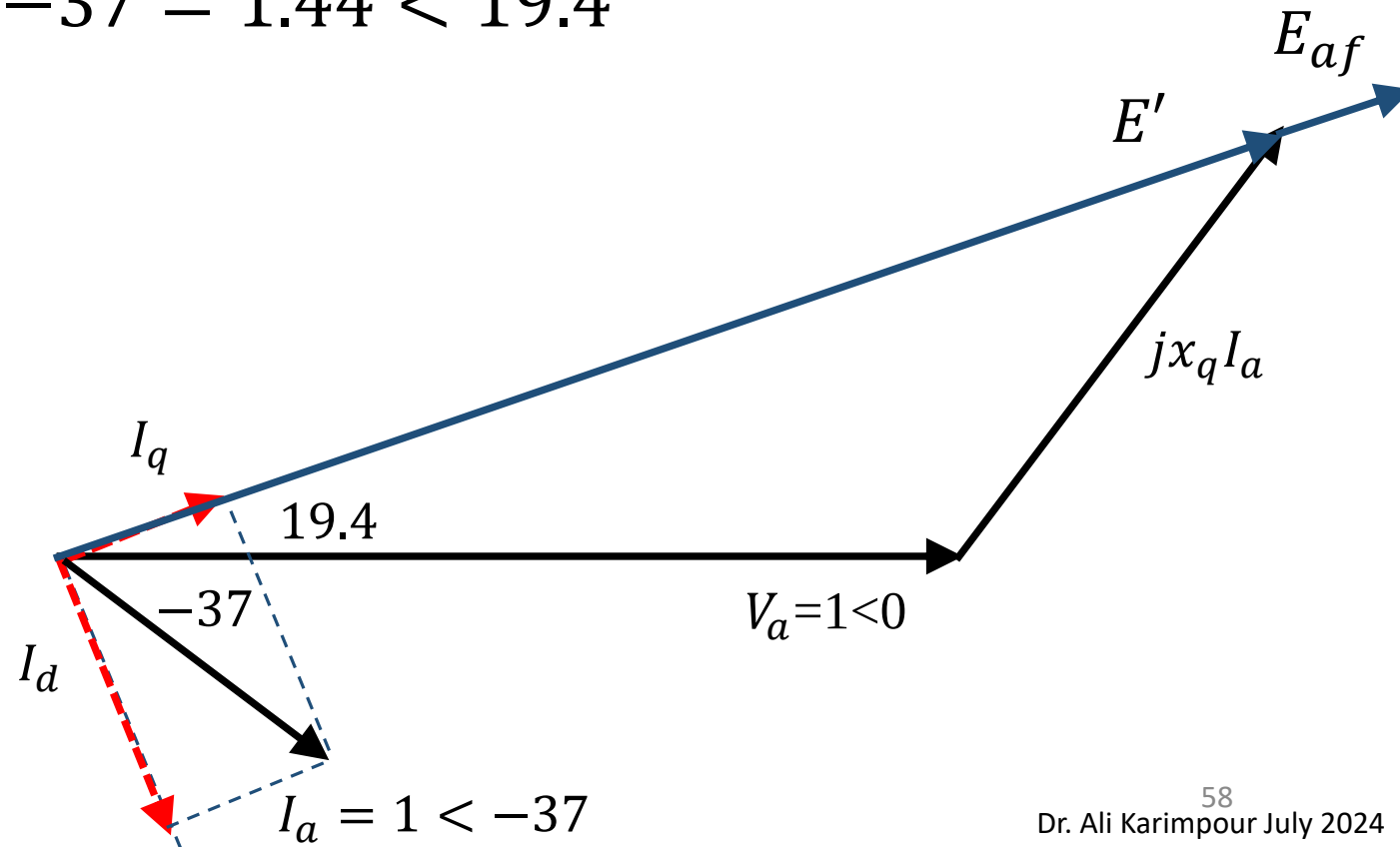
$$E' = V_a + jx_q I_a = 1 + j0.6 * 1 \angle -37^\circ = 1.44 \angle 19.4^\circ$$

$$E_{af} = E' + (jx_d - jx_q)I_d$$

$$I_{ad} = 1. \sin(37^\circ + 19.4^\circ) = 0.83$$

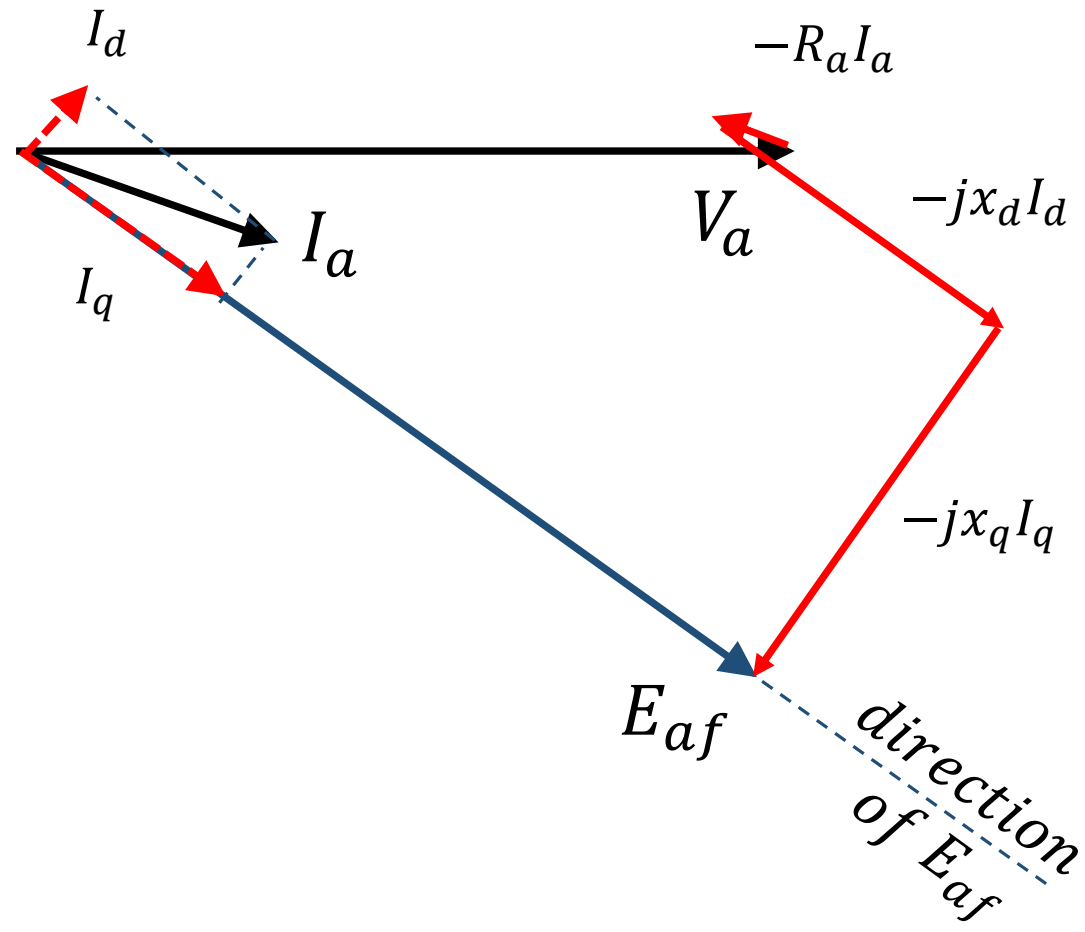
$$E_{af} = (1.44 + 0.83 * (1 - 0.6)) \angle 19.4^\circ$$

$$E_{af} = 1.772 \angle 19.4^\circ \quad R = 77\%$$



Motor Configuration:

$$V_a = E_{af} + R_a I_a + jx_d I_d + jx_q I_q$$

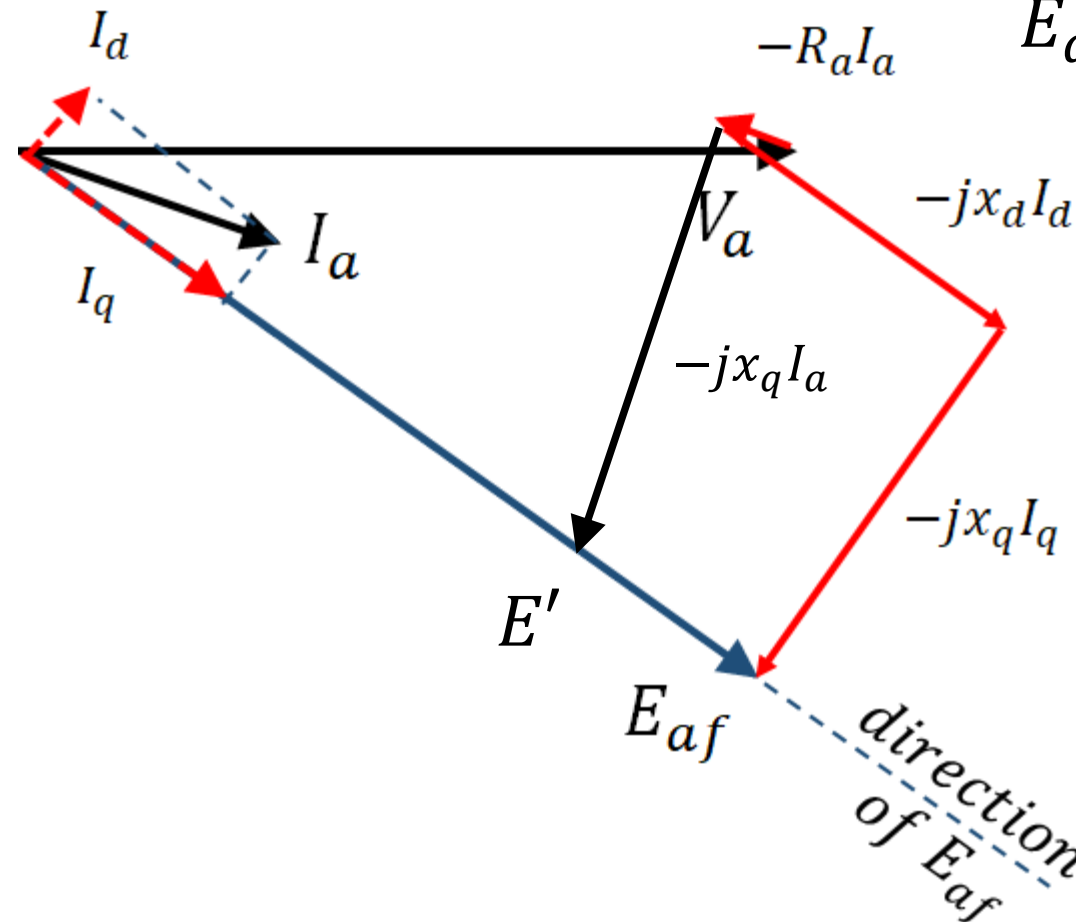


Motor Configuration:

$$V_a = E_{af} + R_a I_a + jx_d I_d + jx_q I_q$$

$$V_a = E_{af} + R_a I_a + jx_q I_d + jx_q I_q + (jx_d - jx_q) I_d$$

$$E_{af} = E' - (jx_d - jx_q) I_d$$



Power-Angle Characteristics of Cylindrical Machines

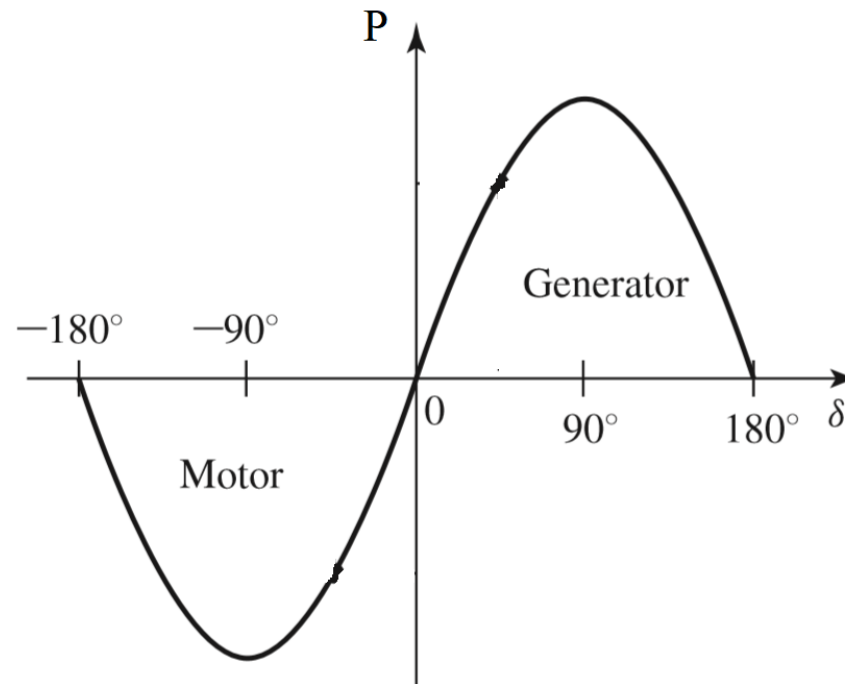
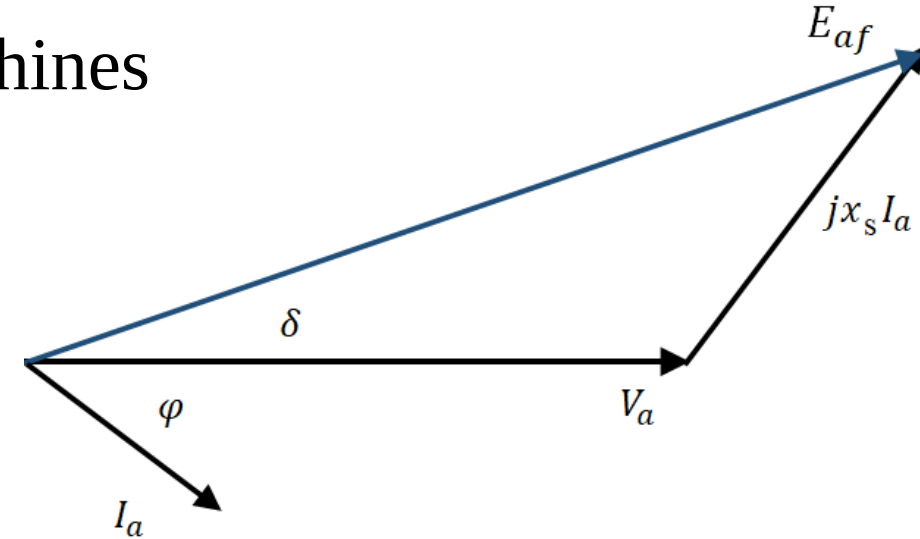
Let $R_a=0$

$$P = \text{Re}[V_a I_a^*]$$

$$P = V_a I_a \cos \varphi$$

$$P = V_a I_a \cos \varphi \frac{x_s}{x_s}$$

$$P = \frac{V_a E_{af}}{x_s} \sin \delta$$



Power-Angle Characteristics of Salient-Pole Machines

Let $R_a=0$

$$P = \text{Re}[V_a I_a^*]$$

$$V_a = -jV_d < \delta + V_q < \delta$$

$$I_a = -jI_d < \delta + I_q < \delta$$

$$P = \text{Re}[(-jV_d < \delta + V_q < \delta)(jI_d < -\delta + I_q < -\delta)]$$

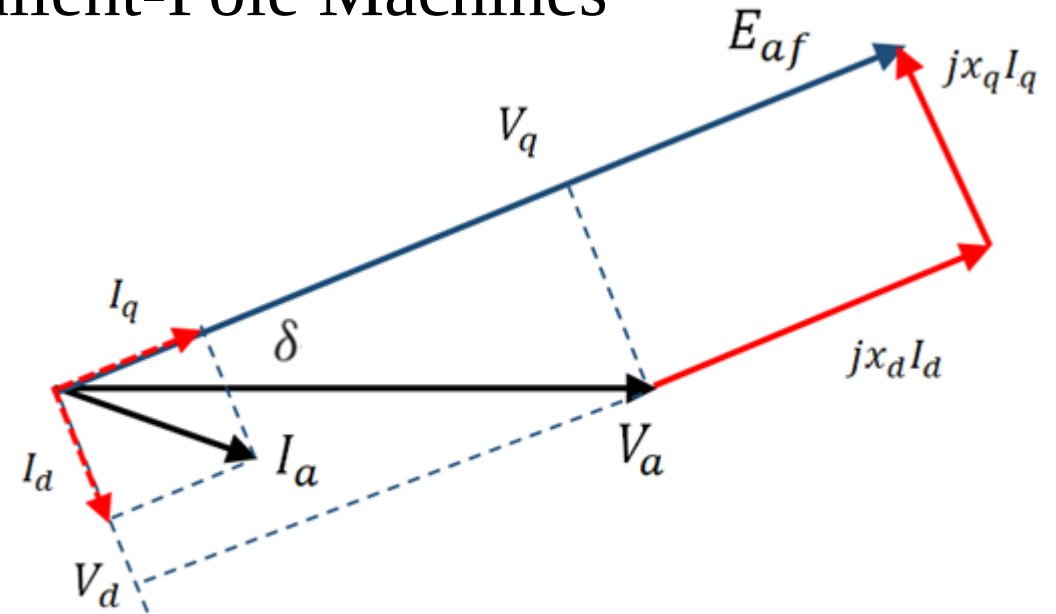
$$P = V_d I_d + V_q I_q$$

$$V_d = V_a \sin \delta$$

$$V_q = V_a \cos \delta$$

$$I_d = \frac{E_{af} - V_q}{x_d}$$

$$I_q = \frac{V_d}{x_q}$$



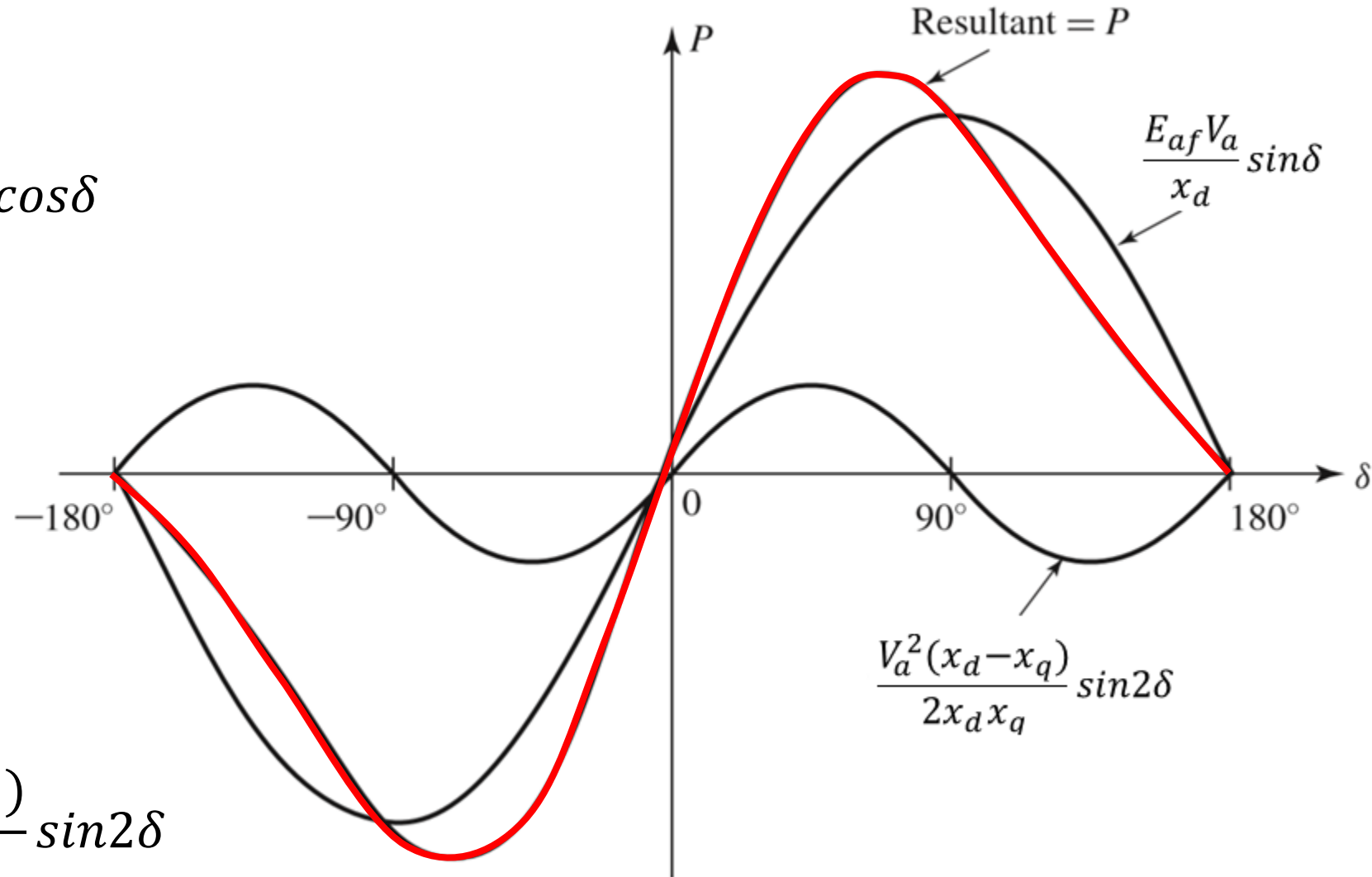
Power-Angle Characteristics of Salient-Pole Machines

$$P = V_d I_d + V_q I_q$$

$$V_d = V_a \sin \delta \quad V_q = V_a \cos \delta$$

$$I_d = \frac{E_{af} - V_q}{x_d} \quad I_q = \frac{V_d}{x_q}$$

$$P = \frac{E_{af} V_a}{x_d} \sin \delta + \frac{V_a^2 (x_d - x_q)}{2 x_d x_q} \sin 2\delta$$



Example 6: A 2000-hp, 2300-V, three-phase, Y-connected, 30-pole, 60-Hz synchronous motor has a synchronous reactance of $x_s=1.95 \Omega/\text{phase}$ and $\text{AFNL}=370 \text{ A}$. Neglecting all losses, compute its excitation current if it is supplied with electric power directly from an infinite bus at rated voltage and frequency and if its field excitation is held constant at that value which would result in unity-power-factor operation at rated load.

$$V_a = E_{af} + r_a I_a + jx_s I_a \quad I_a = ? = \frac{2000 * 746}{\sqrt{3} * 2300} < 0 = 374.5 < 0 \text{ A}$$

$$V_a = ? = 1328 < 0$$

$$E_{af} = V_a - jx_s I_a = 1328 < 0 - 1.95 < 90 * 374.5 < 0 = 1515 < -28.8$$

$$I_f = ? = \frac{1515}{1328} 370 = 422 \text{ A}$$

Example 7: In the previous Example let it is a salient-pole machine with reactances $x_d = 1.95 \, \Omega$ /phase and $x_q = 1.40 \, \Omega$ /phase. Repeat the calculation.

$$E_{af} = V_a - R_a I_a - jx_d I_d - jx_q I_q \quad E_{af} = V_a - jx_q I_a - j(x_d - x_q) I_d$$

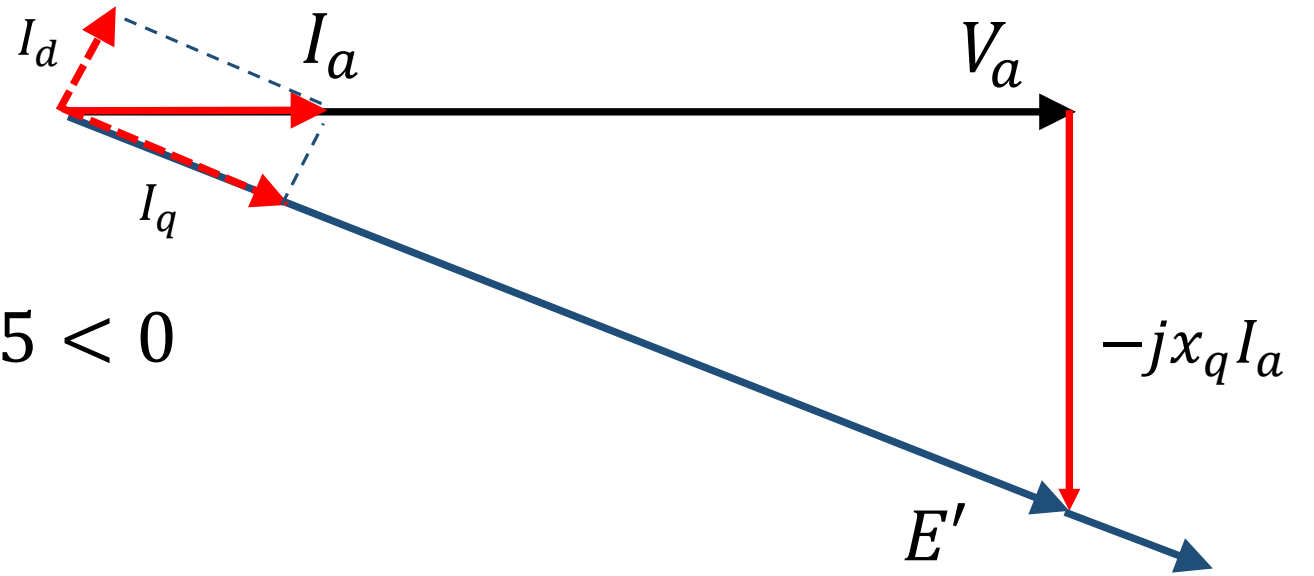
$$E_{af} = E' - j(x_d - x_q) I_d$$

$$E' = 1328 \angle 0 - 1.4 \angle 90 \times 374.5 \angle 0$$

$$E' = 1427 \angle -21.5$$

$$E_{af} = (1427 + (1.95 - 1.4)374.5 \sin(21.5)) \angle -21.5 = 1502 \angle -21.5$$

$$I_f = ? = \frac{1502}{1328} 370 = 418 \, A$$



Compare with previous example

Example 8: The 2000-hp, 2300-V synchronous motor with reactances $x_d = 1.95 \Omega$ /phase and $x_q = 1.40 \Omega$ /phase. Neglecting all losses, compute the maximum mechanical power in kilowatts which this motor can deliver if it is supplied with electric power directly from an infinite bus at rated voltage and frequency and if

- Its field excitation is held constant at that value which would result in unity-power-factor operation at rated load.
- The field is not supplied.
- Draw phasor diagram for condition b and discuss.

$$E_{af} = 1502\sqrt{3} = 1.131 \text{ p.u.} \quad x_d = 1.95 \frac{2000 * 746}{2300^2} = 0.55 \text{ p.u.} \quad x_q = 0.395 \text{ p.u.}$$

$$P = \frac{E_{af}V_a}{x_d} \sin\delta + \frac{V_a^2(x_d - x_q)}{2x_dx_q} \sin 2\delta = \frac{1 * 1.131}{0.55} \sin\delta + \frac{1^2 * 0.155}{2(0.55)(0.395)} \sin 2\delta = 2.056 \sin\delta + 0.357 \sin 2\delta$$

a)

$$P = 2.056 \sin \delta + 0.357 \sin 2\delta \quad \frac{\partial P}{\partial \delta} = 2.056 \cos \delta + 0.714 \cos 2\delta = 0 \quad \delta = -73.2$$

$$P_{max} = -2.166 \text{ p.u.} = -3.23 \text{ Mw} = -4332 \text{ hp} \quad \text{Is it possible ...??!!}$$

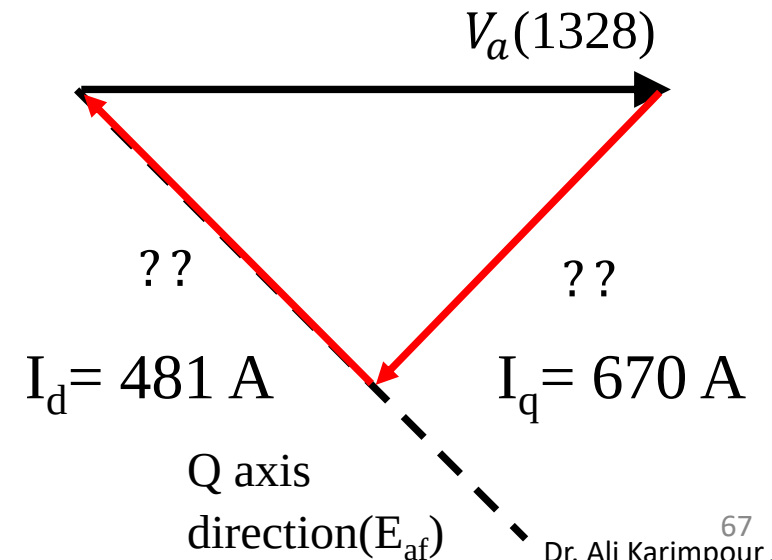
b)

$$P = \frac{E_{af} V_a}{X_d} \sin \delta + \frac{V_a^2 (X_d - X_q)}{2X_d X_q} \sin 2\delta \quad P_{max} = -0.357 \text{ p.u.} = -0.532 \text{ Mw} = -714 \text{ hp}$$

Is it possible ...??!!

c)

$I_a = 825 \text{ A}$ Much bigger than rated current!!



Electrical Machine III Syllabus

1. Introduction

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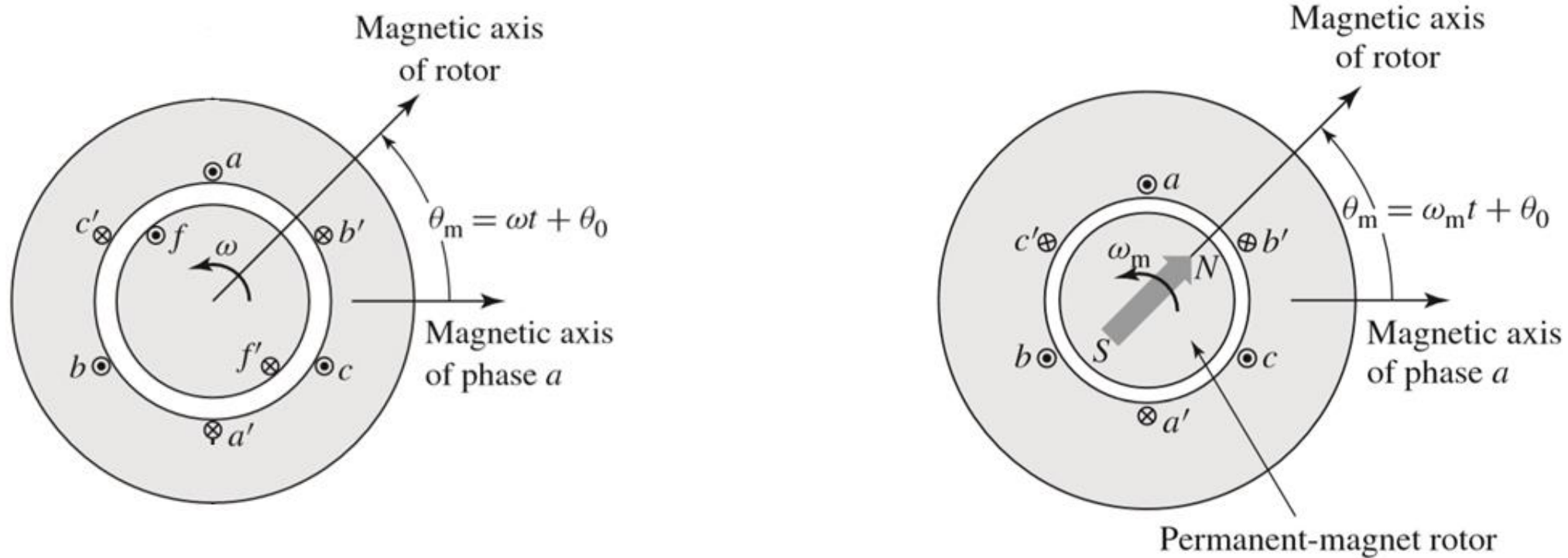
3. Generation of Sinusoidal Voltage in a Synchronous Generator

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Permanent Magnet Synchronous Machines



Small permanent-magnet motors are found in many applications:

disk drives and fans, fuel pumps, wipers, and power windows, doors and seats, etc.

A growing application of larger permanent-magnet machines is:

hybrid-electric vehicles and in generators for large wind turbines.

Permanent Magnet Synchronous Machines

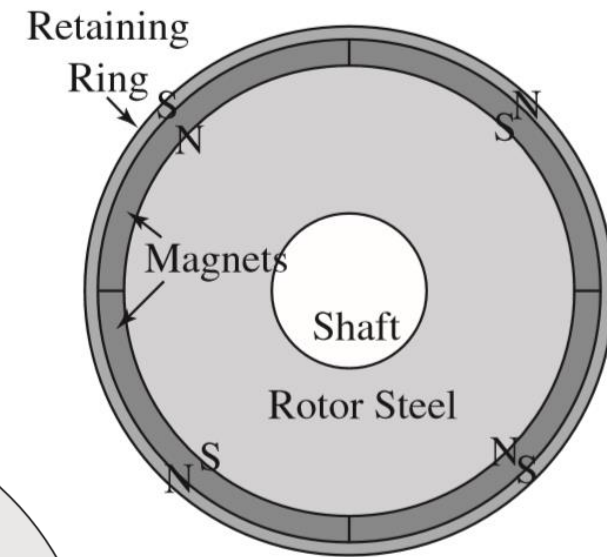
Advantages and disadvantages of PM-SM

- Actual cost of the permanent-magnetic materials themselves.
- The characteristics of permanent-magnets are temperature dependent.
- Unlike a wound-rotor synchronous machine, the rotor excitation in a permanent-magnet machine is fixed.
- Permanent-magnet ac motors must be operated from variable-frequency motor drives.
- No Slip rings.

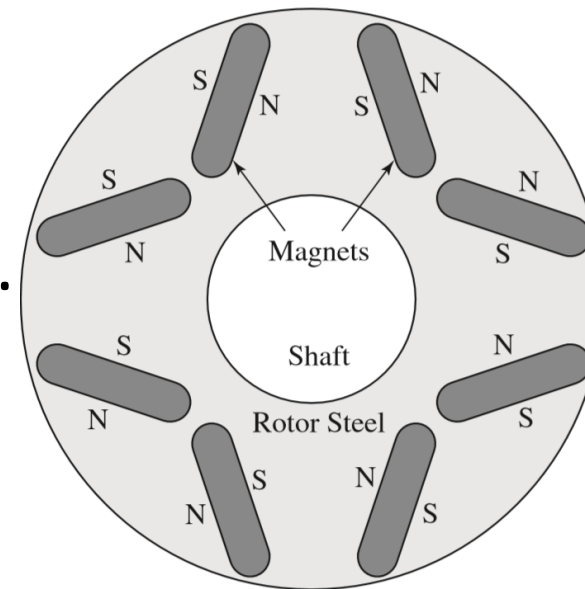
Permanent Magnet Synchronous Machines

The great majority of radial-air-gap permanent-magnet synchronous motors and generators tend to fall into two general classes:

1- Surface-permanent-magnet ac machine.



2- Interior-permanent-magnet ac machine.



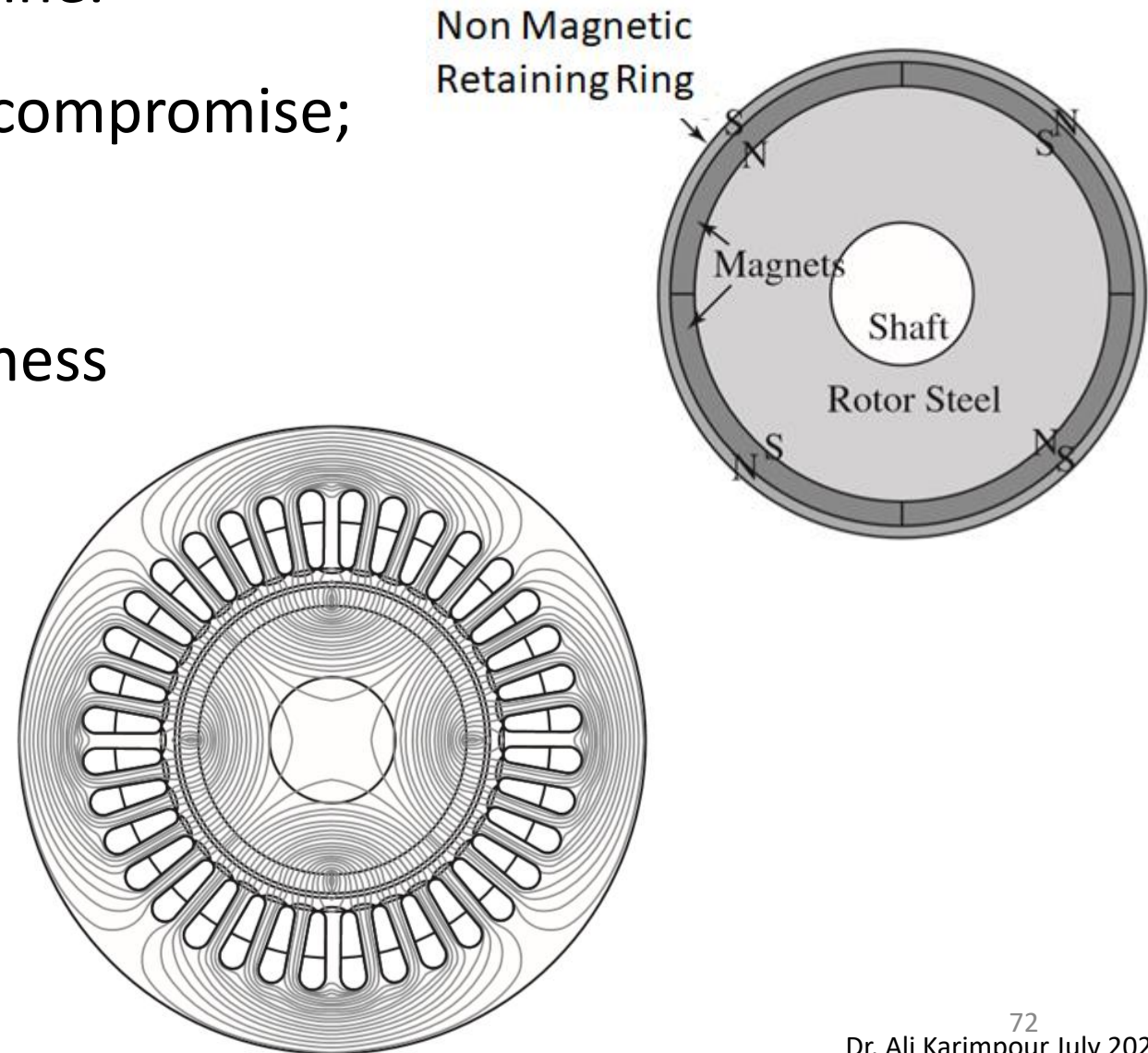
Permanent Magnet Synchronous Machines

1- Surface-permanent-magnet ac machine.

The design of a retaining ring is thus a compromise;

- Thicker ring increases Mechanical integrity of the rotor
- Thinner ring increases the effectiveness of the magnetic circuit.

Open-circuit flux distribution.



Permanent Magnet Synchronous Machines

Example 9: A four-pole, three-phase, surface-permanent-magnet motor has a synchronous inductance of 0.50 mH and a line-line generated voltage of **3.31 V/Hz**. For the purposes of this example, the armature resistance will be neglected.

It is designed for nominal 1800 r/min operation but with over-speed capability to 2400 r/min at **reduced power output**.

The motor can be operated up to a **maximum terminal voltage** of 208 V, line-line and a **maximum terminal current** of 185 A.

In order to insure that the **flux density in the motor remains within acceptable limits**, the drive has a V/Hz **limit** whose function is to insure that the motor terminal voltage does not exceed the rated V/Hz of the motor (in this case, the rated V/Hz is equal to $208 \text{ V}/60 \text{ Hz} = 3.47 \text{ V line-line per Hz}$).

Permanent Magnet Synchronous Machines

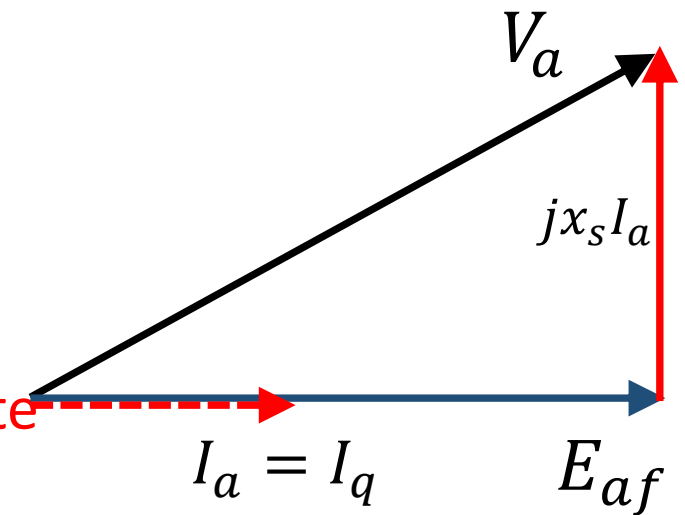
a) With the drive supplying a terminal current of 185 A, calculate the maximum power and torque which the motor can supply at a speed of 1800 r/min. Calculate the d- and q-axis currents under this operating condition. Verify that under this operating condition, the motor terminal voltage will not exceed 208 V, line-line.

$$n = 1800 \text{ rpm} \quad f = \frac{p}{2} \frac{n}{60} = 60 \text{ Hz}$$

$$E_{am} = 3.31 * 60 = 198.6 \text{ V(L-L)} \quad I_a = I_q, I_d = 0,$$

$$V_a = 119.8 \text{ L-n} = 207 \text{ L-L}$$

drive V/Hz limit will not be activate



$$P_{max} = \sqrt{3} E_{am} * I_a = \sqrt{3} * 198.6 * 185 = 63.6 \text{ kW}$$

$$T_{max} = \frac{P_{max}}{\omega_m} = 338 \text{ N.m}$$

Permanent Magnet Synchronous Machines

b) If the motor is operated at speed of 2400 r/min, calculate the maximum output power and torque which can be supplied by the motor assuming that the drive control limits the motor terminal voltage to its maximum value of 208 V line-line and its terminal current to 185 A. Also calculate the d- and q-axis components of the armature current under this operating condition.

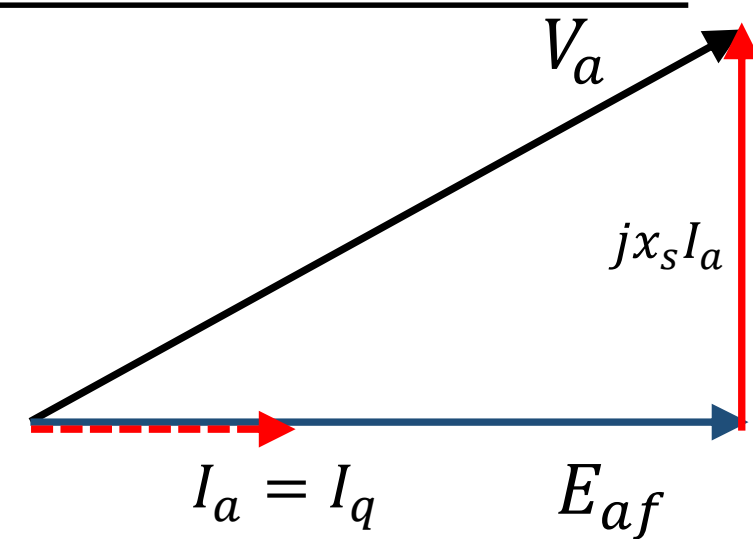
$$n = 2400 \text{ rpm} \quad f = \frac{p}{2} \frac{n}{60} = 80 \text{ Hz}$$

$$E_{am} = 3.31 * 80 = 264.8 \text{ V(L-L)} \quad I_a = I_q, I_d = 0,$$

$$V_a = 159 \text{ L-L} \rightarrow 276 \text{ L-L} \quad \text{drive limit the voltage to 208 V}$$

So,

$$E_{am} = 264.8 \text{ V(L-L)} \quad V_a = 208 \text{ L-L}$$



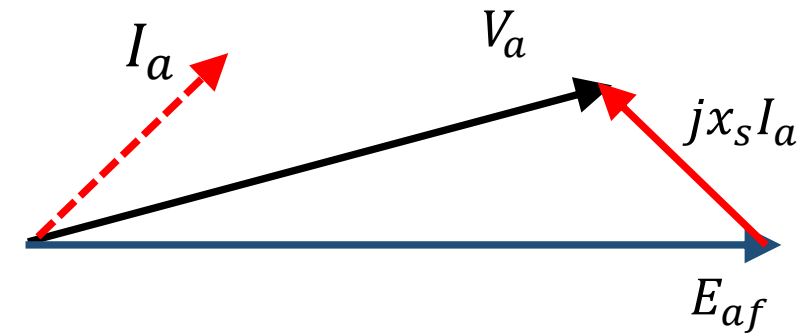
Permanent Magnet Synchronous Machines

So,

$$E_{am} = 264.8 \text{ V} (L - L) = 152.9 \text{ L} - N \quad V_a = 208 (L - L) = 120.1 \text{ L} - N$$

$$x_s I_a = 2 * \pi * 80 * 0.0005 * 185 = 46.5 \text{ V}$$

$$\delta = -\cos^{-1} \left(\frac{V_a^2 + E_{am}^2 - (X_s I_a)^2}{2V_a E_{am}} \right) = -14.0^\circ$$



$$I_a = \frac{V_a \cos \delta - E_{am}}{x_s} = 185 < 51.5$$

$$I_q = 115, \quad I_d = -145$$

$$P = \sqrt{3} E_{am} * I_q = \sqrt{3} * 264.8 * 115 = 52.9 \text{ kW}$$

$$P = 3(V_d I_d + V_q I_q)$$

$$T = \frac{P}{\omega_m} = 211 \text{ N.m}$$

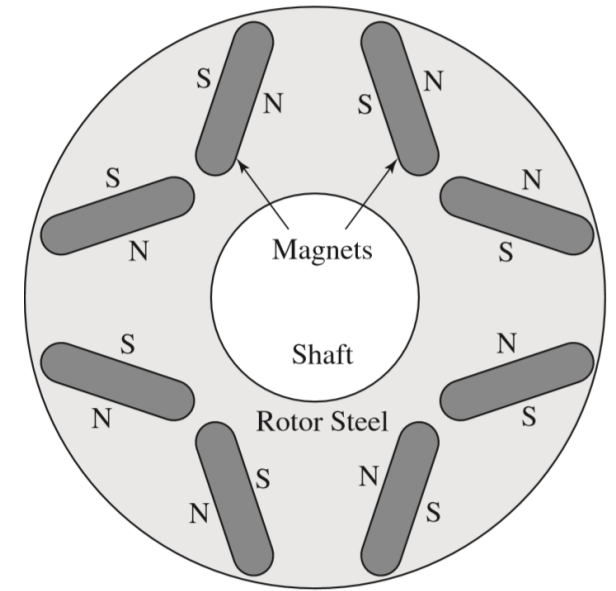
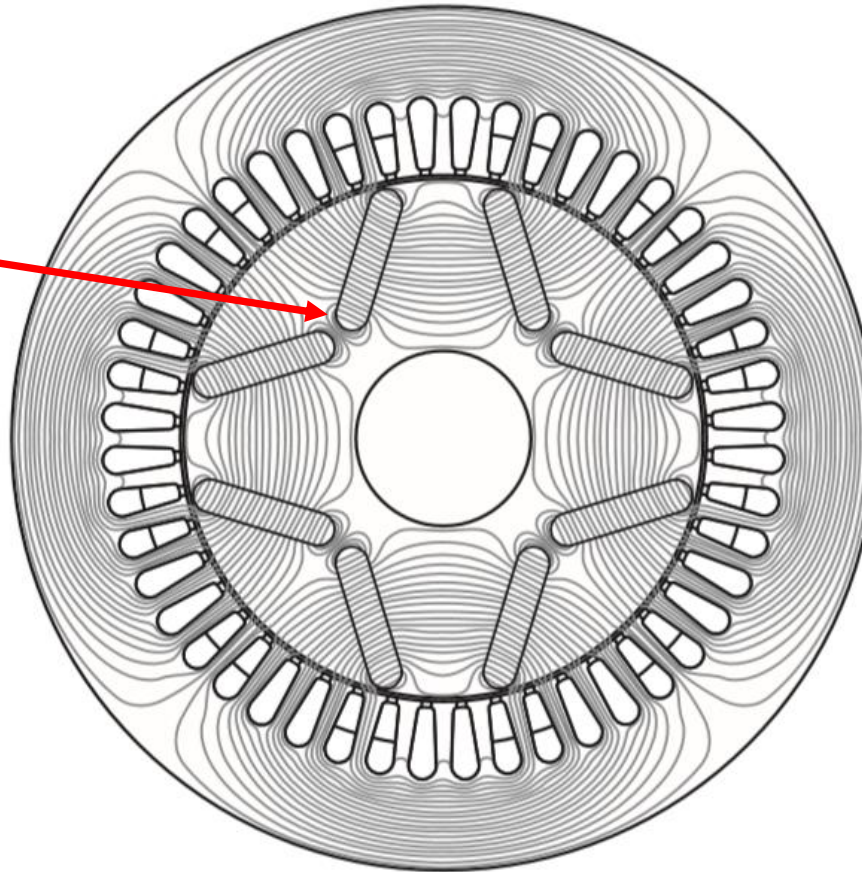
$$P = \sqrt{3} V_a I_a \cos \varphi$$

Permanent Magnet Synchronous Machines

2- Interior-permanent-magnet ac machine.

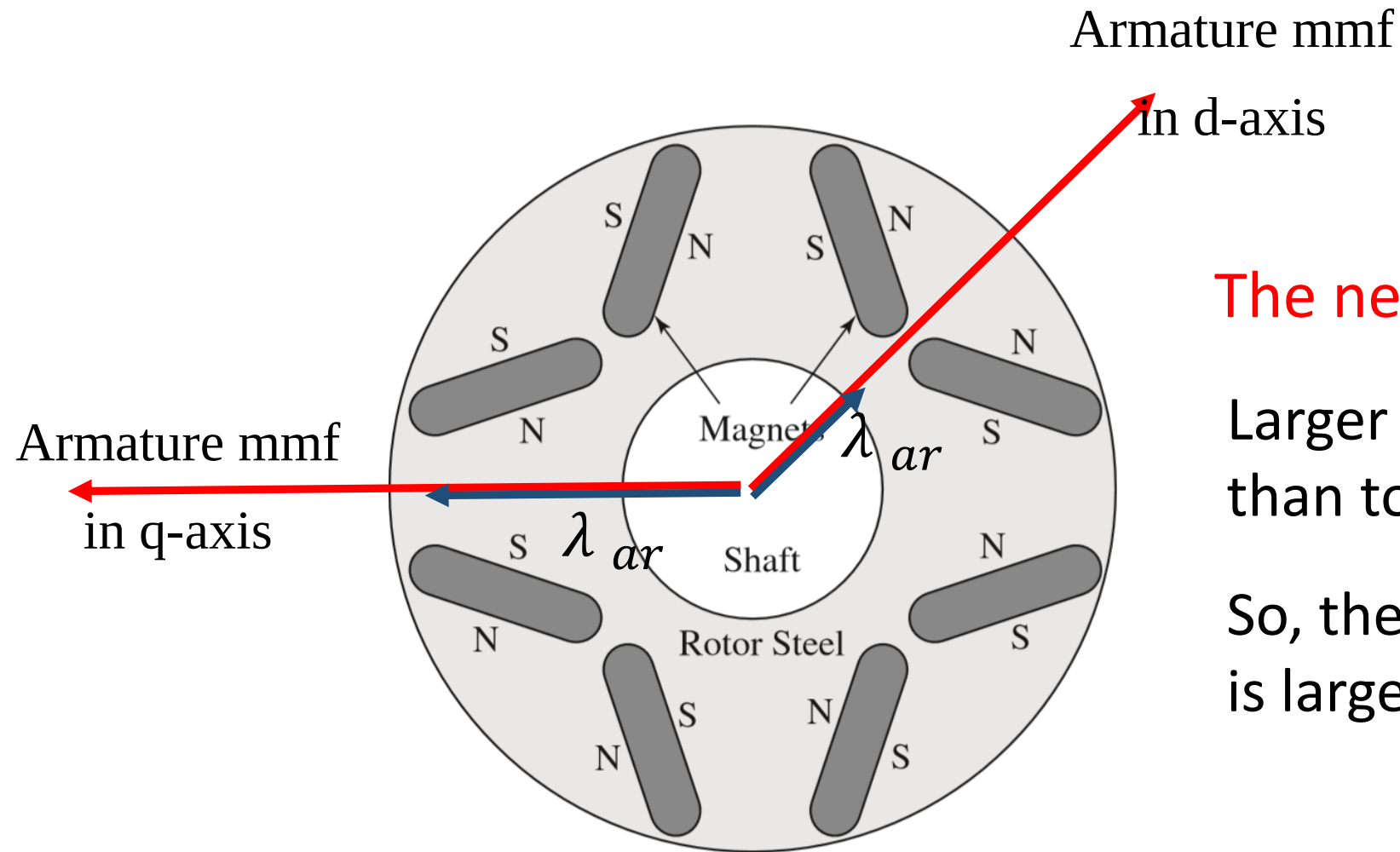
Open-circuit flux distribution.

Design challenge



Permanent Magnet Synchronous Machines

2- Interior-permanent-magnet ac machine.



The net result is that(interior PM):

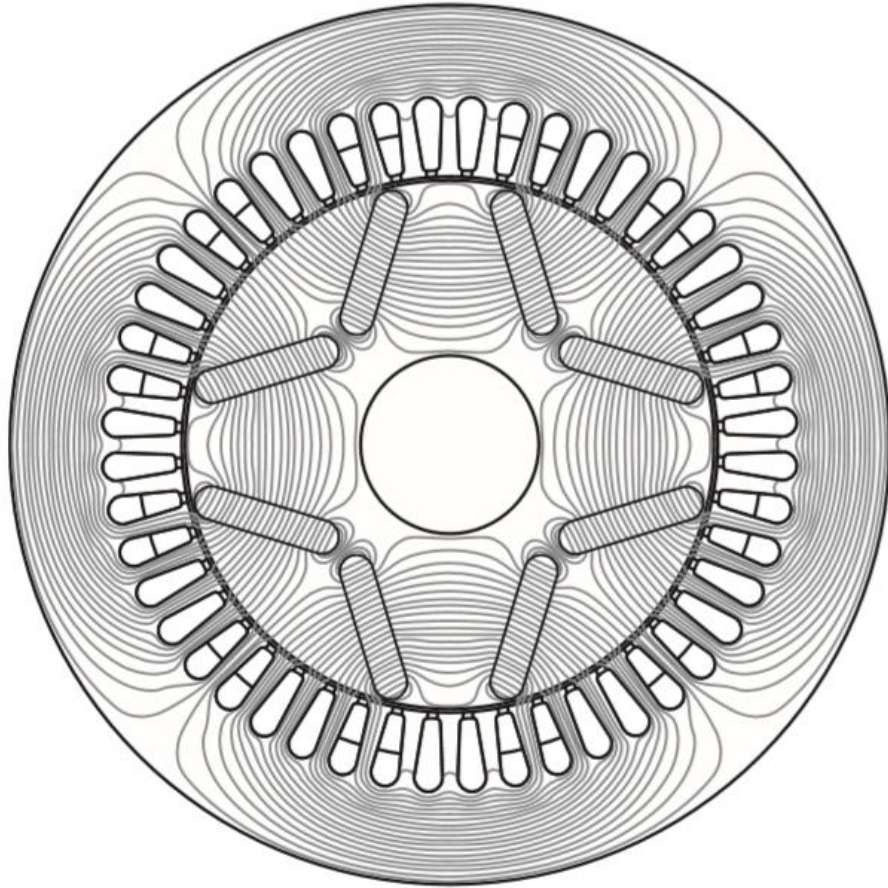
Larger reluctance to direct-axis flux than to quadrature-axis flux

So, the quadrature axis inductance is larger than that of the direct axis.

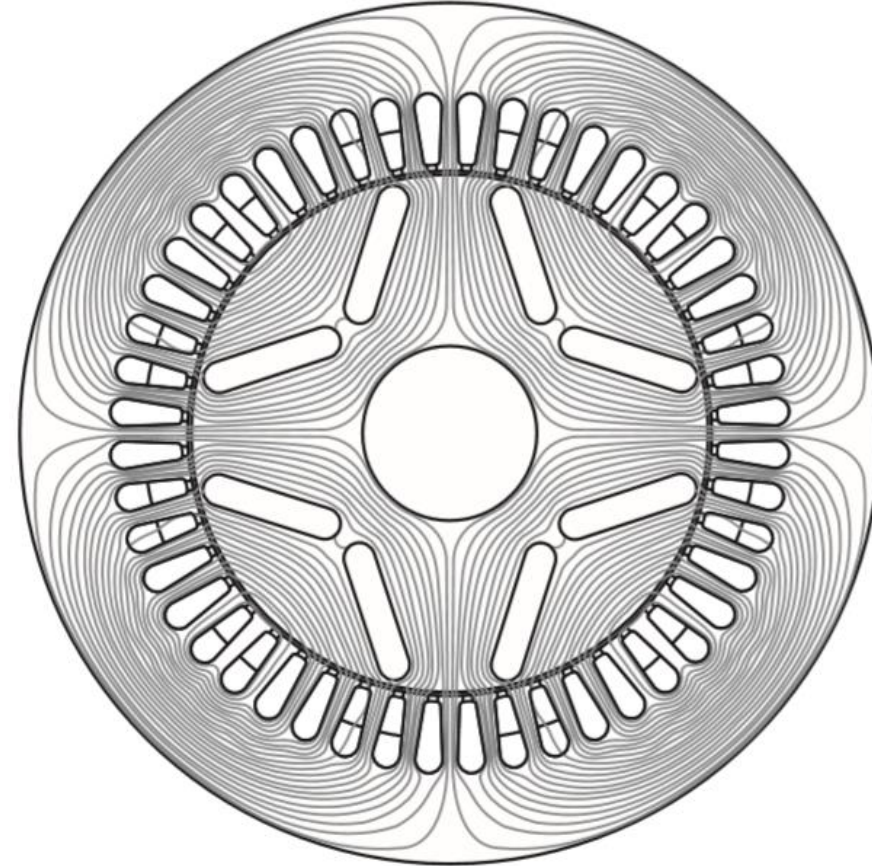
$$x_q > x_d$$

Permanent Magnet Synchronous Machines

2- Interior-permanent-magnet ac machine.



Open-circuit flux distribution



Quadrature-axis flux distribution

Permanent Magnet Synchronous Machines

Example 10: Consider a four-pole, three-phase, interior-permanent-magnet motor which is supplied by the same drive as example 1.

Consider a motor with $L_d=0.50$ mH and $L_q=2.30$ mH.

This motor has a line-line generated voltage of 3.31 V/Hz and it can be operated up to a maximum terminal voltage of 208 V, line-line and a maximum terminal current of 185 A.

- a) Find the maximum output power and torque which the motor can achieve at a speed of 1800 r/min with the terminal voltage and current limited of their maximum values.
- b) Find the corresponding direct- and quadrature-axis currents.

Permanent Magnet Synchronous Machines

$$P = -3 \left(\frac{E_{am} V_a}{X_d} \sin \delta + \frac{V_a^2 (X_d - X_q)}{2 X_d X_q} \sin 2\delta \right)$$

$$V_a = \frac{208}{\sqrt{3}} = 120.1 < 0 \quad E_{am} = 3.31 * \frac{60}{\sqrt{3}} = 114.7 < -\delta \quad I_a = 185 < \varphi$$

$$x_d = 0.0005 * 2\pi f = 0.188 \, \Omega \quad x_q = 0.0023 * 2\pi f = 0.867 \, \Omega$$

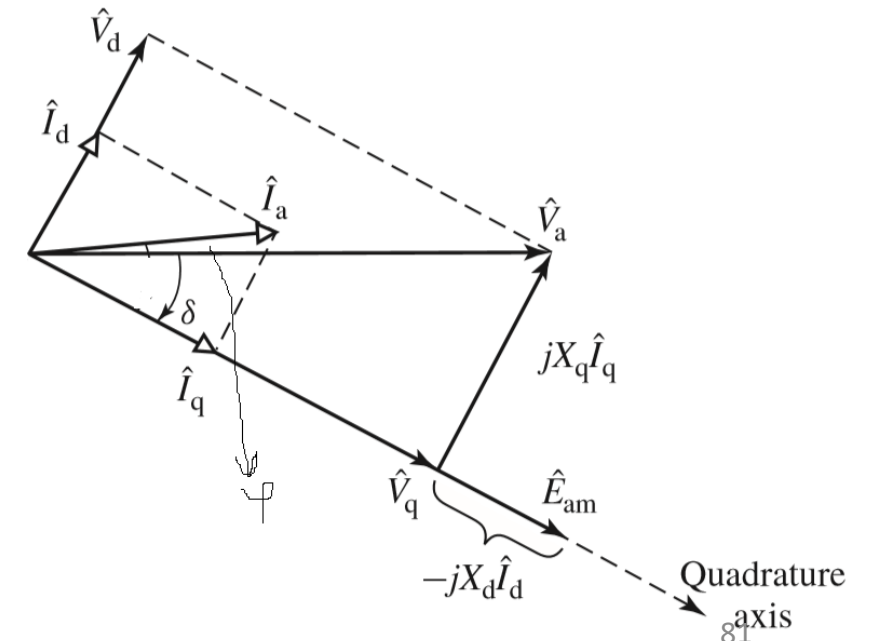
$$120 \cos \delta = 114 - 185 * 0.188 \sin(\varphi + \delta)$$

$$120 \sin \delta = 185 * 0.867 \cos(\varphi + \delta)$$

$$\sin(\varphi + \delta) = 0.8474 \quad I_d = 157 \, A$$

$$\cos(\varphi + \delta) = 0.531 \quad I_q = 98 \, A$$

$$\delta = 45 \quad P_{max} = 64.9 \, \text{kW} \quad T = 344 \, \text{N.m}$$



Electrical Machine III Syllabus

1. Introduction

.....

2. Introduction to Rotating Machines

.....

3. Generation of Sinusoidal Voltage in a Synchronous Generator

.....

4. Synchronous-Machine Analyses

- Synchronous-Machine Inductances; Equivalent Circuits
- Steady-State Power-Angle Characteristic
- Steady-State Operating Characteristic
- Open- and Short-Circuit Characteristics
- Effects of Salient Poles; Introduction to Direct and Quadrature-Axis Theory
- Steady-State Power-Angle Characteristic of Salient-Pole Machines
- Permanent Magnet Synchronous Machine
- **Transient Behavior of Synchronous Machine.**

Transient Behavior of Synchronous Machines

Causes of Transient Conditions
in a Synchronous Generator

Short circuit in the windings

Disconnection or connection of a
large electrical load

Causes of Transient Conditions in a
Synchronous Motor

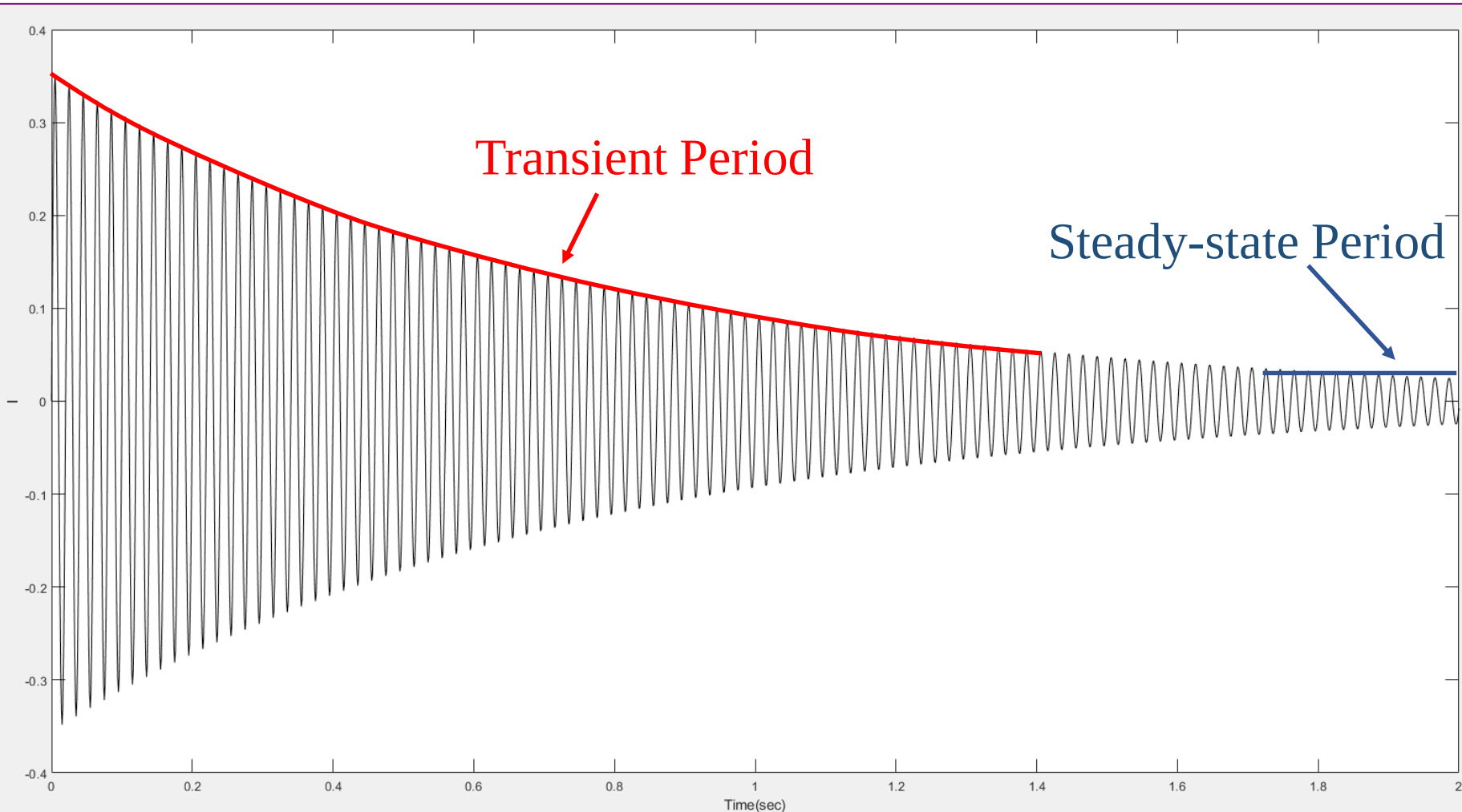
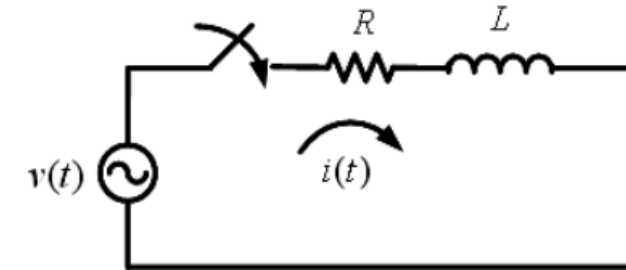
Failure in the windings

Disconnection or connection of a
large mechanical load

Transient Behavior of Synchronous Machines

Comparing an RL circuit with synchronous machine

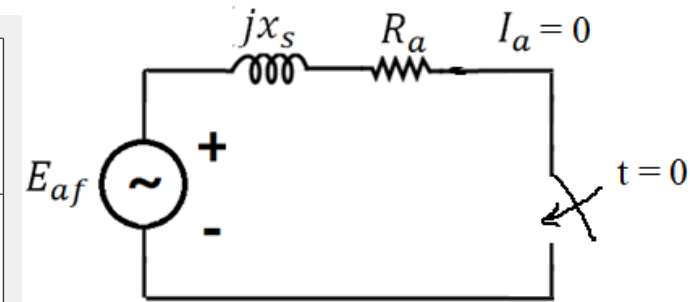
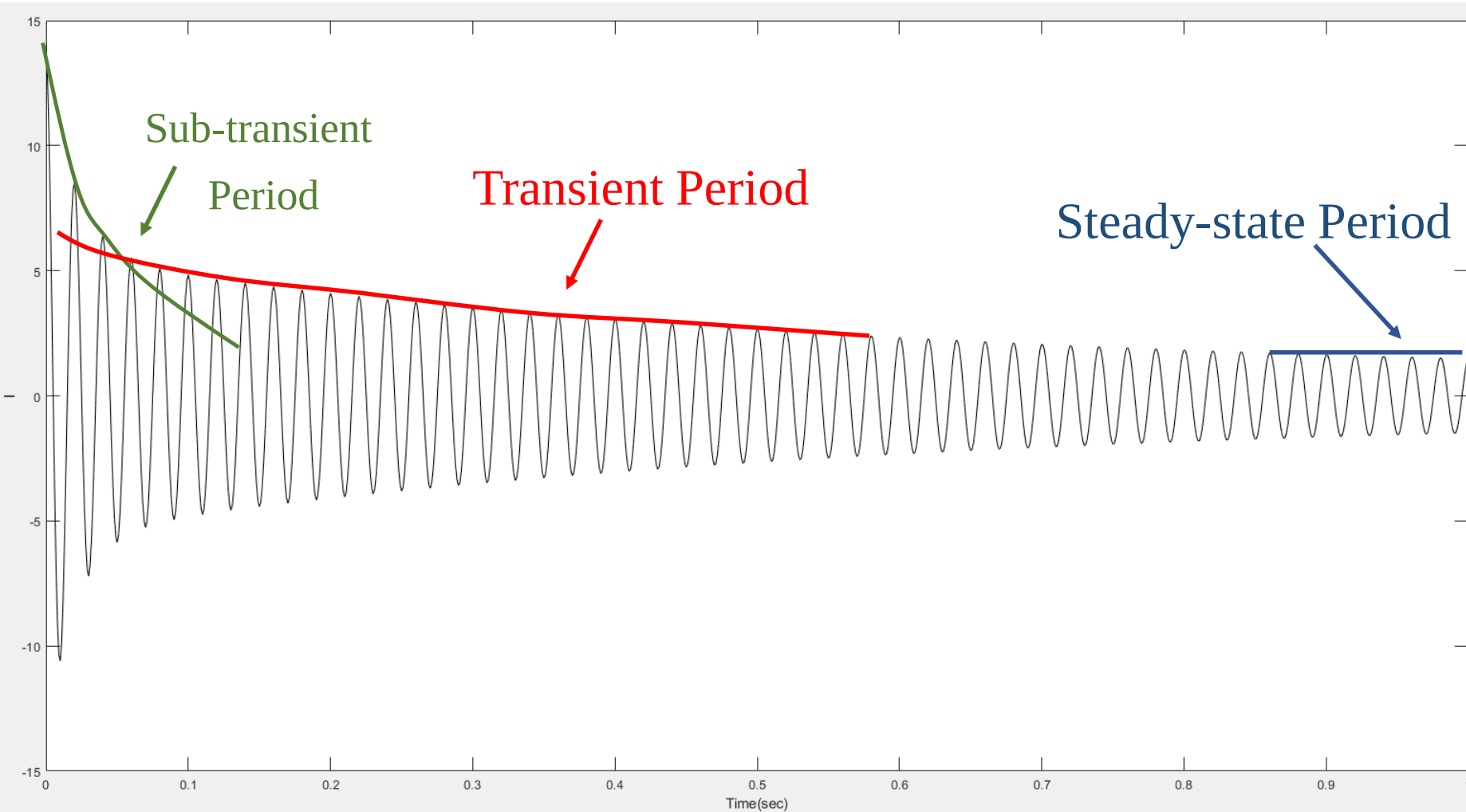
a) Short circuit of an RL circuit with zero initial conditions



Transient Behavior of Synchronous Machines

Comparing an RL circuit with synchronous machine

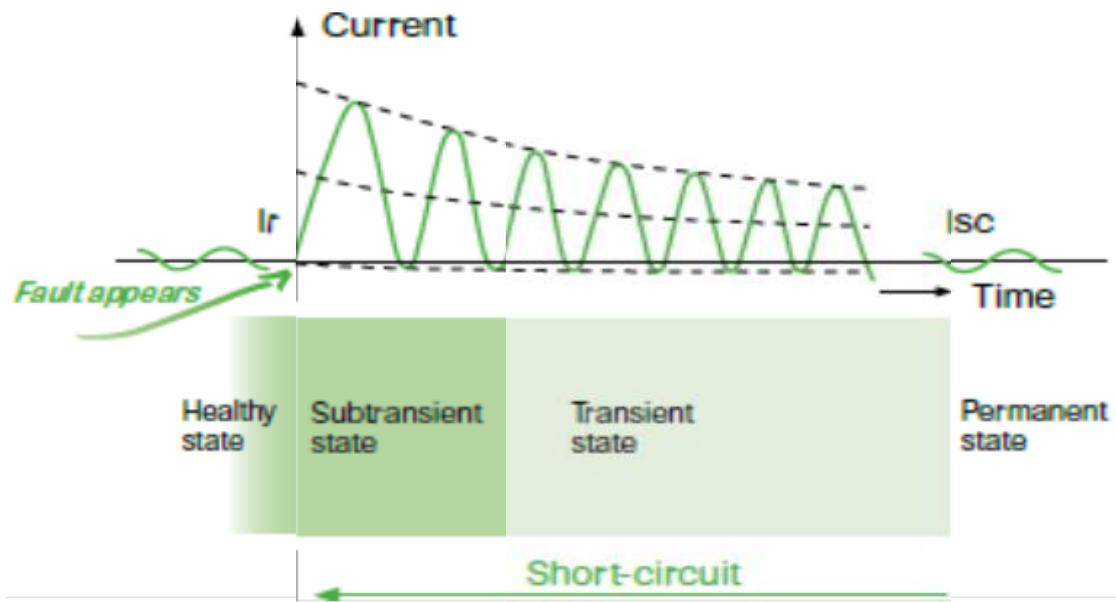
b) Short circuit of a no-load generator



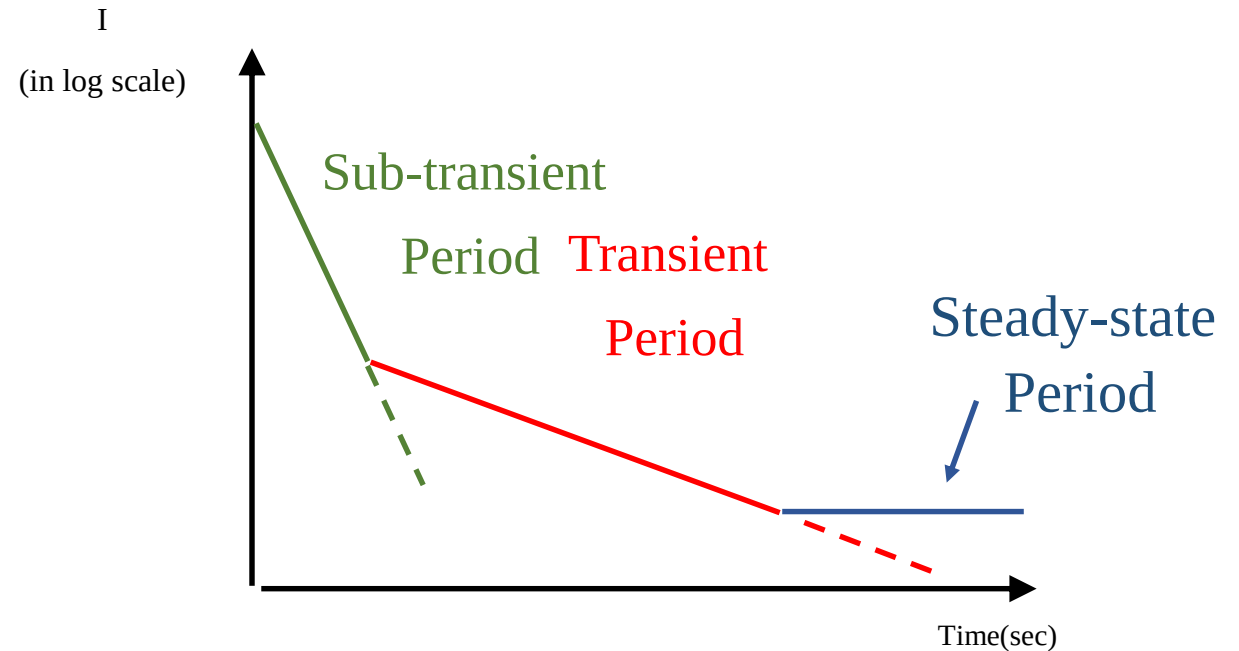
The main reason for difference with RL circuit is variation of inductances

Transient Behavior of Synchronous Machines

Different time periods in transient conditions



Current and the envelope of current amplitude in transient states



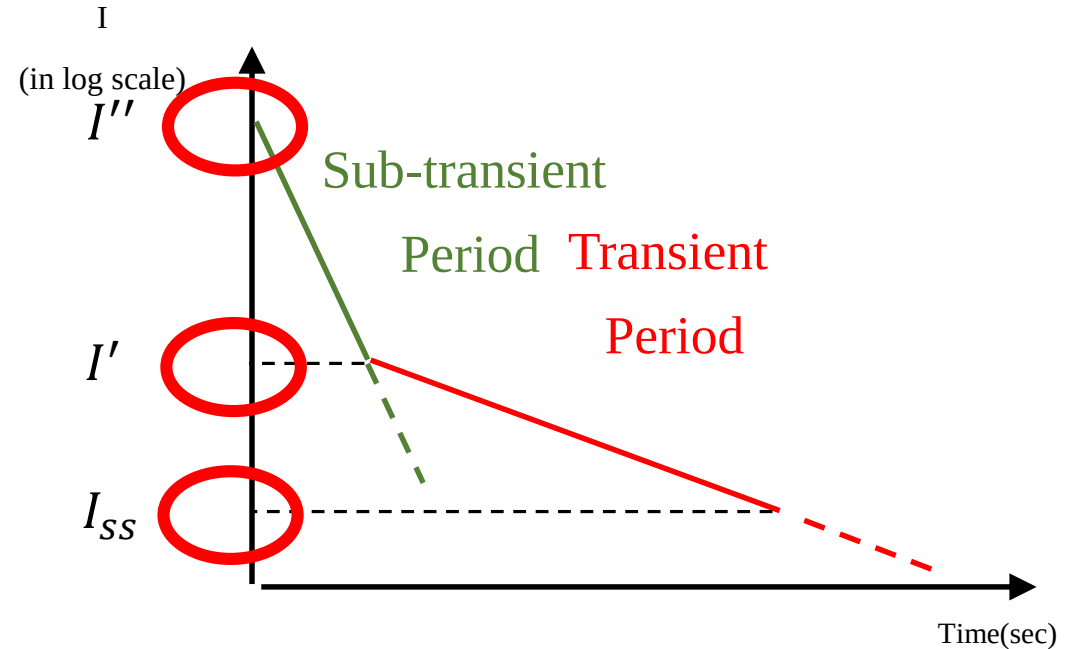
Rate of decrease in current amplitude on a logarithmic scale for each period

Transient Behavior of Synchronous Machines

Envelope of the phase current amplitude after a fault

$$I_a(t) = (I'' - I')e^{-\frac{t}{T''}} + (I' - I_{ss})e^{-\frac{t}{T'}} + I_{ss}$$

Envelope equation

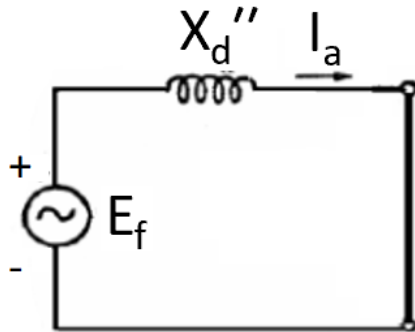


Rate of decrease of current amplitude on a logarithmic scale per period

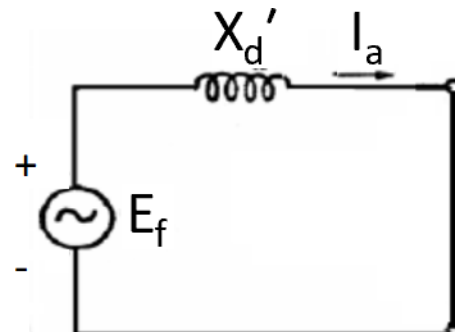
Transient Behavior of Synchronous Machines

Equivalent Circuit of a Salient-Pole Synchronous Generator During Different Time Periods Under No-Load Short-Circuit Conditions

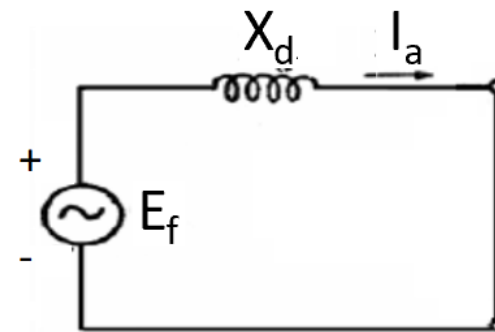
In a machine with salient poles, the reactance X_s transforms into X_d , X_d' , and X_d'' .



a) Subtransient state



b) Transient state



c) Steady state

$$i_a = \frac{\sqrt{2}E_{af0}}{X_d} \cos(\omega t + \theta_0) + \sqrt{2}E_{af0} \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-\frac{t}{T_{d0}'}} \cos(\omega t + \theta_0) + \sqrt{2}E_{af0} \left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-\frac{t}{T_{d0}''}} \cos(\omega t + \theta_0) + Id_{co} e^{-\frac{t}{T_a}}$$

Reason for using X_d in the equivalent circuit and analysis of the d-axis equivalent circuit under transient conditions?

Example 11: A 200-MVA , 15.75-kV, 50-Hz, three-phase synchronous generator has a synchronous reactance of $x_s = 2.64 \Omega/\text{phase}$, and negligible stator resistance. (Ansaldo in Shirvan).

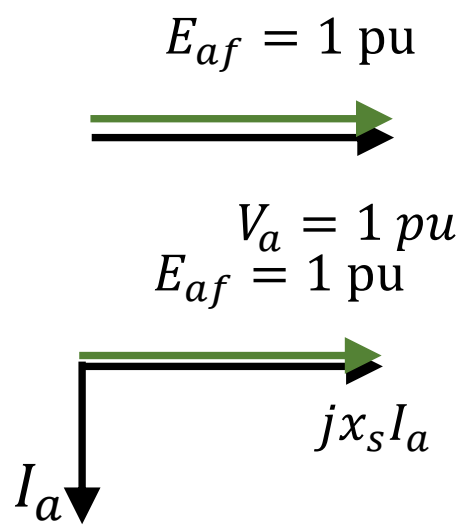
The generator is connected to nominal voltage and operating at no load condition.

- a) Calculate the magnitude and phase angle of the line-to- neutral generated voltage E_{af} , and corresponding phasor diagram.
- b) Calculate the steady state three phase short circuit current in rated condition.

Solution:

a) $X_S = 2.64 \frac{200}{15.75^2} = 2.12 \text{ pu}$ $E_{af} = V_a + jx_s I_a = 1 < 0$

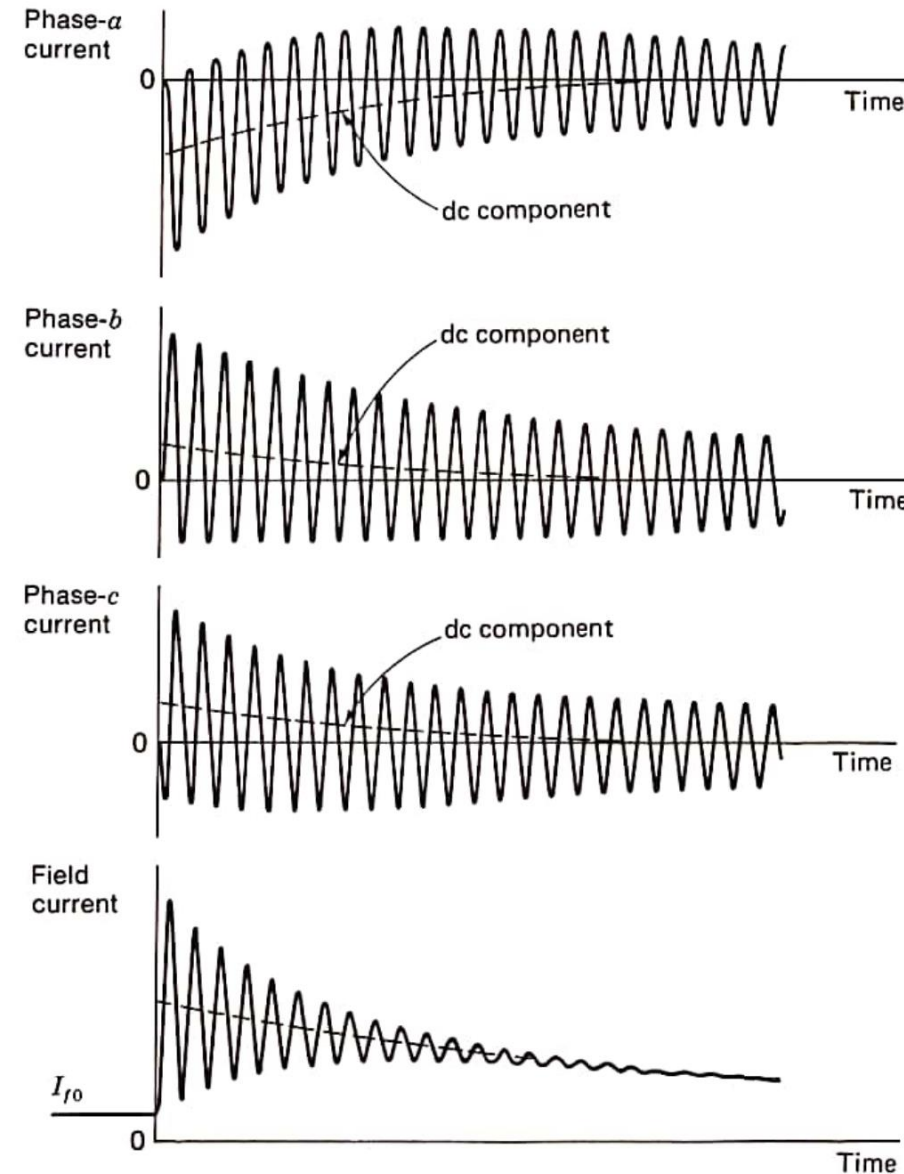
b) $I_a = \frac{|E_{af}|}{|jx_s|} = 0.47 \text{ pu}$



Compare with short circuit in transformer.

Transient Behavior of Synchronous Machines

Armature and field currents following a sudden short circuit on an initially unloaded synchronous machine.



$$\lambda_d = -L_d i_d + L_{af} i_f$$

$$\lambda_q = -L_q i_q$$

$$\lambda_f = -\frac{3}{2} L_{af} i_d + L_{ff} i_f$$

$$\lambda_0 = -L_0 i_0$$

$$v_a = -R_a i_a + p \lambda_a$$

$$v_b = -R_a i_b + p \lambda_b$$

$$v_c = -R_a i_c + p \lambda_c$$

$$v_f = R_f i_f + p \lambda_f$$

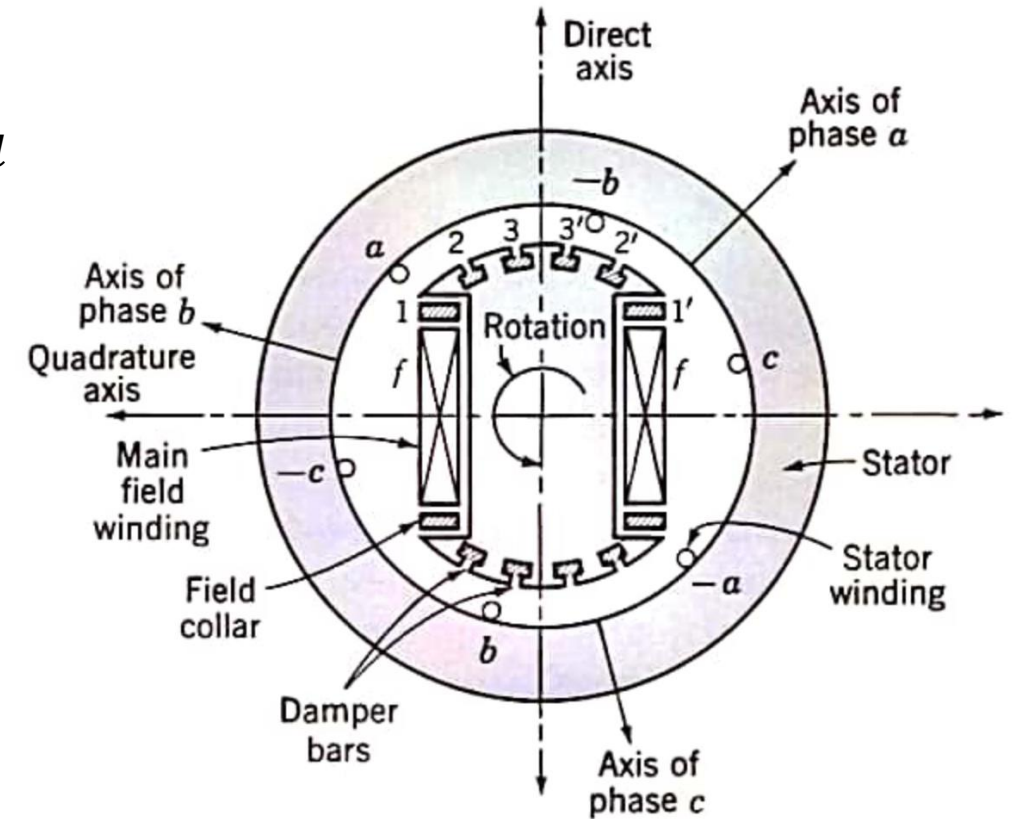
$$\begin{bmatrix} S_d \\ S_q \\ S_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin \theta & -\sin(\theta - 120^\circ) & -\sin(\theta + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix}$$

$$v_d = -R_a i_d + p \lambda_d - \omega \lambda_q$$

$$v_q = -R_a i_q + p \lambda_q + \omega \lambda_d$$

$$v_f = R_f i_f + p \lambda_f$$

$$v_0 = -R_a i_0 + p \lambda_0$$



$$\lambda_d = -L_d i_d + L_{af} i_f$$

$$v_d = -R_a i_d + p\lambda_d - \omega\lambda_q$$

$$\lambda_q = -L_q i_q$$

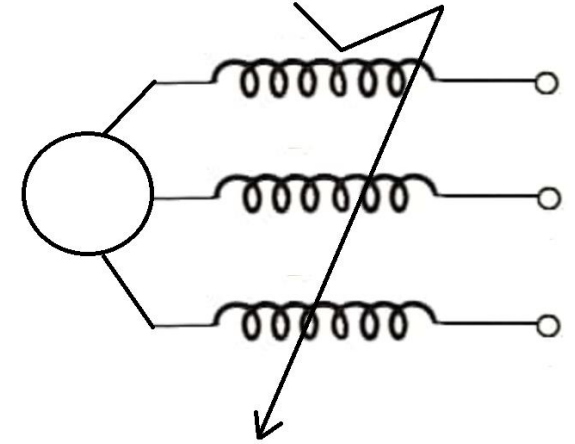
$$v_q = -R_a i_q + p\lambda_q + \omega\lambda_d$$

$$\lambda_f = -\frac{3}{2}L_{af} i_d + L_{ff} i_f$$

$$v_f = R_f i_f + p\lambda_f$$

$$\lambda_0 = -L_0 i_0$$

$$v_0 = -R_a i_0 + p\lambda_0$$



A three phase short circuit in the no-load steady-state condition

At first instances: $\lambda_{ft} = -\frac{3}{2}L_{af} i_{dt} + L_{ff} i_{ft} = 0$ $i_{ft} = \frac{3L_{af}}{2L_{ff}} i_{dt}$

$$\lambda_{dt} = -L_d i_{dt} + L_{af} i_{ft} = -L_d i_{dt} + \frac{3L_{af}^2}{2L_{ff}} i_{dt} = -(L_d - \frac{3L_{af}^2}{2L_{ff}}) i_{dt} \quad \lambda_{dt} = -L_d' i_{dt}$$

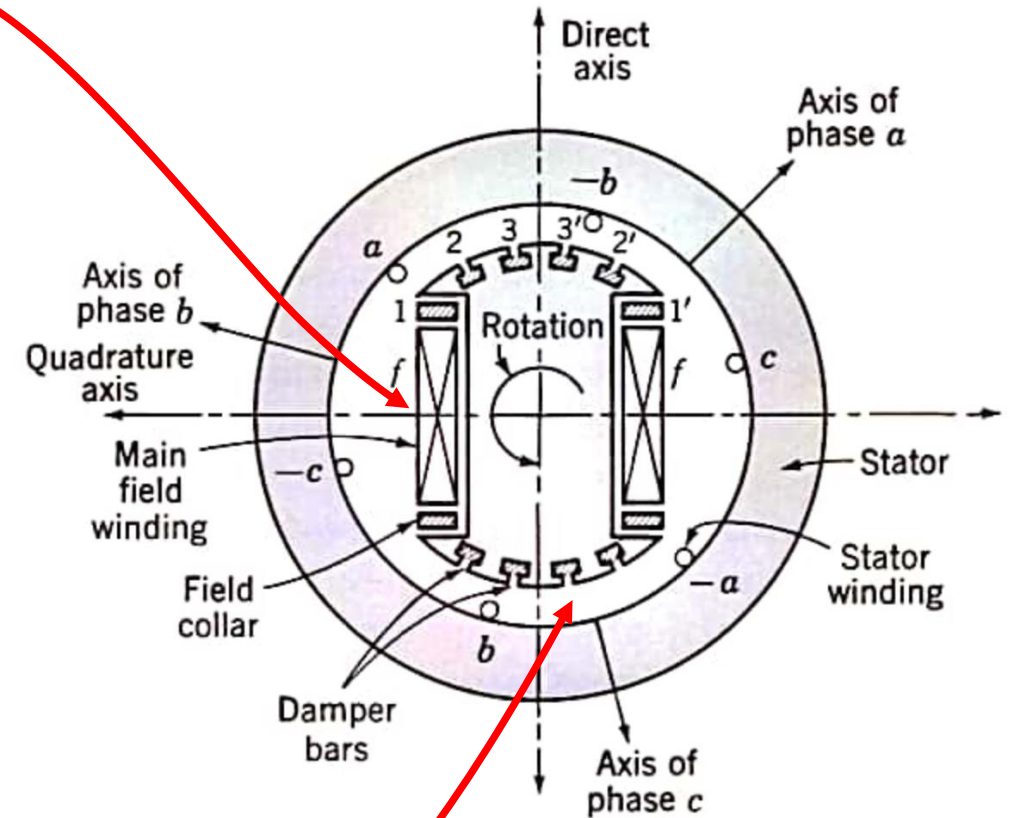
$$L_d' = L_d - \frac{3L_{af}^2}{2L_{ff}}$$

$$L_d' \ll L_d$$

$$L_d' = L_d - \frac{3L_{af}^2}{2L_{ff}}$$

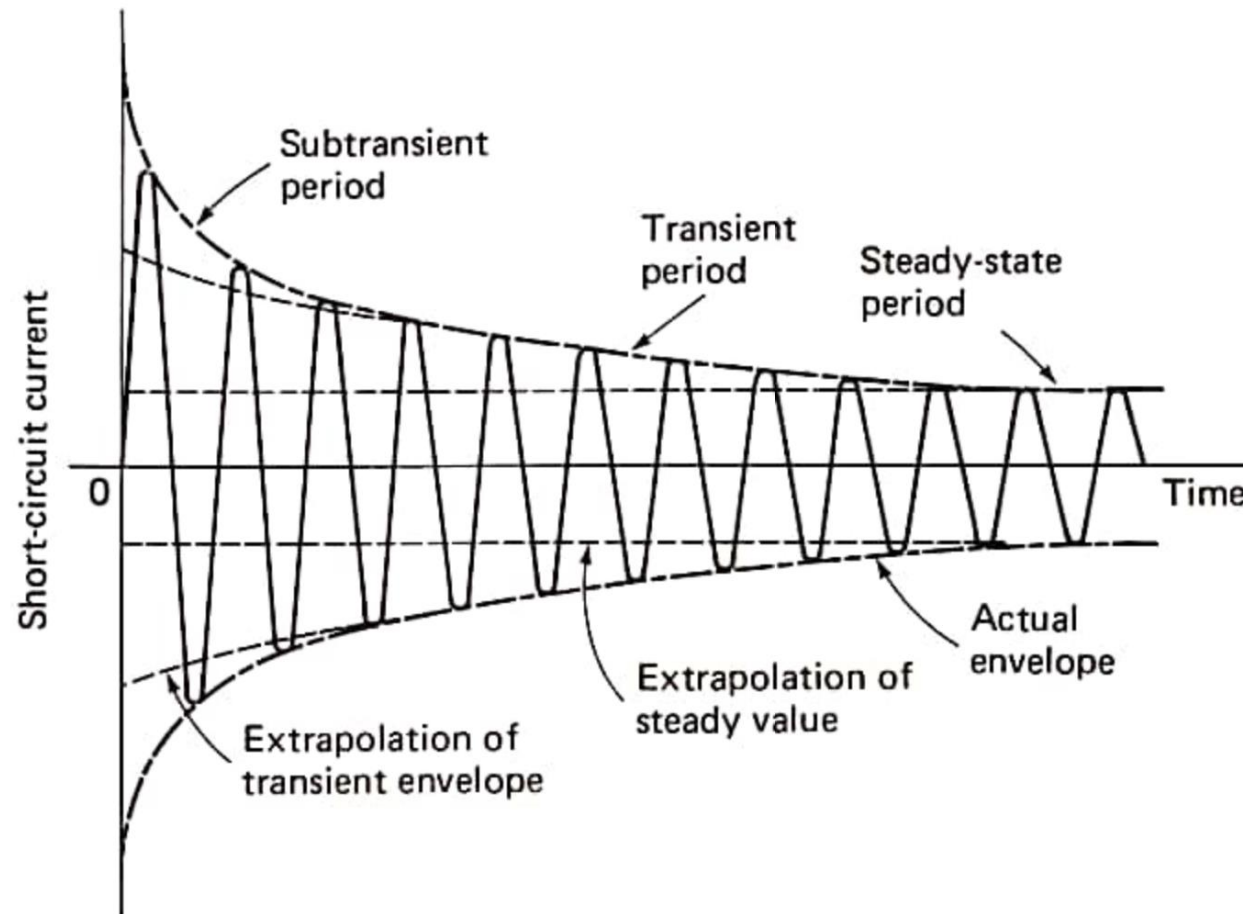
$$L_d' \ll L_d$$

$$L_d'' < L_d'$$



Transient Behavior of Synchronous Machines

$$\frac{\sqrt{2}E_{af0}}{X_d} \cos(\omega t + \theta_0) + \sqrt{2}E_{af0} \left(\frac{1}{X'_d} - \frac{1}{X_d} \right) e^{-\frac{t}{T'_d}} \cos(\omega t + \theta_0) + \sqrt{2}E_{af0} \left(\frac{1}{X''_d} - \frac{1}{X'_d} \right) e^{-\frac{t}{T''_d}} \cos(\omega t + \theta_0)$$



Transient Behavior of Synchronous Machines

TABLE 8-1
TYPICAL VALUES OF MACHINE CONSTANTS†

Machine constant	Generators		Synchronous condensers	Salient-pole motors	
	Solid-rotor	Salient-pole		Low-speed	High-speed
X_d	1.9	1.7	2.2	1.5	1.4
X'_d	0.25	0.30	0.50	0.40	0.35
X''_d	0.20	0.18	0.30	0.25	0.20
X_q	1.85	1.0	1.3	1.0	1.0
X'_q	0.50	1.0	1.3	1.0	1.0
X''_q	0.20	0.25	0.35	0.30	0.25
T'_d	0.55	0.50	1.5	0.35	0.50
T''_d	0.02	0.02	0.03	0.01	0.01
T_a	0.17	0.05	0.25	0.04	0.04

†Reactances are per unit values based on the machine rating; time constants are in seconds.